FTD-MT-24-01-71

AD 727969

FOREIGN TECHNOLOGY DIVISION

NON-STEADY OPERATING REGIMES OF LIQUID ROCKET ENGINES

By

Ye. K. Moshkin

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482

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Technology Division Air Force Systems Command U. S. Air Force	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP
3. REPORT TITLE NON-STEADY OPERATING REGIMES OF LIQUID ROCKET ENGINES	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Translation	
5. AUTHOR(S) (First name, middle initial, last name) Moshkin, Ye. K.	
6. REPORT DATE 1970	7a. TOTAL NO. OF PAGES 459 / 7b. NO. OF REFS 98
8a. CONTRACT OR GRANT NO b. PROJECT NO 6040104 c. d. DIA TASK NO. T65-04-18A	9a. ORIGINATOR'S REPORT NUMBER(S) FTD-MT-24-01-71 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.	
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Foreign Technology Division Wright-Patterson AFB, Ohio

13. ABSTRACT

In the book are examined the operation of liquid-propellant rocket engines under nonstationary conditions and transient conditions, dynamic processes and some questions of statics. There are considered differential equations, which characterize the operation of separate units and the engine on the whole during launching, at midcourse and during shutdown. There are discussed fundamentals of the theory of injectors, some questions of carburetion and burning; there is offered derivation of the formula for determination of combustion delay. The connection is established between this delay and the physical properties of the propellant components. The operation of basic units of the feed system is described, is given derivation of the universal equation of the tank pressurizing system and equations which characterize intra-tank processes are given. There are offered a clear graphic-analytical method and methods of calculations of an engine with the aid of differential equations and with the use of computer technology. The materials discussed in the book can be used during thorough research and calculation of a liquid-propellant rocket engine and its units. The book is intended for scientific workers, designers, instructors and students, specializing in liquid propellant rocket engines. Orig. art. has: 6 tables, 78 ills.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Liquid Rocket Propellant System Liquid Propellant Engine Liquid Rocket Propulsion System						

UNCLASSIFIED
Security Classification

FTD-MT-24-01-71

EDITED MACHINE TRANSLATION

NON-STEADY OPERATING REGIMES OF LIQUID ROCKET ENGINES

By: Ye. K. Moshkin

English pages: 459

Source: Nestatsionarnyye Rezhimy Raboty ZhRD, Moscow, Izd-vo Mashinostroyeniye, 1970, pp. 1-336.

This document is a SYSTRAN machine aided translation, post-edited for technical accuracy by: Robert A. Potts.

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UR/0000-70-000-000

FTD-MT-24-01-71 Date 19 Apr 1971

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* <u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

Translator's note: On several occasions, symbols found in formulae and calculations appear to have been rendered incorrectly in the original document. They will be shown exactly as they appear in the original.

FTD-MT-2

In the book are examined the operation of liquid-propellant rocket engines under non-stationary conditions and transient conditions, dynamic processes and some questions of statics. There are considered differential equations, which characterize the operation of separate units and the engine on the whole during launching, at midcourse and during shutdown. Analysis is given of the possibilities of application of differential equations in partial derivatives under certain boundary conditions during research of nonstationary conditions.

There are discussed fundamentals of the theory of injectors, some questions of carburetion and burning; there is offered derivation of the formula for determination of combustion delay. The connection is established between this delay and the physical properties of the propellant components. There are derived and analyzed the most important equations of the combustion chamber, including the basic differential equation with delay argument. There is given detailed derivation of the equation of thrust during operation of an engine under nonsteady conditions, uncovering of the physical essence of specific thrust. This permits investigating intrachamber processes, including high-frequency instability.

The operation of basic units of the feed system is described, is given derivation of the universal equation of the tank pressurizing system and equations which characterize intratank processes are given. There is described

the motion of elastic liquid in a deformable line and the process of filling of the line with high-boiling and low-boiling components, which together with analysis of the operating conditions of centrifugal and screw pumps gives the possibility of calculating the basic units of feed system.

There are offered a clear graphic-analytical method and methods of calculation of an engine with the aid of differential equations and with the use of computer technology.

The materials, discussed in the book can be used during thorough research and calculation of a liquid-propellant rocket engine and its units.

The book is intended for scientific workers, designers, instructors and students, specializing in liquid-propellant rocket engines. Tables 6, Illustrations 78, Bibliography 98 titles.

PREFACE

The monograph "Nonstationary Operating Conditions of Liquid-Propellant Rocket Engines" is the second edition of the book "Dynamic Processes in Liquid-Propellant Rocket Engines," issued by the publishing house "Mashinostroyeniye" in 1964. For the period of time since this book appeared, interest in questions of dynamics of the processes, which proceed in liquid-propellant rocket engines, increased, and at present their study is a very urgent scientific trend.

The second edition is enlarged, because the theory of non-stationary operating conditions of liquid-propellant rocket engines encompasses a wider circle of questions than the theory of dynamic processes. Specifically, there are examined some problems of nonstationary energy exchange diffusion conditions, features of preparation of propellant for burning and so forth. Simultaneously, from the book are excluded questions well covered in technical literature.

At present there is no sufficiently substantiated procedure of theoretical research of processes in liquid-propellant rocket engines. Therefore, the author, trying to lay the groundwork for the creation of such a procedure, gave detailed derivations of the basic equations, examined the appropriate boundary conditions and made an attempt to take into account not only the main, but also secondary factors, which affect the operation of an engine.

The book consists of three sections, each of them defines the basic problems and directions of research. The first two sections are dedicated to the study of separate units, and in the third on the basis of obtained results there is shown how it is necessary to investigate the interconnection between processes under non-stationary operating conditions of liquid-propellant rocket engines.

In the first section are examined processes of propellant burning: the atomization of components, mixing, preheating, vaporization of liquid propellant and the formation of combustion products; the methods of thermodynamic calculation, widely utilized during research of intrachamber processes, are not given here, inasmuch as most of them are well covered in literature.

Further there is given the derivation of the basic equation of combustion chamber in ordinary derivatives on the basis of the law of conservation of mass on the assumption that processes, proceeding in the chamber, are changed only in time, i.e., the parameters of intrachamber processes at any moment and at any point inside the chamber have the same value.

More detailed research on intrachamber processes requires the utilization of differential equations in partial derivatives. Therefore, there is also provided derivation of these equations and there are shown the possibilities of use of equations of continuity, energy, motion, and momentum. The last equation allows defining the engine thrust during operation both at stationary and nonsteady conditions. The derivation of these equations, which make it possible to deeply and comprehensively investigate intrachamber processes, is given the greatest attention.

In the second section of the book are studied processes in units of the power supply system in the sequence of their power and dynamic actions on each other.

In the examination of operating conditions of accumulators in the beginning there is given the derivation of universal equation, suitable for calculation and research of any type of accumulator. Then there are analyzed the processes in gas, powder and hot (liquid) accumulators, the interaction between gas and liquid, the peculiarities of displacement of liquid from tanks.

Specific attention is allotted to the derivation of the equations, which characterize heat exchange and mass transfer in fuel tanks. The obtained system of equations and boundary conditions can be solved numerically with the aid of digital computers.

In the examination of conditions of overflow of liquid from a tank into the line there is discussed the procedure of application of computers and there is shown the possibility of solution of a two-dimensional, and in certain cases a three-dimensional nonstationary problem with their aid.

The process of filling of lines is described with the aid of equations both with heat exchange between liquid and the wall of the line, and for a case when there is no heat exchange.

In this section there are also examined the delivery characteristics of a centrifugal pump and screw forepump, the motion of liquid under nonsteady conditions and operation of the turbopump assembly as a single unit are investigated.

In the third section there is analyzed the operation of the engine on the whole. In order that the reader would more easily understand the peculiarities of the interconnection between processes, proceeding in the engine, there is conducted the graphic-analytical method of calculation in quasi-static formulation.

Deeper exposure of complex phenomena of the interconnection is given in the form of a system of linearized equations of statics

and dynamics. There is also analyzed the operation of engines, made in various configurations, with the aid of complex systems of nonlinear differential equations, the solution of which is possible only with the use of electronic computers. On a number of examples there is examined the sequence of preparation and solution of equations on analog and digital computers. These examples are based on hypothetical initial data and therefore they bear a purely systematic character.

In the book is used the international system of units (SI). During analysis of dimensionalities of some quantities one should remember that Newton dimensionality (N) $kg \cdot m \cdot s^{-2}$.

The author thanks the people who obligingly gave some materials, included in the book, and, especially, M. V. Arkhipkin, candidate of technical sciences, who wrote Sections 7.17-7.24.

The author is sincerely grateful to honored worker of science and engineering of the RSFSR, Professor I. I. Kulagin, Dr. of Technical Sciences, for the valuable remarks made during review of the manuscript, and to Professor V. M. Kudryavtsev Dr. of Technical Sciences, and also to all people who expressed a number of useful proposals and recommendations during the discussion of the manuscript.

All remarks on the book should be directed to the address: Moscow, K-51, Petrovka, 24, publishing house "Mashinostroyeniye."

INTRODUCTION

The object of research

The theory of nonstationary conditions in liquid-propellant rocket engines is one of the new technical sciences, which studies the regularities of kinematic and dynamic processes, proceeding in units of liquid-propellant rocket engines with change of parameters and functions of working media and structural elements in time.

The liquid-propellant rocket engine includes the combustion chamber, or cluster of combustion chamber, and feed system.

The combustion chamber consists of the injector assembly, on which are located injectors, center section, or actually the combustion chamber, and nozzle. The combustion chamber is equipped with a cooling system.

The systems for feeding propellant components into the combustion chamber are two types - turbopump, most widespread at the present time, and pressurized.

With the application of turbopump feed the engine is made with open configuration or a configuration with afterburning is used. In the first case the waste turbogas is ejected through additional

nozzles into the surrounding medium. In the second - one or both propellant components are gasified, are used for driving the turbine (or two turbines) and then enter the combustion chamber.

The turbopump feed system includes fuel tanks, the tank pressurizing system, fuel pumps, turbine (or two turbines), the generator (or generators) of working medium for the turbine, elements of automation, lines for propellant components, gas pipes and so forth.

In a pressurized feed system the propellant components are supplied with compressed gas, which creates higher pressure in the fuel tanks than in the combustion chamber. Because of the specific features of processes proceeding in the separate units of an engine in nonstationary conditions the intrachamber processes, processes in the units of the feed system and the interconnection between them are considered separately.

Thus, during research of the feed system there are also studied intratank processes, and therefore in the book by a liquid-propellant rocket engine there is acutally meant a rocket power plant (RDU).

The study of dynamic processes encompasses all types of nonstationary conditions, namely: starting operation of the engine, transition from one steady state to another, shutdown of the engine, low-frequency and high-frequency oscillations of parameters (pressures, flow rates, etc.), observed during the entire period of engine operation.

THE COMBUSTION CHAMBER

Intrachamber processes are generated at the moment of exit of propellant components from injectors. For the chamber, as for

the whole engine, there are characteristic starting operation, operation at constant, i.e., steady conditions (at cruise), transient processes in sustainer condition and shutdown of the engine.

During the whole period of operation of the engine the intrachamber processes are accompanied by continuous change of parameters. If we adhere to chronological order, the examination of dynamic processes must begin with the period of starting operation. However, dynamic processes of the same type proceed at cruise, therefore to begin their study it is convenient to conform namely to these conditions.

From an energy point of view the sustainer conditions are most important, inasmuch as their duration is nearly equal to the total time of engine operation.

The nonstationary operating conditions of the combustion chamber are studied in terms of values of parameters, average for the entire chamber, or by their particular values, which are changed in the internal volume of the chamber both in time and in space.

If we are guided by the values of parameters, different for each point of the volume of the chamber, but constant in time, then the dynamics of the processes are reflected in the fact that the rates of motion of liquid and gas masses will be changed mainly lengthwise in the combustion chamber.

Under actual conditions at cruise the dynamic processes lead to a change of parameters both in coordinates of the chamber and in time for any cross section or point of the volume of the chamber. Therefore, for the study of dynamic processes, even under so-called steady conditions, it is necessary to enlist the system of differential equations in partial derivatives. In this case there are

observed low-frequency and high-frequency oscillations. The processes, proceeding at cruise, are characterized by relatively small changes in parameters, therefore here it is sometimes possible to apply equations in small deviations, obtained during linearization of appropriate differential equations.

The change of parameters at the starting operation bears a more complex character than during operation of the engine at cruise. The average pressure in the chamber changes from a certain low pressure before starting to design, nominal, and considerable accelerations of the motion of propellant components and combustion products are observed. These processes proceed the most actively in the initial period, from the moment of arrival of propellant into the chamber to ignition, during ignition and development of burning, sometimes before the chamber establishes approximately 40-70% pressure. After this the pressure in the chamber increases somewhat slower.

Of large value in the organization of processes at cruise operation is the sensitivity of the delay period to pressure and the rate of change of the component ratio. During the starting operation there is fulfilled a series of commands, which complicates the operation of the engine.

The theoretical and experimental study of the dynamics of intrachamber processes with respect to average values of parameters for the whole volume of the chamber is simpler, but allows answering only a limited circle of questions. However, the average values of parameters of the combustion chamber at cruise are not constant. They are changed as a result change of the engine thrust with altitude, intensity of action of external forces, as a result of the change of operating conditions of separate units of the engine and change of the physicochemical properties of the propellant components.

After closing the main fuel valves the feed of the calculated amount of propellant components into the chamber is chased. However, under the effect of a number of factors for a certain period of time into the combustion chamber there still proceeds a small amount of one or both propellant components.

The period of shutdown with consideration of the time of afterburning of propellant is called the period of aftereffect. It is characterized not only by an average magnitude of pressure pulse, but also by its scattering.

The feed system

The operation of the combustion chamber depends on the features of occurrence of processes in the units of the feed system. In the first place, it is necessary to provide the calculated values of parameters of propellant components of a definite character of their change in time at the inlet to the pumps and in the line, connecting the tanks with the pumps. Here, just as during investigation of combustion chambers, we are guided by the average values of parameters of the liquid, moving in the line, or we consider their change both in time and in terms of coordinates. The character of these changes depends on the operating conditions of the turbopump unit, which takes away liquid components from the fuel tanks, and on their feed conditions to the pump inlets.

The operation of the feed system is organized by means of the realization of various programs. The gain of revolutions by the shaft of the turbopump assembly depends on the intensity of occurrence of starting conditions of the starter and the generator of working medium of the turbine, and also on the resistance, being exhibited to the turbine from the pumps, and on the cyclogram of starting, which determines the sequence of execution of separate operations. The bringing of the turbopump unit to operating conditions is possible either with monotonous rapid or rather slow rise of the rpm, or in two stages, when there is noticeable a temporary stabilization of the rpm before arrival at the second, main, stage.

With a pressurized feed system the parameters of the liquid upon exiting the tank are determined by the operating conditions of the pressurizing system and with the specific character of intra-tank processes, which depend in turn upon the conditions of heat exchange between the liquid and gas, filling the tank, or the surrounding medium, on the intensity of mass transfer between the liquid and gas, on the character of motion of liquid in the tank and the actions disturbing the flow.

In the case of rapid starting because of the inertia of the liquid column there is observed a sharp lowering of pressure at the pump inlet, which is accompanied sometimes by cavitation, which frequently leads to the disruption of starting.

In the line, connecting the tank with the pump, there can appear wave processes right up to a water hammer. The intensity of the waves depends on the amount of disturbing factors, on the elastic peculiarities of the liquid, on the construction of the line and on other factors.

The sources of wave processes can be the turbopump unit, aerodynamic loads on the flight vehicle, vibrations of the combustion chamber and so forth.

In this way the chamber and the propellant feed system operate as a unit. Therefore, the completion of research of nonstationary conditions is the examination of the joint operation of all units of the power plant.

SECTION I

COMBUSTION CHAMBER

CHAPTER I

QUESTIONS OF PROPELLANT BURNING

The conversion of propellant into combustion products occurs in the combustion chamber. The propellant components can enter the chamber in liquid or gaseous state. In the first case in the beginning they are atomized, mixed, heated and are vaporized, after which follow the chemical transformations into gaseous phase, as a result giving a mixture of combustible gases, the composition of which depends on their temperature and pressure. The entire complex of these processes is the burning of propellant. With arrival of propellants into the chamber in gaseous form the reactions of interaction between components proceed faster.

In engines, made in the most widespread configuration, one of the components enters the injectors from the upper collecting tank of the chamber injector assembly. The second component, passing along the cooling duct, is directed to another collecting tank of the injector assembly and from there - to the injectors. However, other schemes of feed of components are possible.

In the considered variant, when both components are in liquid phase, the one used for external cooling is preheated in the cooling duct, which facilitates improvement of burning process. When developing the configuration of the engine it is expedient to provide preheating of both components, but it is impossible to allow the start of boiling of liquid before exiting the injectors.

With the selected arrangement of injectors the configuration of collecting tanks and the placing of stiffening ribs in them should provide uniform distribution of components between the appropriate injectors. An exception is the peripheral injectors, to which fuel is supplied for providing protection of the walls of the upper part of the chamber, i.e., the part adjacent to the injector assembly.

In the configuration with afterburning to the injectors are fed components in gaseous form or one component in gaseous, and the other in liquid state. In both cases liquid flows along the cooling duct. In the first case the liquid component from the cooling cavity is directed to the generator, then in gaseous form - to the turbine of the [TNA] (THA) turbopump assembly and after this to injectors. In the second case the liquid component from the cooling cavity enters the injectors and in the internal cavities of bipropellant injectors encounters the second component, exiting the turbine in gaseous form.

1.1. The Process of Propellant Burning in the ZhRD Chamber

The completeness of conversion of propellant into combustion products depends on the state and properties of components, conditions of their inlet into the chamber, gas parameters in the chamber, heat exchange between propellant and combustion products, the specific character of intrachamber processes and so forth.

Contemporary [ZhRD] (ЖРД) liquid-propellant rocket engines are made in the following typical configurations.

1. *Open configuration* (Fig. 1.1) - both components enter the combustion chamber in liquid state.

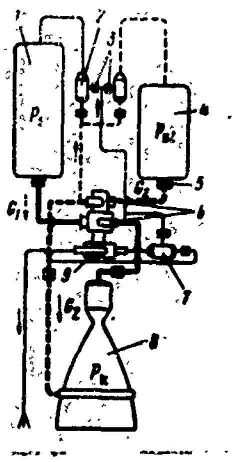


Fig. 1.1. Open configuration of liquid-propellant rocket engine "liquid-liquid": *1, 4* - tanks; *2* - generator (mixer) for tank pressurization; *3* - pressurizing throttle; *5* - component feed valve; *6* - bipropellant pump; *7* - gas generator; *8* - combustion chamber with nozzle; *9* - turbine of turbopump assembly; ——— - main lines; - - - - additional lines.

2. ***Closed configuration***, also called configuration with afterburning, when one of the components (Fig. 1.2) or both components (Fig. 1.3) after the turbine enter the chamber in gaseous form; their temperature at the chamber inlet can be changed in wide limits.

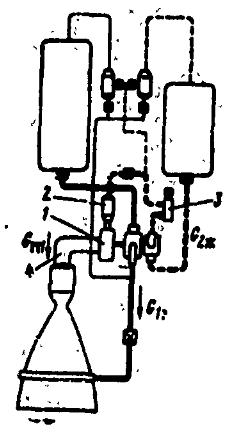


Fig. 1.2. Closed configuration of liquid-propellant rocket engine "liquid-gas": *1* - centripetal turbine; *2* - bipropellant gas generator; *3* - propellant component flow rate regulator; *4* - exhaust turbogas (afterburning) pipe.

Let us examine the processes, proceeding in a combustion chamber, which operates on liquid bipropellant. Burning is completed by the formation of gaseous products, overflowing from the chamber into the nozzle. The burning process is conditionally divided into a number of consecutive stages. However, if at any arbitrary moment of time we consider the volume of the chamber entirely, then it is possible to immediately observe all the stages in it.

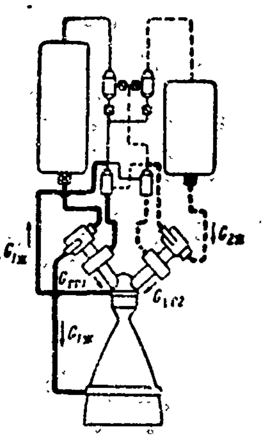


Fig. 1.3. Closed configuration of liquid-propellant rocket engine "gas-gas."

The first stage is atomization of propellant components by injectors, which is characterized by fineness, by the homogeneity of atomization and uniformity of distribution of mixture along the chamber and its cross sections. This depends on the exit character of liquid from the injectors, interaction of the jet with gas medium in the combustion chamber, the properties of liquid and gas. As a result, the jets are disintegrated with the formation of separate drops, which during flight are distorted, continuously changing their shape. The separate drops are broken into finer; some of them when encountering each other are either additionally broken, or merged, moreover the drops being formed again can contain both components.

A number of forces participate in the appearance of drops. Some - forces of surface tension and forces of viscosity - facilitate preservation of the shape of the jet and drops; others - destroy the jet, contribute to breaking down of drops.

The causes, destroying the jet, include turbulent pulsation of particles of liquid in the flow, disturbances of different types in the jet, caused, for example, by low-frequency oscillations of the flow rate of component.

With the presence of a swirl injector (Fig. 1.4) the most important factors, which lead to decomposition of the jet (film), are decrease of its thickness along the spray cone and the wavelike character of motion of liquid. Aerodynamic forces facilitate the decomposition of flow; as a result of friction or disturbances of wave character these forces separate particles of liquid from the surface of film. These forces, affecting the frontal surface of drops, cause their breaking. In the presence of acoustic or low-frequency oscillations there is observed intensive breaking down under the action of pressure waves, additional dynamic loads and so forth.

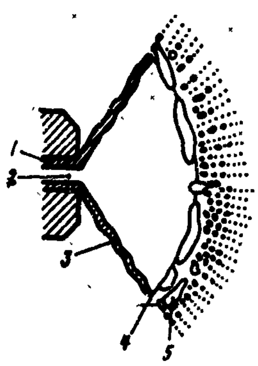


Fig. 1.4. Diagram of decomposition of liquid film: 1 - liquid; 2 - vapors; 3 - liquid film; 4 - start of disintegration of film; 5 - drops.

The fineness of atomization is determined by the average diameter of the drop. The smaller the drops, the faster the transformation of liquid phase into gaseous will be completed. The fineness of atomization depends on the properties of components, the type of injectors, pressure drop on the injectors, the density of gas in the combustion chamber, the character of motion of gas in the atomization zone.

The homogeneity of atomization is determined by the law of distribution, which establishes the connection between the size of drops and their quantity. The sizes of all drops are limited to a

certain range. It is possible to determine the smallest, largest and most characteristic averaged size of drops.

The uniformity of distribution of the mixture through the chamber section essentially depends on the characteristic of injectors and their mutual location on the injector assembly. No matter how the injectors are arranged, how perfect the atomization they provide, nevertheless along the chamber section there will be observed a definite nonuniformity of atomization, quantity of supplied component and, as a consequence, nonuniformity of component ratio. Sometimes, for the purpose of creation of a protective boundary layer, closer to the walls is placed an additional quantity of injectors of some component, more often - fuel.

The distribution of components in the boundary layer along the combustion chamber depends upon the homogeneity of atomization and the range of the jet. The higher the heterogeneity and the larger the maximum size of the drops, the greater the length of the chamber that the transition of liquid phase into gaseous will be observed. Depending on the laws of distribution of each of the components, which will be discussed below, the character of change of the component ratio along the chamber will be determined. The parameters of injectors can be selected so that the burning zone will be concentrated on a small section of the chamber length or will be stretched lengthwise; having combined swirl injectors with spray, it is possible to create several combustion zones in the chamber.

The structure of the zone of propellant preparation and combustion with the prescribed arrangement of injectors is determined by the character of decomposition of the jet, by distribution of drops of components of various sizes in the space of the combustion chamber, which depends on the factors examined above.

Under conditions of transient processes the atomization proceeds according to a more complex scheme, inasmuch as during the starting operation of the engine the flow rate and component ratio, and also pressure, temperature and density of gas are changed with time. The motion of liquid and gases occurs with acceleration, the fineness of atomization, the distribution of masses of drops along the cross section and length of the chamber are changed with time.

The second stage is characterized by rise of temperature of the mass of drop and is finished with the start of vaporization of liquid on its surface. The conditons of heat exchange between drops of components and combustion products are determined by the relative velocity of motion of the liquid, properties and parameters of combustion products and liquids.

The transfer of heat occurs as a result of thermal conductivity, convective and radiant heat exchange; the warming up of the drop depends not only on the heat flow, directed from gases to the drop, but also on the conditions of heat exchange, on properties of components. The intensity of warming up of drops of oxidizer and fuel is different, therefore the areas and the time of their preparation for vaporization are different.

The third stage - vaporization of components. In the course of vaporization of liquid around the drop there is formed a "cloud," consisting of vapors of component. The heat exchange between the cloud and products of burning is caused by thermal conductivity, convection and by the flow of radiant energy. Heat transfer through the cloud from products of burning to the drop occurs as a result of thermal conductivity.

During the second and third stages there occurs accumulation of components in the chamber in liquid state.

The fourth stage is characterized by the thermochemical decomposition of products of vaporization and preflame chemical interaction between the formed products.

The fifth stage - ignition, which can be chemical in the case of application of hypergolic components or thermal, when at least one of the components enters the chamber at rather high temperature.

The sixth stage is burning of the formed gaseous products. In the most common form the burning processes can be described in the following manner.

Homogenous burning is the burning of components, which are in gaseous state. This is typical for liquid-propellant rocket engines, where the propellant burns up after transition from liquid state into gaseous. If pressure in the chamber is higher than critical for both components, then as before there will occur homogeneous burning, inasmuch as the masses of both components diffuse into the medium of combustion products.

With the presence of a boundary between the two media, for example in the case of solid propellant and gaseous oxidizer, the process is called *homogeneous*.

Burning, proceeding at small Reynold's numbers of gas flow, is called *laminar*. In the combustion of liquid-propellant rocket engines the flow of gas is turbulent, therefore burning is called *turbulent*.

With smooth burning the flame is spread with speed from several centimeters to several tens of meters per second. In unusual cases detonation appears, at which the rate of flame propagation reaches several kilometers per second.

We distinguish *kinetic* and *diffusion* burning. In the combustion chambers of liquid-propellant rocket engines diffusion burning at which the rate of the chemical reaction is significantly higher than the rate of mixing is predominant. It appears with separate feed of propellant components into the burning zone. Kinetic burning takes place when the propellant enters the burning zone in an earlier mixed form, for example in engines, which operate on monopropellant, or during the supply of both components in mixed form; moreover mixing can be carried out in the internal cavities of injectors.

Combustion is complete and incomplete. In the first case all the propellant burns up in the burning zone, i.e., in the upper part of the chamber. In the second case a certain quantity of propellant burns out at the approach to the nozzle throat or even past the throat.

Burning can proceed with stoichiometric ratio between components and with excess of fuel or oxidizer. In the combustion chambers of liquid-propellant rocket engines there is provided excess of fuel; in this case the composition of combustion products is somewhat changed in comparison with designed, the intensity of reactions of dissociation is partially suppressed, the quantity of heat being liberated in the chamber is increased and the temperature of combustion products is somewhat lowered.

Chemical transformations bear a complex character. As a result of initial reactions there are developed chains, which are developed in parallel, are branched and finished by the formation of final products.

The seventh stage characterizes the change of the chemical composition of the mixture along the length of the chamber and nozzle. This is caused by two factors. In the first place, during motion of combustion products the pressure and temperature are

continuously changed, which affects the intensity of occurrence of chemical reactions; secondly, with the presence of a protective boundary layer the central nucleus of gases receives an additional quantity of incomplete combustion products.

1.2. Concise Information About the Operation of Injectors

Injectors are selected so that they provide the required flow rate of components. The quality of the injectors is characterized by the accuracy of fuel metering, the fineness of atomization, in certain cases the range of the jet, the angle of atomization, trituration of jet, by the law of distribution of drops with respect to their mass (or sizes).

The arrangement of injectors on the injector assembly should provide distribution of flow rate and ratio of components prescribed by results of experiments or calculation, along the cross section and length of the chamber, the required excess of fuel near the wall for its protection from overheating, protection of the inter-injector space of the injector assembly from overheating, weakening of the dynamic connection between intrachamber space and hydraulic ducts, the creation of conditions of propellant combustion, at which the parameters of flow-frequency and high-frequency oscillations in the chamber will be in permissible limits.

During engine starting the injectors should create the required increase of propellant flow rate and exclude the appearance of unusual phenomena in the combustion chamber (for example, high-frequency acoustic oscillations).

With shutdown of the engine there should not be observed leak of propellant from injectors and the hydraulic areas of the chamber; this facilitates the uniformity of shutdown of engines and decrease of scattering of the period of aftereffect.

In modern engines there are used swirl, spray, composite, gas-liquid and doublet nozzles.

In this paragraph there will be examined only some of the peculiarities of operation of injectors.

Determination of characteristic sizes of drops

Sizes and shapes of drops, being formed into drops of jets in the case of application of liquid injectors, are various. Usually in calculations we are conditionally guided by drops of spherical shape. We consider drops with the smallest radius r_{min} and the largest r_{max}, we find the law of distribution, establishing the connection between size r and number n of drops in a portion of component per second, we determine the average r_{cp} value of the radius of drop.

At the injector exit the initial thickness of the film of spray cone

$$\delta_0 = r_c - r_{\text{в}},$$

where r_c - radius of outlet opening of the injector; $r_{\text{в}}$ - radius of vortex.

In proportion to the distance from the nozzle exit section the radius of the cross section of the spray cone is increased, and the thickness of the film is decreased. By the equation of continuity at the injector exit flow rate

$$G = \pi (r_c^2 - r_{\text{в}}^2) \rho_{\text{ж}} C_0, \tag{1.1}$$

where C_0 - efflux velocity of liquid from the injector.

By using the theory of swirl injector, we find

$$C_0=\frac{\mu}{\varphi}\,\frac{\sqrt{\frac{2}{\varrho_{ж}}\Delta p_{ф}+C_{вх}^2}}{\cos\left[\operatorname{arctg}\frac{\sqrt{8}(1-\varphi)}{(1+\sqrt{1-\varphi})\sqrt{\varphi}}\right]}, \qquad (1.2)$$

where $\Delta p_{ф}$ the pressure drop on the injector; $\rho_{ж}$ - the density of liquid; $C_{вх}$ - inlet velocity of injector. The flow coefficient

$$\mu=\frac{1}{\sqrt{\frac{1}{\varphi^2}+\frac{A^2}{1-\varphi}}}. \qquad (1.3)$$

The geometric characteristic of the injector

$$A=\frac{Rr_c}{i_{вх}r_{вх}^2}\sin\beta, \qquad (1.4)$$

where R - the radius of turbulence chamber; $r_{вх}$ - radius of the inlet opening of injector; $i_{вх}$ - the number of inlet openings; β - angle between the direction of inlet channel and the axis of the nozzle. The loading factor of injector exit section

$$\varphi=1-\left(\frac{r_н}{r_c}\right)^2.$$

The main geometric characteristic and the loading factor allowing for forces of friction are connected by relationship

$$A=\frac{\sqrt{2}}{f}\,\frac{(1-\varphi)}{\varphi^{3/2}},$$

where

$$f=\left[1+\frac{\lambda}{2}\left(\frac{R^2}{ir_{вх}^2}-A\right)\right]; \qquad (1.5)$$

λ - coefficient of friction.

If we do not consider friction, then $f = 1$. When $Re < 32{,}000$

$$\lambda = \frac{380}{Re^{0.88}};$$

if $32{,}000 < Re < 60{,}000$, then

$$\lambda = \frac{85 \cdot 10^{3}}{Re^{5}};$$

In any section of the spray cone, but even before the start of disintegration of flow, the flow rate of liquid

$$G = 2\pi r_{\text{кон}} \delta \varrho_{\text{ж}} C, \tag{1.6}$$

where $r_{\text{кон}}$ - radius of current cone cross section. By simplifying formula (1.2) and considering $C = C_0$, we find

$$\delta = \frac{\varphi}{\mu} \frac{G}{2\pi r_{\text{кон}} \varrho_{\text{ж}} \sqrt{\frac{2}{\varrho_{\text{ж}}} \Delta p_{\Phi}}}. \tag{1.7}$$

When δ becomes rather small, disintegration of the cone sets in, and the forming of drops is begun. Half the spray cone angle

$$\alpha = \operatorname{arctg} \sqrt{8} \frac{1-\varphi}{(1+\sqrt{1-\varphi})\sqrt{\varphi}}. \tag{1.8}$$

Figure 1.5 shows the calculated relationship of parameters of a swirl injector to its geometric characteristic [2]. If we designate the distance from the injector section to the examined section through x, then

$$r_{\text{кон}} = x \operatorname{tg} \frac{\alpha}{2}.$$

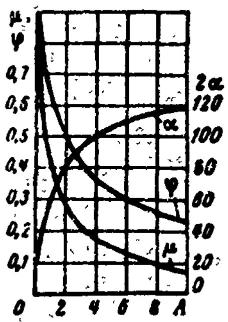


Fig. 1.5. Calculated relationship of parameters of swirl nozzle to its geometric characteristic.

In actuality the efflux process of liquid from injector should be described by more complex equations, especially if we take into account the effect of back pressure and heat flows, directed from the chamber into the region of location of injectors.

There are experimentally established the relationship of the character of liquid efflux to the parameter of injectors and intrachamber processes; they are represented in the form of charts or tables.

The liquid, flowing from the injector, ejects vapors from the center section of the injector and from the space between the injector. As a result of the decrease in pressure there appears the flow of gases and liquid particles from the chamber to the injector assembly and into the injector (Fig. 1.6).

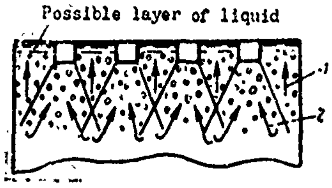


Fig. 1.6. Diagram of motion of liquid particles from the burning zone to the wall of injector assembly (*1*) and into the internal cavities of injectors (*2*).

There exist several approximate methods of determining the characteristics of atomization.

The average diameter of drops at the moment of their formation is determined by the equality of surface tension

$$p_1 = \frac{2\sigma_ж}{r_{0\,ср}} \tag{1.9}$$

and the dynamic pressure of the medium

$$p_2 = \rho_к(|C - W|)^2, \tag{1.10}$$

where $\rho_к$ - the density of gas-air mixture in the combustion chamber; W - the flow velocity of gas.

Hence, with a number of assumptions with consideration of Bernoulli equation we find

$$r_{0\,ср} = \xi \frac{4\sigma_ж}{\Delta p_ф} \frac{\rho_ж}{\rho_к}, \tag{1.11}$$

where ξ - coordination coefficient; $\sigma_ж$ - surface tension of liquid.

Having multiplied and divided the right side of expression (1.11) by δ_0, we obtain

$$r_{0\,ср} = 4\xi\delta_0 \mathrm{We} \frac{\rho_ж}{\rho_к}, \tag{1.12}$$

where Weber criterion

$$\mathrm{We} = \frac{\sigma_ж}{\delta_0 \Delta p_ф}; \tag{1.13}$$

the initial thickness of film

$$\delta_0 = \frac{G}{\pi(r_c - r_в)\rho_ж C_0}. \tag{1.14}$$

Some authors recommend determining the median radius r_m of a drop, i.e., the radius, which separates all the drops into two parts equal in mass which corresponds to condition

$$\sum_{r_{0\,min}}^{r_{0m}} n_i r_{0i} = \sum_{r_{0m}}^{r_{0\,max}} n_i r_{0i}. \qquad (1.15)$$

For the approximate determination of median radius it is possible to recommend the following formula:

$$r_{0m} \approx 50 r_c \left(\frac{\sigma_ж}{\varrho_ж r_c C_0^2}\right)^{0.1} \sqrt{\frac{\eta_ж}{2A\varrho_ж r_c C_0}}, \qquad (1.16)$$

where $\eta_ж$ - coefficient of dynamic viscosity of liquid.

The decomposition of the jet is determined by fluctuations of the liquid in turbulent flow, and there subsequently occur both additional breakdown and connection of drops during collisions, caused by the action of aerodynamic forces. The sizes of drops are determined by the energy of turbulent fluctuations in the liquid, being consumed on overcoming the forces of surface tension and forces of viscosity. Here as criterion it is possible to take the ratio of surface forces to forces of inertia:

$$Z_1 = \frac{\sigma_ж}{2\varrho_ж C^2 r_{0\,max}}.$$

The decomposition should depend on the viscosity of liquid, therefore as the second criterion we take

$$Z_2 = \frac{\eta_ж^2}{2\sigma_ж \varrho_ж \delta_0}.$$

The effect of aerodynamic forces is considered by criterion

$$Z_3 = \frac{\varrho_к}{2\varrho_ж}.$$

For determination of the connection between these criteria let us examine the characteristic boundary conditions. If surface tension is absent, i.e., $\sigma_{ж} = 0$, then a drop cannot be shaped, which corresponds to condition $r_0 = 0$. The lower the efflux velocity of liquid, the larger is the size of drops. Thus, we can write that

$$r_{0\,max}=a_1\left(\frac{\sigma_{ж}}{2\varrho_{ж}C^2}\right)^{n_1},$$

where coefficient a_1 and exponent n_1 determine the intensity of connection between parameters.

The drops are formed even when the liquid does not possess viscosity, i.e., when $\eta_{ж} = 0$. In this instance dimension $r_{0\ max}$ will depend on complex

$$\left(1+\frac{\eta_{ж}^2}{2\sigma_{ж}\varrho_{ж}\delta_0}\right)^{n_2}.$$

With outflow into absolute vacuum, i.e., when $\rho_{к} = 0$, drops will be formed, and increase in the density of medium, into which the liquid flows, will lead to decrease in the size of drops. This means that $r_{0\ max}$ depends on complex

$$\left(1-\frac{\varrho_{к}}{2\varrho_{ж}}\right)^{n_3}.$$

By uniting all three actions and taking into account that hydraulic losses on the injectors

$$\Delta p_{ф}=\varrho_{ж}\frac{C^2}{2},$$

we arrive at relationship

$$r_{0\,max}=a\left(\frac{\sigma_{ж}}{\Delta p_{ф}}\right)^{n_1}\left(1+\frac{\eta_{ж}^2}{2\sigma_{ж}\varrho_{ж}\delta_0}\right)^{n_2}\left(1-\frac{\varrho_{к}}{2\varrho_{ж}}\right)^{n_3}. \qquad (1.17)$$

The treatment of experimental data, provided by various authors, allows us to recommend the following limits of possible changes:

$$10\delta_0^{(1-n_1)} < a < 20\delta_0^{(1-n_1)};\ 0{,}3 < n_1 < 1;\ 0{,}05 < n_2 < 0{,}15;\ n_3 \approx 1.$$

During research and development of new specimens usually the values of coefficient a and the exponents in equation (1.17) are experimentally refined.

Basic equations of efflux of liquid from spray injectors

If through a spray injector liquid is fed to the combustion chamber, to the generator or to another cavity, then in accordance with the equation of continuity the flow rate

$$G = CF_{ж}\varrho_{ж}, \tag{1.18}$$

where the cross-sectional area of liquid flow

$$F_{ж} = \varepsilon F.$$

Here ε - compression factor of the jet, F - cross-sectional area of injector channel.

In a duct of complex shape $F = f(l)$. In injection channels the value of the compression factor can be changed from one to a certain quantity $\varepsilon < 1$; with sufficient extent of the channel the coefficient ε is first decreased, then again increased and can reach the previous value $\varepsilon = 1$. During further motion of liquid along the channel the value of $\varepsilon = 1$, as a rule, is retained. In areas of the channel where $\varepsilon < 1$, the vapor pressure p_c is less than the inlet pressure of the injector p_1 and pressure p_2 in the cavity, into which the liquid flows.

Hydraulic losses in the injector are determined by known formula

$$\Delta p = \xi \varrho_{ж} \frac{C^2}{2}, \qquad (1.19)$$

where ξ - the resistance coefficient. With nonstationary operating conditions the law of conservation of energy for liquid, moving in the flow area of the injector, will be written so:

$$p_1 - p_2 = \varrho_{ж} \frac{C^2}{2} + \Delta p + \frac{m_\phi}{F_2} \dot{C}, \qquad (1.20)$$

where m_ϕ - mass of liquid in the injector. By substituting the value of Δp from equality (1.19) in expression (1.20), after conversions we obtain

$$C = \varphi \sqrt{2 \frac{p_1 - p_2}{\varrho_{ж}}}, \qquad (1.21)$$

where ϕ - the velocity coefficient.

Under nonsteady operating conditions

$$\varphi = \sqrt{\frac{1}{1 + \xi + \beta}}, \qquad (1.22)$$

and in steadied state

$$\beta = \frac{m_\phi}{F_2 \varrho_{ж}} \frac{\dot{C}}{C^2} \approx l_\phi \frac{\dot{C}}{C^2} = 0, \qquad (1.23)$$

where l_ϕ - provided value of the length of injection channel. By substituting the value of velocity from formula 1.21 in equation 1.18, we find

$$G=\mu F\sqrt{2g_{ж}(p_1-p_2)}, \qquad (1.24)$$

where the flow coefficient

$$\mu=\varepsilon\varphi. \qquad (1.25)$$

The examined relationships are suitable even for calculation of gas-spray injectors. The efflux conditions of working medium from injectors are characterized, as was shown above, by coefficient ε, ξ, and β, which are determined experimentally or, when this is possible, by reference formulas, recommended in the corresponding courses.

The character of motion of liquid in channels of complex and cylindrical shape in the presence of back pressure was investigated by B. N. Siov [83]. It was shown that the flow rate of liquid from the injector is characterized by ratio p_2/p_1 and is determined by the difference of $p_1 - p_2$, or $p_1 - p_c$ depending on the quantity of criterion S. Moreover

$$S=\frac{p_1-p_c}{p_1-p_2}=\frac{\varphi_2^2}{\varepsilon^2}(1-\xi_c), \qquad (1.26)$$

where ϕ_2 – velocity coefficient, calculated in terms of parameters at the injector exit; ξ_c – resistance coefficient for the part of the channel where contraction of flow is observed.

Some information about the operation of gas-liquid injectors

In engines with afterburning of gas, exhausted in the turbopump assembly turbine, in the main combustion chamber are applied gas-liquid injectors or injectors feeding both components into the chamber in gaseous form. The first type of nozzles received the

greatest application. Here as a rule, the oxidizer proceeds from the turbine in gaseous state, and liquid fuel is supplied to injectors from the coolant passage of the chamber. Such an injection system is possible to apply in engines, which develop tens of tons of thrust at chamber pressure on the order of 10.0-20.0 MN/m^2. The configuration, which provides the gasification of both components, corresponds to engines with thrust several hundreds of tons and at chamber pressure exceeding 20.0 MN/m^2.

The question of decomposition of a jet was examined for the first time in 1878 by Rayleigh. He considered that a liquid jet represents a cylinder of infinite length, which the environment does not affect. During solution it was also proposed that as a result of the development of wave processes from the jet are separated drops, the size of which depends on the wavelength. From all the disturbances one, "optimum," wave was separated. The problem was solved by the method of small disturbances. The amplitude of oscillations was small in comparison with the diameter of jet. It was assumed that in the nonlinear area the length of the optimum wave is retained (Rayleigh hypothesis). It was determined that long-wave oscillations are unstable.

K. Weber established that the development of instability depends on the relative efflux velocity of liquid.

On a sheet, when it has not been broken, two types of waves are observed. The first type of waves separates the conical sheet into a system of rings; the second type is directed tangentially, across the motion, and separates the rings into jets. As a result a spectrum of drops of various sizes is formed.

With the increase of the relative efflux velocity the wavelength of unstable disturbance is reduced, which leads to decrease in the size of drops. At rather high velocities before the ring will have time to be shaped a flow of the finest drops is formed. This

condition is characteristic for modern gas-liquid injectors. The formed drops are additionally broken down in a gas flow. M. S. Volynskiy established that the breakdown of rather course drops is evaluated with the aid of deformation criterion [75]

$$D=\frac{\rho_{\text{г}} W_{\text{г}} d_{\text{к}}}{\sigma_{\text{к}}}, \tag{1.27}$$

where $\rho_{\text{г}}$, $W_{\text{г}}$ – density and velocity of gas; $d_{\text{к}}$, $\sigma_{\text{к}}$ – diameter of drop and coefficient of surface tension.

The behavior of the drop depending on the deformation criterion is illustrated in the following manner: $D < 10.7$ – the drop is deformed; $D = 10.7$ – 10-20% of drops are split; $D = 10.7$-14 – the percentage of drops being disintegrated increases; from one drop there are formed several, often 3-5 drops; $D > 14$ – all drops are crushed into many fine ones.

To the values of critical phase of deformation at the lower and upper stability limits correspond $D_{\text{н}} = 10.7$ and $D_{\text{в}} = 14$. According to M. S. Volynskiy the breakdown of drops, being absorbed by gas flow, depends on the criterion of deformation and is described by the equation, which is obtained as a result of equating mass and aerodynamic forces together.

For every liquid it is possible to determine such a diameter of drop $d_{\min}$ which drops with diameter $d < d_{\min}$ will not be broken down by gas flow. The smaller the diameter of drop, which fell into flow, the greater is its acceleration and the less its deformation under action of flow.

Some peculiarities of operation of injectors in liquid-propellant rocket engines

The existing theory of injectors is constructed under the assumption of efflux of liquid from injectors into a medium, which

possesses normal pressure and temperature. Under actual conditions the liquid enters dense and highly heated gas. This leads not only to change in the character of atomization of liquid and its preheating after outflow, but also to the appearance of specific processes in the internal cavity of the injector.

Research on the determination of flow characteristics of spray, spray-swirl and swirl injectors with efflux of liquid into a medium with back pressure showed the following.

At the efflux of liquid through a spray injector there can be observed stall and stall-free operating conditions.

The reason for stall involves the development of cavitation in the nozzle of injector. Stall flow condition is characterized by decrease of the flow coefficient.

In conditions of stall-free flow with increase of back pressure the flow rate increase because of the ejection action of the liquid jet on gas in the turbulence cavity of the injector.

During the study of spray-screw injectors there is detected the earlier unobserved phenomenon of adhesion of the spray cone with the central liquid jet with back pressure of air more than 0.2 MN/m^2.

During hydraulic pressure drop tests of swirl injectors it is established that for injectors with small geometric characteristic $A < 1$ at certain relationships between pressure drop on the injector Δp_{ϕ} and pressure p_{κ} of the medium, into which the liquid flows, there is possible the disappearance of the gas vortex, which is accompanied by an abrupt change of the flow coefficient. It is assumed that the reason for this phenomenon involves the ejection of gas by liquid from the internal cavity of the injector.

Analysis of experimental data and visual observations showed that the main factors, which affect the disappearance of the gas vortex, which is accompanied by intermittent change of the flow coefficient, are not only the mentioned parameters, but also the properties of liquid and the construction of the injector, especially, the length of its nozzle.

Tests of transparent models of swirl injectors with geometric characteristics $A < 1$ permitted observing the filling of the cavity of vortex with liquid, i.e., the disappearance of gas vortex. Thus, for instance, for a swirl nozzle with $A = 0.69$ the gas vortex disappeared when $\Delta p_{\text{ф}} = 0.4$ MN/m^2 and $p_{\text{к}} = 3.0$ MN/m^2, and for $\Delta p_{\text{ф}} = 0.8$ MN/m^2 and $\Delta p_{\text{ф}} = 1.2$ MN/m^2 - when $p_{\text{к}} = 2.0$ MN/m^2.

The hydraulic pressure drop test of these injectors as a group showed that filling of the gas cavity by liquid and abrupt change of the flow coefficient occur considerably earlier (at achievement of $p_{\text{к}} = 0.2$-0.6 MN/m^2 and $\Delta p_{\text{ф}} = 0.15$-$0.25$ MN/m^2) and do not depend on the length of the nozzle.

This fact is an essential argument of the fact that the determining conditions for the disappearance of gas vortex are conditions in the injection chamber (pressure chamber) behind the nozzle section of injectors.

By experiments on a single injector with hydraulic pressure drop test it is established that pressure in the vortex cavity, filled with liquid, is less than pressure in the main flow. Furthermore, as measurements along the axis of the injector showed, pressure is reduced from the edge of the injector to its bottom. The presence of rarefaction facilitates the creation of the flow of liquid, which flows inside the injector toward the main flow. There was advanced the hypothesis about the presence in a swirl injector, operating in a gas medium, of gas-liquid counter flow, which appears in view of the ejecting action of liquid flowing from the injector

on the gas vortex. The ejection of gas leads to the appearance of rarefaction in the injector cavity (in the turbulence chamber) in comparison with pressure in the injection chamber.

The ejecting power of the flow of liquid is increased with decrease of the geometric characteristic of the injector, with increase of the developed pressure drop and with increase of the gas density, i.e., increase of back pressure. With increase of ejection the pressure drop between the vortex cavity and the pressure chamber is increased, as a result of which a flow of gas is formed, which moves from the injection chamber inside the injector, seizing the liquid in its path and carrying it into the cavity of the injector. This gas-liquid flow, moving inside the injector, rotates in the direction of rotation of the main flow. Simultaneously the angle of spray cone is decreased, and its trituration is increased. With increase of pressure in the pressure chamber, which simulates the combustion chamber during experiments, the size of the gas vortex is reduced, and at pressure above $p_{\text{к}} = 50$ MN/m^2 the cavity of the injector is filled up with liquid.

On the injector edge in this case there is observed a small volume of gas, which fluctuates with high frequency. As a result of separation of liquid from the gas-liquid flow the radial size of the gas vortex is reduced, and there comes a moment when the gas vortex is intersection by a liquid bridge and the whole vortex cavity is filled up with liquid. At low values of Δp_{ϕ} this bridge is formed at the bottom of the swirl injector on the worm of a screw injector. With increase of the pressure drop the bridge moves toward the nozzle. After its formation the vortex inside the injector is divided into two gas volumes, between which an intensive exchange occurs, and the gas vortex highly fluctuates.

Figure 1.7a shows the operation of the injector with drop $\Delta p_{\phi} = 1.2$ MN/m^2 and back pressure $p_{\text{к}} = 0.1$ MN/m^2; Fig. 1.7b - when $\Delta p_{\phi} = 1.2$ MN/m^2 and $p_{\text{к}} = 3.0$ MN/m^2. One can well see that the vortex was divided into two parts and between them was formed a liquid bridge.

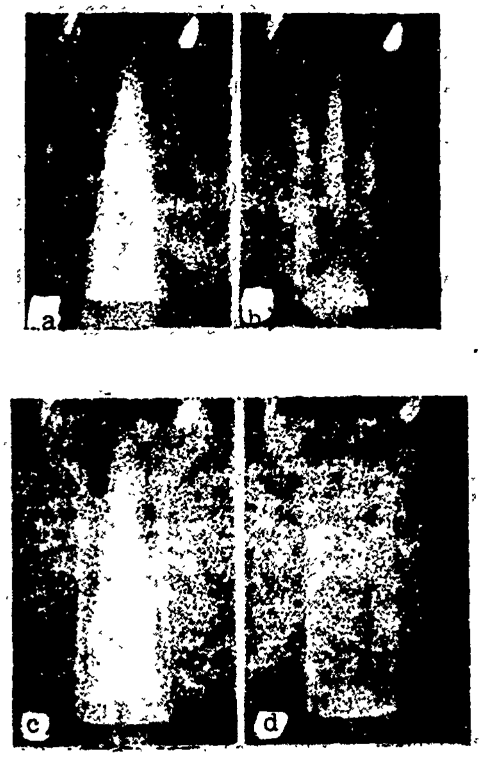


Fig. 1.7. The flame of atomization of injectors, operating under various conditions.

On Fig. 1.7c there is seen the operation of the injector when $\Delta p_{\Phi} = 0.1$ MN/m^2 and $p_{\kappa} = 0.1$ MN/m^2.

Here the gas vortex occupies almost the entire visible volume.

With the same pressure drop, but when $p_{\kappa} = 5.0$ MN/m^2 (Fig. 1.7d) the gas vortex moves toward the injector exit section.

As a result of the effect of ejection there appear gas flows, directed from the chamber to the injector assembly. The presence of such countercurrents was indicated at one time by M. V. Mel'nikov. There is experimentally shown the possibility of transfer of liquid particles by the shown gas flows.

For confirmation of the advanced hypothesis about the mechanism of disappearance of gas vortex special experiments were conducted. It was assumed that if the spray cone will not be changed (is stabilized), then there will not be conditions for seizure of drops by gas counterflow. During tests of injectors with stabilized spray angle there should not be observed phenomena, which lead to the disappearance of vortex.

The experiment, conducted with special adapters for the injector, confirmed this position. On the device a model of an injector with geometric characteristic $A = 0.69$ was investigated. The model has a profiled surface, because of which the cone angle of the injector is not changed. The experiments showed that under any conditions with respect to $\Delta p_{\text{ф}}$ and $p_{\text{к}}$ the vortex does not disappear, although a gas counterflow exists. Besides this, with the presence of a profiled surface of pressure drop, as the experiments of V. V. Logushkov showed, the conditions of protection of chamber walls and injector assembly from the action of heat flows are improved.

Returning to the earlier described experiment of hydraulic pressure drop test of injectors with $A < 1$, it can be said that the abrupt change of the flow coefficient for the group of injectors is begun at smaller $\Delta p_{\text{ф}}$ and $p_{\text{к}}$ than with hydraulic pressure drop test of a single injector. This is explained by the mutual penetration of spray cones, as a result of which the quantity of liquid in the gas counterflow sharply increases. Thereby it was established that the disappearance of gas vortex for swirl injectors occurs because of filling of the cavity of gas vortex by liquid, introduced by gas-liquid counterflow from the injection chamber, and not by liquid supplied to the injector through tangential channels.

Hydraulic pressure drop tests of the group of injectors, placed in the injector assembly, showed that their flow coefficient

is somewhat greater than during a single test (by 1-3%), moreover the higher the geometric characteristic, the smaller is this difference. For injectors with $A < 1$ the "sudden changes" of flow rate, connected with the disappearance of vortex, are displaced toward lower pressures and gas densities in the pressure chamber.

An X-ray photograph of atomization of several injectors, placed in an injector, showed that their spray angles are changed insignificantly in comparison with the spray angles measured during hydraulic pressure drop test of singly operating injectors.

The insignificance of the action of surrounding injectors on the spray cone angle of the investigated injector is explained by the approximate equality of the ejecting action on the part of internal and external cavities, surrounding the spray cone of the injectors.

1.3. Laws of Distribution

In a rather general form the law of distribution of the quantity of drops with respect to the values of their mass can be written so:

$$\frac{dn}{dm_0} = N_0 m_0^k \exp(-\beta m_0^\nu)_0 = f_0(m_0), \tag{1.28}$$

where N_0 - the normalizing factor, which characterizes the flow rate of drops per second:

$$N_0 = \frac{n_0}{\int\limits_0^\infty f_0(m_0)\, dm_0}; \tag{1.29}$$

β, k, ν - parameters characterizing the law of distribution; n_0 - number of drops being formed per second; m_0 - mass of drop after completion of the process of division, but before the beginning of its vaporization or diffusion into combustion products:

$$m_0=\frac{4}{3}\,\frac{\pi}{\xi_m}\,\rho_ж r_0^3.$$

Here r_0 - characteristic dimension of the drop, moreover the coordination coefficient

$$\xi_m=\left(\frac{r_0}{r_{ш0}}\right)^3,$$

where $r_{ш0}$ - radius of spherical drop, the mass of which is equal to the mass of the considered drop. If distribution complys with Maxwell law with two-dimensional initial dispersion, then

$$N_0=\frac{n_0}{\sigma_0^2};\ k=1;\ \beta=\frac{1}{2\sigma_0^2};\ \nu=2,$$

where σ_0 - mean square deviation of circular Gaussian dispersion. With two-dimensional initial dispersion the random quantity is the geometric sum of the other two random quantities, subordinate to Gauss law.

With three-dimensional initial dispersion

$$N_0=\sqrt{\frac{2}{\pi}}\,\frac{n_0}{\sigma_0^3};\ k=2;\ \beta=\frac{1}{2\sigma_0^2};\ \nu=2.$$

During calculation of engines we frequently use the formula of N. Tresh, for which $k = -3$; $\nu=-1$; $\beta \approx 0.2$. During investigation of particular specimens the law of distribution or parameters k, β, ν in formula (1.28) should be found by results of processing the statistical data. For determinations of the overall number of drops, being formed in a unit of time, we use equation (1.28). After division of variables and integration we find

$$n_0\approx N_0\int\limits_{m_{0\,\min}}^{m_{0\,\max}} m_0^k\exp(-\beta m_0^{\nu})\,dm_0. \qquad (1.30)$$

The values, obtained with integration from 0 to $m_{0\ \min}$ and from $m_{0\ \max}$ to ∞, are commensurable with the error, which appears

as a result of the inaccurate determination of exponents k, ν and β. Therefore, instead of formula (1.30) it is possible to use formula

$$n_0 \approx N_0 \int_0^\infty m_0^k \exp(-\beta m_0^\nu)\, dm_0. \tag{1.31}$$

The flow rate of liquid is determined by the product of the number of drops by their mass. By introducing the expression of mass under the integral sign, we obtain:

$$G(0) = N_0 \int_0^\infty m_0^{k+1} \exp(-\beta m_0^\nu)\, dm_0. \tag{1.32}$$

If the time the drops stay in the chamber before the beginning of vaporization τ_s is known, then the quantity of liquid, which is in the chamber in the vaporization preparation stage, will be

$$Y_{ж s} = \int_0^{\tau_s} G(0)\, dt. \tag{1.33}$$

The total number of drops, located in the chamber in the vaporization preparation stage

$$n_{\tau_s} = \frac{G(0)\,\tau_s}{m_{0\,cp}}, \tag{1.34}$$

where $m_{0\ cp}$ - average value of initial mass of drop.

With the aid of the chart of function (1.28) it is possible to judge the fineness and homogeneity of atomization. The smaller $m_{0\ max}$ is, the finer the atomization. The closer the value of $m_{0\ max}$ is to $m_{0\ min}$, the more uniform the atomization.

The uniformity of distribution of component along the cross section and along the length of the chamber depends on the number of injectors of the injector assembly, their arrangement and on the law of distribution of drops, created by one injector.

For a unit of time through area $dydz$, located perpendicular to axis x, the elementary flow rate from one injector will be [17]

$$dG(x,y,z)=\frac{9}{2\pi}\frac{G_{\text{ф}}}{x^2\operatorname{tg}^2\alpha}\exp\left(-\frac{9}{2}\frac{r^2}{x^2\operatorname{tg}^2\alpha}\right)dydz, \qquad (1.35)$$

where α - half the spray cone angle.

As a result of the intersection of spray cones and encounter of liquid flows there are formed secondary sources of atomization and mixing, moreover around the axis of the secondary source the mass of components is distributed by normal law.

The quantity of liquid, falling on the elementary area, will be

$$G=G_i\int_{y_1}^{y_2}\varphi(y)dy\int_{z_1}^{z_2}\varphi(z)dz, \qquad (1.36)$$

where G_i - experimentally determined flow rate of liquid from the secondary source.

Let us write the values of functions of density of distribution so:

$$\varphi(y)=\frac{3}{\sqrt{2\pi}x\operatorname{tg}\alpha}\exp\left(-\frac{9y^2}{2x^2\operatorname{tg}^2\alpha}\right); \qquad (1.37)$$

$$\varphi(z)=\frac{3}{\sqrt{2\pi}x\operatorname{tg}\alpha}\exp\left(-\frac{9z^2}{2x^2\operatorname{tg}^2\alpha}\right). \qquad (1.38)$$

By substituting the values of functions from formulas (1.37) and (1.38) in expression (1.36) and summarizing flow rates G_i through all the injectors, for any k-th component we find

$$G_k=\frac{9}{2\pi x^2}\sum_1^{n_k}\frac{G_{ik}}{\mathrm{tg}^2\,\alpha_{ik}}\left[\int_{y_{1ik}}^{y_{2ik}}\exp\left(\frac{-9y_k^2}{2x_k\,\mathrm{tg}^2\,\alpha_{ik}}\right)dy\right]\times\left[\int_{z_{1ik}}^{z_{2ik}}\exp\left(\frac{-9z_k^2}{2x_k^2\,\mathrm{tg}^2\,\alpha_{ik}}\right)dz\right]. \qquad (1.39)$$

By using formula (1.39) for oxidizer and then for fuel, it is possible to establish the law of distribution of the component ratio in the form

$$K(x,y,z)=\frac{G_1(x,y,z)}{G_2(x,y,z)}. \qquad (1.40)$$

During the experimental determination of the character of distribution of components along the length and cross section of the combustion chamber various methods are used. In certain cases one of the components is tinted or such components are selected, which after mixing are easily separated (for example, water and kerosene). Promising application is the method of tagged atoms; it is possible into one component to add an isotope, which possesses beta-activity, and into the other - gamma-activity. When photographing the process of mixing the application of X-ray instruments is not excluded.

In the accepted scheme of calculation the flow of liquid is diverging, moreover the peripheral injectors will circumflow the chamber walls. In actual conditions during the motion of liquid there takes place ejection of gas, as a result of which the pressure on the axis of the chamber should be distinguished from pressure in the cavities, adjacent to walls. Besides this, pressure will be changed along the axis length of the chamber axis. All this leads to the unique location of liquid phase in the internal volume of the chamber.

1.4. Heating of a Drop to the Beginning of Vaporization of Liquid

Under the effect of heat flow, directed from combustion products, the mass of the drop is warmed up. Let us examine the period of time, when the surface layer of the drop reaches the boiling point, which corresponds to the prescribed pressure in the chamber.

For determination of the character of distribution of temperature along the radius of drop and in time we should use the differential equation of thermal conductivity, which in spherical coordinates has the form

$$\frac{\partial}{\partial t}[r,T(r,t)]=a_{ж}\frac{\partial^2}{\partial r^2}[r,T(r,t)], \tag{1.41}$$

where $a_{ж}$ - coefficient of thermal conductivity of liquid, moreover

$$a_{ж}=\frac{\lambda_{ж}}{c_{ж}\varrho_{ж}}.$$

At the initial moment of time the temperature of liquid in the drop at any distance from the center of mass is constant and is equal to a certain prescribed temperature $T_{ж0}$, i.e.,

$$T(r,0)=T_{ж0} \tag{1.42}$$

Heat flow q supplied to the surface of the drop is completely transferred to liquid; this condition is written so:

$$q=\lambda\frac{\partial}{\partial r}[T(r_{н},t)], \tag{1.43}$$

where $r_{н}$ - radius of external surface of the drop.

For a spherical drop according to conditions of symmetry of warming and distribution of temperatures along the radius

$$\frac{\partial}{\partial r}[T(0,t)]=0. \qquad (1.44)$$

Such are the boundary conditions of the problem. The solution of differential equation (1.41) at initial (1.42) and boundary (1.43) and (1.44) conditions has the form [59]

$$T(r,t)=T_{ж0}+q\frac{r_н}{\lambda_ж}\left[\frac{3a_ж t}{r_н^2}-\frac{3r_н^2-5r^2}{10r_н^2}-\right.$$
$$\left.-\sum_{1}^{\infty}\frac{2}{\mu_n^2\cos\mu_n}\,\frac{r_н\sin\mu_n\cdot\frac{r}{r_н}}{r\mu_n}\exp\left(-\mu_n^2\frac{a_ж t}{r_н^2}\right)\right], \qquad (1.45)$$

where μ_n - the roots of characteristic equation, moreover, if Biot criterion $Bi \to 0$, then $\mu_1 = 0.0000$; $\mu_2 = 4.4934$; $\mu_3 = 7.7253$; $\mu_4 = 10.9041$; $\mu_5 = 14.0662$; $\mu_6 = 17.2208$. With increase of Biot criterion the numerical values of roots are increased [59].

For determination of the temperature of liquid on the surface of the drop in expression (1.45) instead of the current value of r one should substitute the radius of the external surface $r_н$. In this case we obtain

$$T(r_н,t)=T_{ж0}+q\frac{r_н}{\lambda_ж}\left[\frac{3a_ж t}{r_н^2}-\frac{1}{5}-\sum_{1}^{\infty}\frac{2}{\mu_n^3}\operatorname{tg}\mu_n\exp\left(-\mu_n^2\frac{a_ж t}{r_н^2}\right)\right]. \qquad (1.46)$$

At large values of Fourier number

$$\mathrm{Fo}=\frac{a_ж t}{r_н^2} \qquad (1.47)$$

series (1.46) converges rapidly.

The heating period of the drop to the beginning of vaporization is finished at moment of time τ_s, which corresponds to achievement of boiling point $T_{\text{ж} s}$ by the surface of the drop. To this moment of time corresponds condition

$$T(r_н, \tau_s) = T_{\text{ж} s}. \tag{1.48}$$

By substituting in equation (1.46) τ_s instead of t and considering equality (1.48), the value of τ_s is found graphically. For approximate determination of the value of τ_s let us use formula

$$\tau_s = \varepsilon c_{\text{ж}} \varrho_{\text{ж}} r_0 \left[\frac{T_{\text{ж} s} - T_{\text{ж}0}}{3q} + \frac{r_0}{15\lambda_{\text{ж}}} \right], \tag{1.49}$$

where ε - the coefficient, considering the effect of discarded terms and unconsidered factors of heat exchange; $c_{\text{ж}}$ - the heat capacity of liquid.

For determination of heat flow from combustion products to liquid in the period, with still no mass removal from the drop, it is possible to use formula

$$q = \frac{\lambda_к}{r_н} (1 + 0{,}3\, \mathrm{Re}^{0{,}5} \mathrm{Pr}^{0{,}33}) (T_к - T_{\text{ж} s}). \tag{1.50}$$

Here

$$\mathrm{Re} = \frac{2|W - C| r_н}{\nu_к}, \tag{1.51}$$

where C - velocity of motion of drop; W - velocity of gas flow, circumflowing the drop;

$$\mathrm{Pr} = \frac{\varrho_к \nu_к c_к}{\lambda_к}; \tag{1.52}$$

$T_{\text{к}}$ – temperature of gas in the combustion chamber in the region of preheating of liquid propellant; $\lambda_{\text{к}}$ – coefficient of thermal conductivity of drop.

If $200 < \text{Re} < 3000$, then according to D. N. Vyrubov

$$q \approx 0.26 \frac{\lambda_{\text{к}}}{r_{\text{н}}} \text{Re}^{0.5} (T_{\text{к}} - T_{\text{ж}s}). \qquad (1.53)$$

Now instead of relationship (1.49) we will have a relationship which allows approximately determining the time

$$\tau_s = c_{\text{ж}} \rho_{\text{ж}} \frac{r_0}{\lambda_{\text{к}}} \left[\sqrt{\frac{r_0 \nu_{\text{к}}}{|W - C|}} \frac{T_{\text{ж}s} - T_{\text{ж}0}}{T_{\text{к}} - T_{\text{ж}s}} + \frac{r_0}{15} \right]. \qquad (1.54)$$

For determination of τ_s at conditions there are widely used the recommendations given in [48].

For determination of $T_{\text{ж}s}$ one should use the results of processing experimental data or formula [15]

$$T_{\text{ж}s} = \frac{(T_{\text{ж}s})_{(p=1)} r_{\text{ж}}^{*}}{r_{\text{ж}}^{*} - R (T_{\text{ж}s})_{(p=1)} \ln \frac{p_{\text{к}}}{1}}, \qquad (1.55)$$

where $(T_{\text{ж}s})_{(p=1)}$ – the boiling point at pressure $p = 1$; R – gas constant.

When using formula (1.54) the greatest difficulties appear when determining the gas parameter in the preheating region of drops. Some experiments give $T_{\text{к}} = 800\text{–}1200°\text{K}$. Considering the presence of countercurrents in the region of the injector assembly, on the average we obtain $|W - C| = 20\text{–}50$ m/s.

For all practical purposes the numerical values of parameters should be determined by calculation and by experiment for each separate case.

1.5. Change of the Mass of Drop in the Period of Its Vaporization

The mass transfer between drops and gaseous products is characterized by equation

$$\mathrm{Nu}_{д}=B\left(2+0{,}6\,\mathrm{Re}^{0{,}5}\,\mathrm{Pr}_{д}^{0{,}33}\right), \qquad (1.56)$$

where B – proportionality factor; $\mathrm{Nu}_{д}$ – Nusselt diffusion criterion, moreover

$$\mathrm{Nu}_{д}=2\frac{q_m r_к}{D\rho_к}, \qquad (1.57)$$

where q_m – specific flow of mass from the surface of drop; D – diffusion coefficient.

The Reynolds number

$$\mathrm{Re}=2\frac{|W-C|\,r_к}{\nu_к}. \qquad (1.58)$$

The Prandtl diffusion criterion

$$\mathrm{Pr}_{д}=\frac{\nu_к}{D}. \qquad (1.59)$$

At large Re numbers the addend in formula (1.56) can be disregarded, in this case we obtain

$$q_m\approx 0{,}425BD\rho_к\sqrt{\frac{|W-C|}{r_к\nu_к}}. \qquad (1.60)$$

The mass of drop

$$m_к=\frac{4}{3}\,\frac{\pi}{\xi_m}\,\rho_ж r_н^3; \tag{1.61}$$

specific flow of mass

$$q_m=-\frac{\dot{m}_к}{4\pi r_н^2}, \tag{1.62}$$

consequently,

$$q_m=-\frac{\rho_ж}{\xi_m}\,\dot{r}_н. \tag{1.63}$$

By equating expressions (1.60) and (1.63), after division of variables we find

$$-r_н^{0,5}dr_н=0{,}425B\xi_m D\,\frac{\rho_к}{\rho_ж}\sqrt{\frac{|W-C|}{\nu_к}}\,dt. \tag{1.64}$$

By integrating the left side from r_0 to r_H and the right side from 0 to t, we obtain

$$r_н=(r_0^{3/2}-\phi_д t)^{2/3}, \tag{1.65}$$

where

$$\phi_д\approx 0{,}638\,B\xi_m D\,\frac{\rho_к}{\rho_ж}\sqrt{\frac{|W-C|}{\nu_к}}. \tag{1.66}$$

The equation of law of change of the mass of drop can be obtained by using a criterial equation in the form

(1.61)

$$\mathrm{Nu}=2+0,6\mathrm{Re}^{0,5}\mathrm{Pr}^{0,33}, \tag{1.67}$$

where

(1.62)

$$\mathrm{Nu}=2\frac{\alpha r_{\text{к}}}{\lambda_{\text{к}}}; \tag{1.68}$$

$$\mathrm{Re}=2\frac{|W-C|r_{\text{к}}}{\nu_{\text{к}}}; \tag{1.69}$$

(1.63)

$$\mathrm{Pr}=\frac{\nu_{\text{к}}\varrho_{\text{к}}c_{\text{к}}}{\lambda_{\text{к}}}. \tag{1.70}$$

of

In this case the heat flow from gases to the drop

$$q=\frac{\lambda_{\text{к}}}{2r_{\text{к}}}(2+0,6\,\mathrm{Re}^{0,5}\,\mathrm{Pr}^{0,33})(T_{\text{к}}-T_{\text{жs}}). \tag{1.71}$$

(1.64)

By disregarding the addend in expression (1.71), we find

side from

$$q=\frac{0,425\lambda_{\text{к}}}{r_{\text{к}}^{0,5}}\sqrt{\frac{|W-C|}{\nu_{\text{к}}}}(T_{\text{к}}-T_{\text{жs}})\,\mathrm{Pr}^{0,33}. \tag{1.72}$$

(1.65)

The specific heat flow, consumed in vaporization of the drop,

$$q=-\frac{\varrho_{\text{ж}}}{\xi_{m}}r_{\text{ж}}^{\circ}\frac{dr}{dt}. \tag{1.73}$$

(1.66)

By equating expressions (1.72) and (1.73), after integration and conversions we obtain

$$r_{ж}=(r_0^{3/2}-\psi t)^{2/3}, \qquad (1.74)$$

where for a drop, already preheated to $T_{ж\,s}$,

$$\psi=0{,}638\xi_{ж}\lambda_{к}\sqrt{\frac{|W-C|}{\nu_{к}}}\,\frac{T_{к}-T_{жs}}{\varrho_{ж}r_{ж}^{*}}\mathrm{Pr}^{0,33}.$$

If we approximately take into account the warming up of the entire mass of the drop to temperature $T_{ж\,s}$, then

$$\psi=0{,}638\xi_{ж}\lambda_{к}\sqrt{\frac{|W-C|}{\nu_{к}}}\,\frac{T_{к}-T_{жs}}{\varrho_{ж}\left[r_{ж}^{*}+c_{ж}(T_{жs}-T_{ж0})\right]}\mathrm{Pr}^{0,33}. \qquad (1.75)$$

The diffusion coefficient is approximately numerically equal to the coefficient of thermal conductivity, i.e.,

$$D\approx\frac{\lambda_{к}}{c_{к}\varrho_{к}}. \qquad (1.76)$$

All parameters in equation (1.76) correspond to identical condition of heat-mass transfer.

By substituting the value of D from equation (1.76) in formula (1.66) and equating the right sides of (1.66) and (1.75), we find

$$B=\frac{c_{к}(T_{к}-T_{жs})}{r_{ж}^{*}}. \qquad (1.77)$$

If we perform calculation with allowance for time, consumed on preheating the drop, then approximately

$$B=\chi\frac{c_{к}(T_{к}-T_{жs})}{r_{ж}^{*}+c_{ж}(T_{жs}-T_{ж0})}.$$

The coordination coefficient χ in engineering practice is refined experimentally.

1.6. Transition Processes of the Mass of a Drop into Products of Burning

Subcritical pressure

In the period of vaporization the drop is surrounded by vapors and by transformation products, which form a "cloud." The size of the cloud continuously changes; the quantity of vapors is increased because of vaporization of liquid and is decreased as a result of their diffusion into products of burning. If $C = W$, then the cloud has the shape of a sphere. If $C > W$, then the cloud follows a drop, forming a "tail." If $W > C$, then the drop, surrounded by a cloud, moves with the tail forward.

The penetration of vapors into combustion products is described by the equation of diffusion, which in spherical coordinates for a spherical cloud has the form

$$\frac{\partial}{\partial t}[R_{\text{об}}(t)\,c(R,t)] = D\,\frac{\partial^2}{\partial R^2}[R_{\text{об}}(t)\,c(R,t)], \tag{1.78}$$

where $R_{\text{об}}(t)$ – radius of external surface of the cloud, which changes with time; $c(R, t)$ – concentration, which depends on the current value of radius R and is changed with time; D – diffusion coefficient, moveover in the most general case it is equal to the sum of coefficients of molecular and turbulent diffusion.

At moment of time $t = 0$ for any value of $R - r_{\text{Н}}$ the concentration of vapors is equal to zero; therefore the initial condition will be written so:

$$c[(R-r_0),0]=0, \tag{1.79}$$

where r_0 – radius of drop at moment $t = 0$.

Being guided by the law of conservation of mass, the first boundary condition can be written so:

$$\frac{4}{3}\pi\frac{\varrho_ж}{\varepsilon_ж}(r_0^3-r^3)=4\pi\int_0^\infty R_{об}^2(t)\,c(R,t)\,dR. \qquad (1.80)$$

Condition (1.80) shows that at any moment of time the decrease of mass of drop (the left side of the equation) is equal to the mass of products in the cloud and in the combustion products. Under actual conditions the value of the upper limit of integration is determined by the geometry of the chamber. Integration within limits to infinity will not introduce noticeable error, inasmuch as the concentration of diffusing products is rapidly decreased with removal of the drop from the surface.

At any moment of time the derivative from concentration with respect to the radius of cloud on the surface of the drop is equal to zero, therefore the second boundary condition is written so:

$$\frac{\partial}{\partial R}[c(r_н,t)]=0. \qquad (1.81)$$

The equation of the law of conservation of mass of vaporizing drop has the form

$$m_0=m_н+m_{об}+m_D, \qquad (1.82)$$

where m_0 – mass of drop before the start of vaporization; $m_н$ – current value of mass of drop; $m_{об}$ – mass of cloud; m_D – mass of vapors, diffused into products of burning.

Having taken the time derivatives from expressions (1.82) and (1.61), we find

$$\dot{m}_{\text{н}}+\dot{m}_{\text{об}}+\dot{m}_D=0; \tag{1.83}$$

$$\dot{m}_{\text{н}}=4\pi\frac{\varrho_{\text{ж}}}{\xi_m}r_{\text{н}}^2\dot{r}_{\text{н}}, \tag{1.84}$$

and also

$$\dot{m}_{\text{об}}=4\pi\frac{\varrho_{\text{об}}}{\xi_{\text{об}}}(R_{\text{об}}^2\dot{R}_{\text{об}}-r_{\text{н}}^2\dot{r}_{\text{н}}). \tag{1.85}$$

It is known that

$$\dot{m}_D=-4\pi R_{\text{об}}^2 D\frac{\partial c}{\partial R_{\text{об}}}. \tag{1.86}$$

Thus,

$$\left(\frac{\varrho_{\text{ж}}}{\xi_m}-\frac{\varrho_{\text{об}}}{\xi_{\text{об}}}\right)r_{\text{н}}^2\dot{r}_{\text{н}}+\frac{\varrho_{\text{об}}}{\xi_{\text{об}}}R_{\text{об}}^2\dot{R}_{\text{об}}-R_{\text{об}}^2 D\frac{\partial c}{\partial R}=0. \tag{1.87}$$

To investigate the process of vaporization there is used system of equations (1.74), (1.78) and (1.87).

Supercritical pressure

For a spherical drop the diffusion of mass into products of burning is described by equation

$$\frac{\partial}{\partial t}[r_{\text{н}}(t)c(R,t)]=D\frac{\partial^2}{\partial R^2}[r_{\text{н}}(t)c(R,t)]. \tag{1.88}$$

At the initial moment of time, i.e., when $t = 0$, at any distance of $R - r_H$ from the surface of the drop the concentration of substance of the drop is equal to zero. Consequently, the initial condition will be written so:

$$c[(R-r_{н}),0]=0. \qquad (1.89)$$

At a certain moment of time t_0 and at any other moment of time $t > t_0$ the whole mass of the drop will pass into the area of products of burning, which is conditionally assumed infinite, therefore the boundary condition will be written so:

$$\frac{4}{3}\pi r_{н}^3\varrho_{ж}=4\pi\int_0^\infty c(R,t)R^2dR. \qquad (1.90)$$

For a spherical drop the second boundary condition is determined by the fact that at any moment of time in the center of the drop derivative

$$\frac{\partial}{\partial r}c(0,t)=0. \qquad (1.91)$$

Diffusion of the mass of drop into the area of products of burning proceeds very rapidly. Therefore, calculation can be performed under the assumption of instantaneous separation of mass. Considering the small sizes of the drop, it is possible to be guided by a point source. In this case the solution of equations (1.88) will be written so:

$$c(R,t)=\frac{r_{н}^3\varrho_{ж}}{3\sqrt{4\pi}(RD)^{3/2}}\exp\left(-\frac{R^2}{4Dt}\right). \qquad (1.92)$$

1.7. Characteristic of Vaporization

The character of growth of products of burning with time or along the length of the chamber is frequently called the burnout curve.

Let us assume $G(0)$ will characterize the instantaneous inflow of liquid propellant into the chamber, moreover as the beginning of timing we will take the moment of entry of propellant into the combustion chamber.

The instantaneous inflow per second of gaseous vaporization products into the chamber

$$G_{0-t}=G(0)-G(t), \tag{1.93}$$

where $G(t)$ – the instantaneous quantity of propellant per second, which entered the chamber at moment of time $t = 0$ and remained in liquid state to moment of time t.

The characteristic of vaporization $\phi(t)$ is the relationship of instantaneous quantity of vaporization products per second, formed at moment of time t, to the instantaneous quantity of propellant per second, which entered the chamber at moment of time $t = 0$, i.e.,

$$\varphi(t)=\frac{G(0)-G(t)}{G(0)}=1-\frac{G(t)}{G(0)}. \tag{1.94}$$

As was shown,

$$\frac{dn}{dm_0}=f_0(m_0). \tag{1.95}$$

At the initial moment of time the density of distribution (dn/dm_0) is characterized by function $f_0(m_0)$, depending on the initial values of masses of drops m_0.

The mass flow rate per second at moment of time $t = 0$ is determined by the product of the number of drops by their initial mass. The number of drops when $t = 0$

$$n_0 = \int_0^\infty f_0(m_0)\,dm_0. \tag{1.96}$$

Inasmuch as mass m_0 – variable quantity, then for determination of the flow rate of liquid the value of mass should be introduced under the integral sign:

$$G(0) = \int_0^\infty m_0 f_0(m_0)\,dm_0. \tag{1.97}$$

At any arbitrary moment of time t the equation of the density of distribution will be written so:

$$\frac{dn}{dm_{н}} = f(m_{н}), \tag{1.98}$$

where m_{H} – the current value of the mass of drop, moreover, by using equality (1.74), let us find

$$m_{н} = (m_0^{0.5} - \varphi_m t)^2, \tag{1.99}$$

where

$$\varphi_m = \frac{4\tau}{3\xi_m}\varphi. \tag{1.100}$$

Instead of expression (1.99) there can be taken, of course, another law of vaporization, if it is more suitable for the given particular conditions.

According to formula (1.95) we have

$$dn = f_0(m_0)\,dm_0. \tag{1.101}$$

According to formula (1.98) we find

$$dn = f(m_H)\,dm_H. \tag{1.102}$$

By equating the right sides of expressions (1.101) and (1.102), we find the law of distribution for any moment of time t in the form

$$f(m_H) = f_0(m_0)\frac{dm_0}{dm_H}. \tag{1.103}$$

By analogy with expression (1.97) it is possible to write the formula for determination of the flow rate at any moment of time:

$$G(t) = \int_0^\infty m_H f(m_H)\,dm_H. \tag{1.104}$$

By using equality (1.103), we find

$$G(t) = \int_0^\infty m_H f_0(m_0)\frac{dm_0}{dm_H}\,dm_H. \tag{1.105}$$

Complex $f_0(m_0)\dfrac{dm_0}{dm_H}$ can be expressed through the value of current mass m_H. Masses m_0 and m_H are connected by equation (1.99), so that

$$f_0(m_0)\frac{dm_0}{dm_H} = f_0\left[(m_H^{0,5}+\psi_m t)^2\right]\left(1+\frac{\psi_m t}{m_H^{0,5}}\right). \tag{1.106}$$

Now equation (1.105) takes this form:

$$G(t)=\int_0^{\infty} m_{\text{н}} f_0 \left[(m_{\text{н}}^{0,5}+\psi_m t)^2\right] \left(1+\frac{\psi_m t}{m_{\text{н}}^{0,5}}\right) dm_{\text{н}}. \tag{1.107}$$

Inasmuch as equation (1.99) is valid for a drop of any size, we find that the mass of the coarsest drop will be changed so:

$$m_{\max}=(m_{0\max}^{0,5}-\psi_m t)^2. \tag{1.108}$$

Thus, under our conditions expression (1.107) takes the form

$$G(t)=\int_0^{m_{\max}} m_{\text{н}} f_0 \left[(m_{\text{н}}^{0,5}+\psi_m t)^2\right]\left(1-\frac{\psi_m t}{m_{\text{н}}^{0,5}}\right) dm_{\text{н}}. \tag{1.109}$$

The flow rate at any moment of time can be expressed through the value of initial mass, in this case we will have

$$G(t)=\int_{(\psi_m t)^2}^{m_{0\max}} (m_0^{0,5}-\psi_m t)^2 f_0(m_0)\, dm_0. \tag{1.110}$$

An expression for determination of the characteristic of vaporization $\phi(t)$ with timing from moment τ_s can be written, by using equations (1.94), (1.97) and (1.110).

1.8. Relationships Between the Flow Rates of Propellant Components Per Second

Depending on the purpose of research, the relationship between the flow rates of propellant components per second, or briefly – the component ratio, is determined differently.

1. If there is studied the connection between the combustion chamber and feed systems and low-frequency oscillations are investigated, then the ratio of oxidizer G_1 and fuel G_2 at the injector exit is designated so:

$$k_1^* = \frac{G_1(0)}{G_2(0)}. \tag{1.111}$$

The flow rates of components change with time as a result of the nonuniformity of operation of units of the power plant, the action of control elements, fluctuations of pressure in the combustion chamber, wave processes in the hydraulic elements of the feed system and so forth. In rather general form the flow rate of component G_i can be represented so:

$$G_i = \overline{G}_i(t) + \overline{A}_i(t)\exp\{-i[\omega_i(t)t + \varphi_i]\}. \tag{1.112}$$

Here time t characterizes the moment of arrival of the component into the combustion chamber. The amplitude of oscillations $A_i(t)$, just as frequency $\omega(t)$, are changed with time depending on the design and operating conditions of the engine. Phase shift ϕ_i is determined mainly by the geometry of flow ducts and the operating conditions of the power plant. The exponent in formula (1.112) is a complex number.

If nominal flow rate $G_i(t)$, amplitude $A_i(t)$, frequency $\omega_i(t)$ and phase shift $\phi_i(t)$ are not changed with time, then, being guided by their average values $\overline{G}_i$, $\overline{A}_i$, $\overline{\omega}_i$, $\overline{\phi}_i$, we obtain

$$G_i = \overline{G}_i + \overline{A}_i\exp[-i(\overline{\omega}_i t + \overline{\varphi}_i)]. \tag{1.113}$$

In the case of sinusoidal oscillations

$$G_i = \overline{G}_i + \overline{A}_i\sin(\overline{\omega}_i t + \overline{\varphi}_i). \tag{1.114}$$

2. The burning conditions of propellant are determined by the current value of the component ratio

$$k_t = \frac{G_1(t)}{G_2(t)}. \qquad (1.115)$$

3. The conditions of occurrence of chemical reactions in gaseous phase depend on relationship

$$k_{0-t} = \frac{G_1(0) - G_1(t)}{G_2(0) - G_2(t)}. \qquad (1.116)$$

1.9. The Rate of Change of Component Ratio

The component ratio determines the value of a number of parameters, including the rate of burning U and the efficiency of gas. For the prescribed propellant by calculation or from results of processing of experimental data it is possible to determine the relationship of gas efficiency to the component ratio, i.e.,

$$RT = RT(k_1). \qquad (1.117)$$

By differentiating expression (1.117), we find

$$\frac{d}{dt}(RT) = \frac{\partial}{\partial k_1}[RT(k_1)]\dot{k}_1. \qquad (1.118)$$

Consequently, with change of the component ratio in time the power being expended or consumed by the burning flow [67]

$$N = \frac{d}{dt}(RT) \qquad (1.119)$$

depends on the properties of components and the burning conditions. According to equality (1.118) the considered power depends on the rate of change of the component ratio and on the absolute value of the component ratio. The total derivative of component ratio

$$\dot{k}_1 = \frac{\partial k_1}{\partial t} + \bar{C}\,\mathrm{grad}\,k_1. \tag{1.120}$$

Local derivative $\partial k_1/\partial t$ characterizes the change of component ratio with time at the combustion chamber inlet and is determined by the operating conditions of the feed system.

As a result of the different rate of vaporization (or diffusion) of components there appears the need to consider coordinate derivatives from component ratio — the second term of the right side of equation (1.120).

As was noted above, the efficiency of gases depends on the component ratio, moreover to the largest value of RT corresponds some optimum value of $k_{1\text{опт}}$, at which $\frac{\partial}{\partial k_1}(RT) = 0$. If $k_1 < k_{1\text{опт}}$, then $\frac{\partial}{\partial k_1}(RT) > 0$; when $k_1 > k_{1\text{опт}}$, then $\frac{\partial}{\partial k}(RT) < 0$.

By using equations of state and (1.118), we find

$$\dot{p} = \frac{Y_{\text{к}}}{V_{\text{к}}}\frac{\partial}{\partial k_1}(RT)\dot{k}_1 + RT\frac{d}{dt}\left(\frac{Y_{\text{к}}}{V_{\text{к}}}\right), \tag{1.121}$$

where $Y_{\text{к}}$, $V_{\text{к}}$ — the amount of gases in the chamber and the chamber volume. By integrating equation (1.121), we find

$$\Delta p = \int_{k_{10}}^{k_1} \frac{Y_{\text{к}}}{V_{\text{к}}}\frac{\partial}{\partial k_1}(RT)\,dk_1 + \int_{\varrho_1}^{\varrho_2} RT\,d\varrho. \tag{1.122}$$

Thus, with change of the component ratio with time a change of pressure in the chamber can be observed. If derivative $\dot{k}$ appears in the limited part of the chamber volume, then the change in pressure will bear a local character. In this case a pressure wave can appear, which is propagated along the chamber volume. Upon reaching the walls it will be reflected from them and intersect the area of burning. As a consequence acceleration of burning can occur, which under certain conditions will lead to intensification of wave processes.

Above we examined the change of flow rate of component with time:

for oxidizer

$$G_1=\bar{G}_1+\bar{A}_1\exp[-i(\bar{\omega}_1 t+\bar{\varphi}_1)], \tag{1.123}$$

for fuel

$$G_2=G_2+\bar{A}_2\exp[-i(\bar{\omega}_2 t+\bar{\varphi}_2)]. \tag{1.124}$$

By differentiating, we find

$$\frac{\partial G_1}{\partial t}=-i\bar{\omega}_1\bar{A}_1\exp[-i(\bar{\omega}_1 t+\bar{\varphi}_1)]; \tag{1.125}$$

$$\frac{\partial G_2}{\partial t}=-i\bar{\omega}_2\bar{A}_2\exp[-i(\omega_2 t+\bar{\varphi}_2)]. \tag{1.126}$$

Inasmuch as

$$\frac{\partial}{\partial t}k_1=\frac{\frac{\partial}{\partial t}G_1-k_1\frac{\partial}{\partial t}G_2}{G_2}, \tag{1.127}$$

then after conversions, having accepted $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}$, we obtain

$$\frac{\partial}{\partial t} k_1 = \frac{i\bar{\omega}}{\bar{G}_2 \exp(i\bar{\omega}t) + \bar{A}_2 \exp(-i\bar{\varphi}_2)} \Big[-A_1 \exp(-i\bar{\varphi}_1) + \\ + \frac{\bar{G}_1 \exp(i\bar{\omega}t) + \bar{A}_1 \exp(-i\bar{\varphi}_1)}{\bar{G}_2 \exp(i\bar{\omega}t) + \bar{A}_2 \exp(-i\bar{\varphi}_2)} \bar{A}_2 \exp(-i\bar{\varphi}_2) \Big]. \tag{1.128}$$

In the case of sinusoidal oscillations

$$\frac{\partial}{\partial t} k_1 = \frac{i\bar{\omega}}{\bar{G}_2 \cos\bar{\omega}t + i\bar{G}_2 \sin\bar{\omega}t + \bar{A}_2 \cos\bar{\varphi}_2 - i\bar{A}_2 \sin\bar{\varphi}_2} \Big[-\bar{A}_1 \cos\bar{\varphi}_1 + \\ + i\bar{A}_1 \sin\bar{\varphi}_1 + (\bar{A}_2 \cos\bar{\varphi}_2 - i\bar{A}_2 \sin\bar{\varphi}_2) \times \\ \times \frac{\bar{G}_1 \cos\bar{\omega}t + i\bar{G}_1 \sin\bar{\omega}t + \bar{A}_1 \cos\bar{\varphi}_1 - i\bar{A}_1 \sin\bar{\varphi}_1}{\bar{G}_2 \cos\bar{\omega}t + i\bar{G}_2 \sin\bar{\omega}t + \bar{A}_2 \cos\bar{\varphi}_2 - i\bar{A}_2 \sin\bar{\varphi}_2} \Big]. \tag{1.129}$$

In order to guarantee $k_1 = 0$ during operation at steady state, it is necessary to safisfy conditions

$$\left. \begin{aligned} \bar{A}_1 &= k_1 \bar{A}_2, \\ \bar{\omega}_1 &= \bar{\omega}_2, \\ \bar{\varphi}_1 &= \bar{\varphi}_2. \end{aligned} \right\} \tag{1.130}$$

Example 1.

Investigate the character of change of the component ratio of propellant in time at the injector edge during cruise operation of the engine [67].

Given.

Disturbance of liquid flows appears at the exit from the flow areas of centrifugal pump impellers, moreover the rpm of the shaft of the turbopump unit is equal to 470. Each impeller has 6 blades each. Spectrographic research of oscillations showed that the oscillations are close to sinusoidal.

By the oscillograms, obtained during engine test, it has been established that amplitudes in the oxidizer and fuel chains are equal to $A_1 = 0.4$ kg/s and $A_2 = 0.75$ kg/s respectively.

By comparing the oscillograms, on which the character of change of flow rates at pump outlets and nozzle inlets is recorded, we managed to determine the phase shifts: on the oxidizer chain $\phi_1 =$ $= 0.25$ rad and on the fuel chain $\phi_2 = 0.55$ rad.

Solution.

To get the complete idea about the character of change of the component ratio in time and the chamber volume it is necessary to examine a system of three equations.

The first equation should characterize the distribution of flow rates of propellant components along the cross section of the chamber depending on the location of injectors and their characteristics. The second equation should give the possibility of determining the change in the component ratio along the length of the chamber depending on the character of propellant burnout (see 1.7). As the third equation it is necessary to use equation (1.129), allowing the determining of the change of the component ratio with time at the injector edge.

Let us determine the frequency of disturbing oscillations

$$f = \frac{nz}{60} = \frac{470 \cdot 6}{60} = 47 \text{ Hz}.$$

With sinusoidal oscillations [formula (1.129)] let us calculate by the following scheme.

1. Let us assign interval Δt.
2. Let us fix current time t.
3. Let us calculate circular frequency $\omega = 2\pi f$.
4. Let us determine (3)·(2).
5. On the table we find cos (4).
6. On the table we find sin (4).
7. On the table we find $\cos \phi_2$.
8. On the table we find $\sin \phi_2$.
9. $\overline{G}_2 \cdot (5)$.
10. $\overline{G}_2 \cdot (6) \cdot i$.
11. $\overline{A}_2 \cdot (7)$.
12. $\overline{A}_2 \cdot (8) \cdot i$.
13. $(9) + (10)$.
14. $(10) - (12)$.
15. $(3) \cdot (13)$.
16. $(3) \cdot (14)$.
17. $(13)^2$.
18. $(14)^2$.
19. $(17) + (18)$.
20. $(16) : (18)$.
21. $(15) : (19)$.
22. $\cos \varphi_1$.
23. $\cos \psi_2$.
24. $\overline{A}_1 \cdot (22)$.
25. $\overline{A}_1 \cdot (23) \cdot i$.
26. $\overline{G}_1 \cdot (5)$.
27. $\overline{G}_1 \cdot (6)$.
28. $(26) + (24)$.
29. $(27) - (25)$.
30. $(28) \cdot (13)$.
31. $-(28) \cdot (14) \cdot i$.
32. $(29) \cdot (13) \cdot i$.
33. $(29) \cdot (14)$.
34. $(30) + (33)$.
35. $(31) + (32) \cdot i$.
36. $(34) : (19)$.
37. $(35) : (19) \cdot i$.
38. $(36) \cdot (11)$.
39. $-(36) \cdot (12) \cdot i$.
40. $(37) \cdot (11) \cdot i$.
41. $(37) \cdot (12)$.
42. $(38) + (41)$.
43. $(39) + (40) \cdot i$.
44. $(42) + (24)$.
45. $(43) + (25) \cdot i$.
46. $(20) \cdot (44)$.
47. $(20) \cdot (45) \cdot i$.
48. $(21) \cdot (44) \cdot i$.
49. $-(21) \cdot (45)$.
50. $(46) + (49)$.
51. $(47) + (48) \cdot i$.
52. $(50)^2$.
53. $(51)^2$.
54. $(52) + (53)$.

55. We find the modulus, which will characterize the sought value of derivative $\dot{k}_1 = (54)^2$.

We determine the phase, which characterizes the displacement of values $\dot{k}_1$ in comparison with initial values of flow rates, moreover $\phi_{\dot{k}_1} = \text{arc tg}\,[(51):(50)]$.

The given algorithm is used for compiling a program for a digital computer. Thus, for instance, for the accepted values of parameters on an M-20 machine in the initial step of calculation the following results were obtained:

— 00 000000000	+ 03 111299008	+ 01 441836457
— 03 999999999	+ 03 105583680	+ 01 452495143
— 02 200000000	+ 03 104122163	+ 01 462804138
— 02 300000000	+ 03 106698268	+ 01 155151365
— 02 400000000	+ 03 113696824	+ 01 144321547

etc.

In the first lines of each column there is recorded the time, in the second – the value of modulus and in the third – phase.

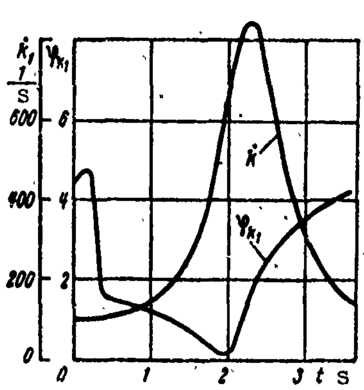


Fig. 1.8. For the example of calculation.

Research and calculations showed that during engine operation k_1 and $\phi_к$ on the injector edge are continuously changed in time, as is seen on Fig. 1.8, constructed from calculation results.

1.10. Characteristic Times of Conversions

The total time the burning propellant stays in the combustion chamber

$$T=\tau_в+t_и+t_х+\varepsilon, \tag{1.131}$$

where $\tau_в$ – time from the moment of entry of liquid propellant into the combustion chamber to the beginning of its vaporization; $t_и$ – time of vaporization; $t_х$ – time of chemical transformations; ε – time of passage of combustion products through the chamber to the critical cross section, called the time of stay of combustion products in the chamber.

The determination of the time of chemical transformations presents considerable difficulties. To this question are dedicated special sections of the course of kinetics of chemical gas reactions [43], [96]. When performing engineering calculations we use approximate relationships.

The reaction rate

$$W_р=-\int_V \frac{dc}{dt}\,dV. \tag{1.132}$$

In the examination of the element of volume the reaction rate

$$\omega=-\frac{dc}{dt}. \tag{1.133}$$

According to the law of mass action the reaction rate of the first power

$$\omega = -kc, \tag{1.134}$$

where k – the reaction rate constant.

After separation of variables and integration for the current concentration we find

$$c = c_0 \exp(-kt_x). \tag{1.135}$$

For the reaction of n-th power

$$\omega = -kc_1^{\nu_1} c_2^{\nu_2} \ldots c_k^{\nu_n}, \tag{1.136}$$

where ν – the stoichiometric coefficient of the reaction.

Equation (1.135) is even suitable for calculation of complex processes, if the calculations are done in the first approximation, and values of c_0 and k are taken on the basis of experimental data [43], [96].

The time of stay of combustion products in the chamber is determined in the following manner.

The gas velocity in the section of the chamber with length l

$$W = \frac{dl}{dt}.$$

By the equation of continuity, considering the cross-sectional area of the chamber variable, we find

$$W=\frac{G}{F(l)\varrho_{\text{к}}}. \qquad (1.137)$$

According to the equation of state

$$\varrho_{\text{к}}=\frac{p_{\text{к}}}{RT_{\text{к}}}.$$

The specific pressure pulse

$$\beta=\frac{p_{\text{к}}}{G}F_{\text{кр}}=\frac{\sqrt{RT_{\text{к}}}}{a}, \qquad (1.138)$$

where

$$a=\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\left(2\frac{n}{n+1}\right)^{0.5}; \qquad (1.139)$$

n – the politropic index. After conversions we find

$$\varepsilon=\frac{1}{F_{\text{кр}}}\int_0^l\frac{F(l)}{a\sqrt{RT_{\text{к}}}}\,dl. \qquad (1.140)$$

Being guided by the average values of a and $RT_{\text{к}}$, we find

$$\varepsilon=\frac{V_{\text{к}}}{aF_{\text{кр}}\sqrt{RT_{\text{к}}}}=\frac{\beta V}{F_{\text{кр}}RT_{\text{к}}}. \qquad (1.141)$$

Formula (1.141) is used for determination of the combustion chamber volume. The value of ε sufficient for complete combustion depends on the properties of components, the carburetion conditions, the components ratio, pressure in the chamber; it is determined experimentally.

Concise information about chain reactions

Chemical reactions are simple and complex. A simple reaction proceeds in one elementary act; the chemical equation of such a reaction is identical to stoichiometric equation. A complex reaction is the totality of acts, which proceed in series or parallel, or both in series and parallel. In a complex chemical reaction intermediate substances are formed, which as a rule are more active than initial. We distinguish ordinary and chain complex reactions.

In an ordinary complex reaction an active particle (active center) produces the intermediate substance, changing into the reaction product.

A chain reaction is distinguished from complex by the fact that here simultaneously with the product of flow rate there appear active particles, i.e., continuous regeneration of active particles occurs. The reaction, started by one active particle, because of the repetition of cycles, caused by reappearing active particles, is not ceased until the disturbance of the sequenc. of cycles, caused by the destruction of active particles. During the study of chain reactions two sources of active particles are considered – an external source and the appearance of particles during the reaction itself.

Since the chain reaction is characterized by the fact that in one link of the chain for every vanished active particle on the average there appears more than one new active particle, such a chain reaction is called branched.

Gas reactions do not conform to the classical laws of chemical kinetics – to the law of mass action and to Arrhenius temperature law. This is understandable, since the mechanism of gas burning in actuality proves to be more complex than follows from ordinary stoichiometric equations.

The chain reaction is characterized by the length of the chain, which is the ratio of the reaction rate to the rate of thermal generation of active particles. This ratio is equal to the number of molecules of the reaction product, which are on every active particle, forming under the action of heat.

Experimental data show that active particles are chemically unsaturated products - free atoms and radicals. Consequently, the initial formation of these products is a necessary condition of the appearance of a chain reaction. They appear under the influence of various actions, including as a result of thermal dissociation of gases, but at temperatures below 1000°K their energy and concentration are usually inadequate for the chain reaction to acquire a noticeable rate. Because of thermal dissociation the nucleation of radicals plays a noticeable role at rather high temperatures, characteristic for the central part of the combustion chamber.

The process of nucleation of active particles as a result of chemical interaction is most often realized when one of the interacting molecules is a molecule containing multiple bonds, as, for instance, a molecule of oxygen, olefin, alde yde, etc.

The initiating action of surfaces plays a large role in the kinetics of chain reactions. The radicals, being mucleated on the surface, can move into the basic volume. The intensity of nucleation of chain reactions depends upon the presence and properties of catalysts. Thus, a heterogeneous reaction on a silver catalyst in a gas volume causes a homogeneous chain reaction.

Breaking of chains -- this is the destruction of active particles; it is caused by chemical processes, which proceed with the participation of atoms and radicals and do not lead to their regeneration. These processes can proceed both in the volume and on the surface.

The nucleation of a chain can occur because of collision with some unsaturated molecule M; in this instance the nucleation of a chain can be illustrated by reaction:

$$H_2+M=2H+M.$$

Here an active particle was formed – atomic hydrogen.

The breaking of the chain of an active particle, caused, for example, by the disappearance of atomic oxygen:

$$O+CO+M=CO_2+M.$$

This reaction characterizes a triple collision and the diappearance of atomic oxygen.

Let us consider for example the reaction of hydrogen burning. The main features of the mechanism of this reaction are also peculiar to the reactions of burning of other gases, therefore the reaction of hydrogen burning is taken as the model reaction.

In the beginning there occurs nucleation of chains with the formation of active particles – radical OH and atomic hydrogen [43]:

$$H_2+O_2=2OH-16\ \text{kcal}; \qquad (a)$$
$$H_2+M=2H+M-104{,}2\ \text{kcal}. \qquad (b)$$

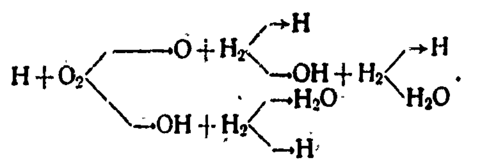
The continuation of the chain is characterized by the appearance of atomic hydrogen:

$$OH+H_2=H_2O+H+13{,}7\ \text{kcal}. \qquad (c)$$

The reactions, terminating with the formation of active particles, correspond to branching of the chain:

$$H+O_2=OH+O-15 \text{ kcal}; \quad (d)$$
$$O+H_2=OH+H-1 \text{ kcal}. \quad (e)$$

Reactions (c), (d), (e), as a consequence of the formation of active particle – atomic hydrogen – can be represented in the form of the following scheme:



(f)

Simultaneously with the generation of active particles there occurs breaking of the chains in the volume and on the wall. To breaking of the chain in the volume corresponds reaction

$$H+O_2+M=HO_2+M+47 \text{ kcal}, \quad (g)$$

and to breaking of the chain on the wall – reaction

$$H \rightarrow {}^1/_2 H_2. \quad (h)$$

In the kinetics of hydrogen burning there participates one additional active particle HO_2, called a low-activity radical. Continuation of the chain with the aid of this low-activity radical is described by equations [43]

$$HO_2+H_2=H_2O+H-15\ \text{kcal}, \qquad (i)$$
$$HO_2+H_2O=H_2O_2+OH-29\ \text{kcal}. \qquad (j)$$

but the breaking of chains on the wall proceeds so:

$$HO_2\rightarrow\left\{\begin{matrix}{}^1/_2\,H_2O_2+{}^1/_2\,O_2\\ {}^1/_2\,H_2O+{}^3/_4\,O_2\end{matrix}\right\}. \qquad (k)$$

Under certain conditions the rate of some reactions is increased according to law $\exp(\phi t)$; after a short interval of time the rate can turn out to be so large that the reaction acquires the character of an explosion. In liquid-propellant rocket engines there are known low-probability explosive reactions.

One of the peculiarities of branched chain reactions is the presence of an induction period, which is manifested in the fact that during a certain interval of time no noticeable increase of pressure is revealed. After this time the pressure begins to noticeably build up.

The speed of the entire process on the whole is determined by the speed of the slowest stage; during hydrogen in the process of branching of the chain the endothermic reaction (d) is limiting.

The interaction between oxygen and carbon monoxide is more complex than the interaction between oxygen and hydrogen. In this instance, if the temperature is above 1000°K, a slow reaction is begun on the walls. The excitation of the reaction through the whole volume is attained by the introduction of small quantities of H_2O or H_2. Here the exciter of the reaction is, apparently, atomic hydrogen, which is formed because of dissociation; the active particles include, as before, atomic oxygen and hydroxyl.

CHAPTER II

COMBUSTION CHAMBER EQUATIONS

In the combustion chamber there occurs a complex mass and energy exchange between the propellant components entering the chamber, the vaporization products and gaseous maxture being formed as a result of burning.

Let us examine the element of volume *abcdefjh*, shown in Fig. 2.1. During the burning of liquid propellant this element will be filled partically with liquid, partially with gas, representing a mixture of vaporization products with products of burning. The separate drops of propellant components, having various sizes, moving in the internal cavity of the chamber, will have axial, radial and tangential velocity components, the quantities of which depend on the location of the considered element and are changed with time. As a result of vaporization of drops or diffusion of liquid into gas the flow rate of liquid, velocity, density and other parameters of gas in the element will be changed with respect to coordinates of the element and will also depend on the location of the element.

The greatest quantity of liquid will be in those elements, which are closer to the injection assembly; in the beginning, where preparation of the propellant for vaporization is observed, the size of drops is practically unchanged. Further, in the area of intensive vaporization, the size of drops is decreased rapidly. If an element is located at a sufficient distance from the injection assembly, its

volume is filled only by gas, the composition of which can continuously change as a result of the reactions proceeding in gaseous phase.

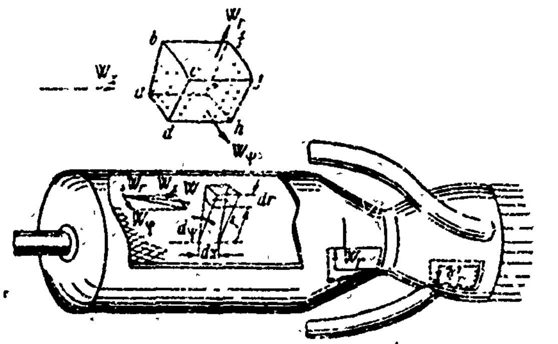


Fig. 2.1. Diagram of the element of burning flow in the internal cavity of the combustion chamber.

During the study of intrachamber processes the calculation equations are written for the whole volume of the chamber, being guided by averaged values of parameters, or for the elementary volume examined above with subsequent integration with respect to the whole volume of the chamber.

2.1. The Basic Equation of the Chamber

By using the law of conservation of mass of a burning flow and considering the entire volume of the combustion chamber on the whole, we write a differential equation – the basic equation of the combustion chamber.

For arbitrary moment of time t into the internal cavity of the chamber proceeded Y kg of propellant (Fig. 2.2). Through the nozzle throat from the chamber flowed $Y_{\text{кр}}$ kg of combustion products.

Consequently, in the chamber there was accumulated $Y_ж$ kg of liquid propellant, not yet heated to the boiling point. $Y_и$ kg of liquid vaporizing components and $Y_к$ kg of vaporization products. In such a formulation of the problem the equation of the law of conservation of mass takes the form

$$Y - Y_ж - Y_и - Y_к - Y_{кр} = 0. \tag{2.1}$$

By differentiating expression (2.1) with respect to t, we obtain

$$\dot{Y} - \dot{Y}_ж - \dot{Y}_и - \dot{Y}_к - \dot{Y}_{кр} = 0. \tag{2.2}$$

Any total quantity Y_i, entering equation (2.1), can be determined by mass flow rate per second (time) G_i, since

$$Y_i = \int_i G_i dt. \tag{2.3}$$

The reading is taken from moment of time τ [48] of the beginning of vaporization (or diffusion) of propellant, being guided by some average value of the delay, i.e., time τ_s of heating of liquid to the start of boiling on the surface; in this case the overall quantity of propellant, which entered the chamber for the considered time, will be

$$Y = \int_{-\tau_{s0}}^{t} G_\Sigma dt, \tag{2.4}$$

where $G_\Sigma = G_1 + G_2$ – the inflow of propellant per second (time) into the combustion chamber; τ_{s0} – delay, which corresponds to initial feed conditions of propellant into the chamber. During time $t = \tau_{s0}$ the liquid propellant is still not converted into gaseous products, pressure in the chamber is not changed and therefore τ_{s0} = const.

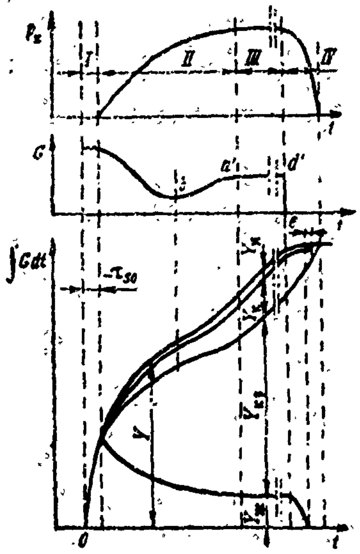


Fig. 2.2. Change of the chamber parameters with time: I - initial period of preparation of propellant; II - starting; III - operating; IV - period of aftereffect.

By differentiating expression (2.4), we find

$$\dot{Y} = G_\Sigma = G_1 + G_2. \qquad (2.5)$$

The quantity of propellant, which is in the chamber in liquid phase, in the vaporization preparation stage,

$$Y_{ж} = \int_{t-\tau_s}^{t} G_\Sigma dt. \qquad (2.6)$$

By differentiating expression (2.6), we find

$$\dot{Y}_{ж} = G_\Sigma - (G_\Sigma)_{-\tau_s} \cdot (1 - \dot{\tau}_s). \qquad (2.7)$$

Here subscript $(-\tau_g)$ shows that the inflow of propellant G_Σ corresponds to time $t - \tau_g$, so that designation $(G_\Sigma)_{-\tau_g}$ is equivalent to entry $G_\Sigma(t - \tau_g)$.

The total quantity of propellant, accumulated in the chamber at the beginning of vaporization,

$$Y_{\text{ж}0}=\int\limits_{-\tau_{g0}}^{0} G_\Sigma dt. \tag{2.8}$$

If we are guided by the average value of propellant inflow, then

$$Y_{\text{ж}0}=G_{\Sigma\,\text{cp}}\tau_{g0}. \tag{2.9}$$

By equations (2.5) and (2.7) we find

$$\dot{Y}-\dot{Y}_{\text{ж}}=(G_\Sigma)_{-\tau_g}(1-\dot{\tau}_g). \tag{2.10}$$

The equations for determination of τ_g were obtained in the first chapter. There are examined the methods of determination of

$$\dot{Y}_{\text{и}}=G_{\text{и}}. \tag{2.11}$$

In the simplest case, if we are guided by one characteristic initial size of drops r_0, then [67]

$$\dot{Y}_{\text{и}}=\frac{2r_0^2}{5\xi_m\psi}\dot{G}_\Sigma, \tag{2.12}$$

or

$$\dot{Y}_{\text{и}}=\chi\dot{G}_\Sigma, \tag{2.13}$$

where

$$\chi=\frac{2r_0^2}{5\varepsilon_m\psi}. \tag{2.14}$$

The quantity of gas in the chamber, consisting of vaporization products and combustion products, is determined by equation of state

$$Y_\kappa=\frac{p_\kappa V_\kappa}{RT_\kappa}, \tag{2.15}$$

where $V_к$ – free volume of the chamber, i.e., the volume not occupied by liquid.

By designating the constant, determined by geometric dimensions (according to a drawing), volume of the chamber through $V_{к0}$, we obtain

$$V_\kappa=V_{\kappa 0}-\frac{Y_ж+Y_н}{\varrho_ж}. \tag{2.16}$$

Having taken the derivative with respect to t and assuming the density of liquid constant, we find

$$\dot V_\kappa=-\frac{1}{\varrho_ж}(\dot Y_ж+\dot Y_н). \tag{2.17}$$

By using equalities (2.7) and (2.13), we arrive at equation

$$\dot V_\kappa=-\frac{1}{\varrho_ж}\left[G_\Sigma-(G_\Sigma)_{-\tau_s}(1-\dot\tau_s)+\chi\dot G_\Sigma\right]. \tag{2.18}$$

Now let us differentiate equality (2.15) with respect to t; taking into account that efficiency $RT_к$ is a variable quantity, depending on the component ratio, we obtain

$$\dot{Y}_{к} = \frac{V_{к}}{RT_{к}}\dot{p}_{к} + \frac{p_{к}}{RT_{к}}\dot{V}_{к} - \frac{p_{к}V_{к}}{(RT_{к})^2}\frac{d}{dt}(RT_{к}). \quad (2.19)$$

For determination of the last term in expression (2.2) let us use equation

$$Y_{кр} = \int_{0}^{t+\varepsilon} G_{кр}\,dt, \quad (2.20)$$

where ε — time from the termination of propellent vaporization to the moment of passage of gaseous products through the nozzle throat.
By differentiating equality (2.20), we find

$$\dot{Y}_{кр} = (G_{кр})_{t+\varepsilon}(\dot{t} + \dot{\varepsilon}). \quad (2.21)$$

If ε = const, then

$$\dot{Y}_{кр} = (G_{кр})_{t+\varepsilon}. \quad (2.22)$$

Flow rate through the nozzle throat

$$G_{кр} = \frac{F_{кр}a}{\sqrt{RT_{к}}}p_{к} = \frac{F_{кр}}{\beta}p_{к}, \quad (2.23)$$

where β — specific pressure pulse; a — function of politropic index n:

$$a = \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\left(2\frac{n}{n+1}\right)^{0.5}. \quad (2.24)$$

Now instead of equation (2.21) we have

$$\dot{Y}_{кр} = F_{кр}\left(\frac{p_к}{\beta}\right)_{+\varepsilon}(1+\dot{\varepsilon}). \qquad (2.25)$$

If ε = const, then

$$\dot{Y}_{кр} = F_{кр}\left(\frac{\dot{p}_к}{\beta}\right)_{+\varepsilon}. \qquad (2.26)$$

Thus derivative $\dot{Y}_{кр}$ corresponds to moment of time $t + \varepsilon$, i.e., relative to the accepted beginning of timing lags behind the period of "transporting" of products from the zone of propellant combustion to the region of the nozzle throat.

Formulas for determination of quantity ε were given in Chapter I. Let us compute its time derivative

$$\dot{\varepsilon} = \frac{1}{F_{кр}}\frac{\partial}{\partial t}\int_0^l \frac{F(l)}{a\sqrt{RT_к}}\,dl. \qquad (2.27)$$

By substituting the obtained values of derivatives, mass time flow rates, in expression (2.2), after conversions we arrive at the basic equation of the combustion chamber:

$$\varepsilon\dot{p}_к + f_1(p_к)\,p_к - \frac{\beta}{F_{кр}} f_2(p_к)\,G_\Sigma = 0, \qquad (2.28)$$

where

$$f_1(p_к) = \varepsilon\frac{\dot{V}_к}{V_к} - \frac{\varepsilon}{RT_к}\frac{d}{dt}(RT_к) + \frac{\beta}{p_к}\left(\frac{p_к}{\beta}\right)_{+\varepsilon}(1+\dot{\varepsilon}); \qquad (2.29)$$

$$f_2(p_к) = \frac{(G_t)_{-\tau_s}}{G_\Sigma}(1-\dot{\tau}_s) - \chi\frac{\dot{G}_\Sigma}{G_\Sigma}. \qquad (2.30)$$

By the results of processing of experimental data it is possible to determine the law of vaporization of liquid propellant components and to construct so-called burnout curve.

The buildup of the quantity of vaporization products occurs immediately after the arrival of components at the internal cavity of the chamber. However, the intensity of vaporization is sharply changed with time.

The formation of components is characterized by an equation, which in general form is written so:

$$G_{0-t}=G_{\Sigma}(0)\varphi(t), \tag{2.31}$$

where $G_{\Sigma}(0)$ – time inflow of propellant into the chamber; $\phi(t)$ – vaporization characteristic, determined from experiment.

For the purpose of simplification of the entry we will again designate term $G_{\Sigma}(t)$ through G_{Σ}. As a result of the vaporization of the entire mass of propellant the inflow of gaseous products will be

$$G=\int_0^{t_0} G_{\Sigma}\varphi(t)\,dt, \tag{2.32}$$

where t_0 – the total time of vaporization.

Inasmuch as the numerical values, obtained with integration of expressions (2.32) from t_0 to ∞ are small, this integration when performing approximate calculations can be from zero to infinity; thus

$$G=\int_0^{\infty} G_{\Sigma}\varphi(t)\,dt. \tag{2.33}$$

Now instead of equation (2.28) we will have

$$\varepsilon \dot{p}_{\text{к}} + f_1(p_{\text{к}}) p_{\text{к}} - \frac{\beta}{F_{\text{кр}}} \int_0^\infty f_2(p_{\text{к}}) G_\Sigma \varphi(t) dt = 0. \quad (2.34)$$

Let us return to equations (2.28), (2.29) and (2.30). The first term in equation (2.28) is caused by change of pressure in the chamber. The change of free volume is considered by term

$$\varepsilon \frac{\dot{V}_{\text{к}}}{V_{\text{к}}} p_{\text{к}}.$$

With change of pressure in the chamber the intensity of heating of liquid components is changed; as a consequence of this - the rate of warming and vaporization of liquid propellant is changed, the absolute value of the free volume of the chamber is changed. The second term of the right side of equation (2.29) reflects the effect of change of the efficiency of combustion products. The product of $RT_{\text{к}}$ at prescribed properties of propellant components depends basically on the component ratio and on the combustion efficiency of propellant.

Let us note that with consideration of the peculiarities of the interconnection between the combustion chamber and the feed system the component ratio, as will be shown below, depends on the pressure in the chamber.

The last component, which enters equation (2.29), carries a correction, caused by the time of movement of combustion products along the axis of the chamber.

Propellant is introduced into the chamber with delay, which in equation (2.28) together with equation (2.30) is considered by variable

$$\frac{\beta}{F_{\text{кр}}} (G_\Sigma)_{-\tau_3}.$$

Inasmuch as $\tau_{\text{в}} = \tau_{\text{в}}(p_{\text{к}})$, and $p_{\text{к}}$ changes with time, then during the study of intrachamber processes, as follows from the obtained equations, derivative $\dot{\tau}_{\text{в}}$ is considered, i.e., the effect of pressure in the chamber on the amount of delay.

The last term of equation (2.30) characterizes the effect of the rate of change of the propellant feed under conditions of gas formation.

During calculation of an engine it is frequently necessary to solve problems in a simpler formulation. Thus, for instance, if propellant is fed to the chamber in gaseous state, then $\tau_{\text{в}} = 0$.

With change of pressure within small limits or in cases when low-frequency oscillations are not considered, it is sometimes possible to consider $\tau_{\text{в}}$ = const and, consequently, $\dot{\tau}_{\text{в}} = 0$.

If $\tau_{\text{в}}$ = const the vaporization rate is constant, then derivative $\dot{V} = 0$.

In some engines in steady state there is provided a constant value of component ratio. In this case $RT_{\text{к}}$ = const and, consequently, $\frac{d}{dt}(RT_{\text{к}}) = 0$.

When conducting approximate calculations we take time of transportation $\varepsilon = 0$; then $\dot{\varepsilon} = 0$ and the last component in equation (2.29) becomes one, i.e.,

$$\frac{\beta}{p_{\text{к}}}\left(\frac{p_{\text{к}}}{\beta}\right)_{+\varepsilon}(1+\dot{\varepsilon})=1. \qquad (2.35)$$

During the study of initial period of operation of an engine it is necessary to compute the quantity of propellant, which is collected in the chamber before the start of ignition. Inasmuch as during this period the initial pressure $p_{\text{к}0}$ does not change, then

τ_{s0} = const. Before the start or ignition in the internal cavity of the chamber there is propellant, the quantity of which will be equal to

$$Y_{ж} = \int_{-\tau_{s0}}^{0} G_{\Sigma} dt - Y'_{ж}, \tag{2.36}$$

where $Y_{ж}'$ – the quantity of propellant, which fell in the nozzle throat region and flowed from the chamber in liquid state during time τ_{s0}. If G_{Σ} = const and $Y_{ж}'$ = 0, then

$$Y_{ж} = G_{\Sigma}\tau_{s0}. \tag{2.37}$$

After ignition of propellant an increase of pressure in the chamber is begun, in this case almost all the engine parameters are changed.

As a result of the increase in pressure period τ_s is decreased and the process of vaporization is accelerated. In this case the quantity of propellant, which is in the combustion chamber in liquid state, is decreased. Gaseous products are formed not only as a result of vaporization of components, entering from the feed system, but also under the action of additional feeding, caused by decrease of period τ_s. While pressure in the chamber grew from $p_{к0}$ to $p_{к}$, the delay was decreased from τ_{s0} to τ_s. As a result of only additional feeding in the internal volume of the chamber gaseous products are formed to the extent of

$$Y_{п} = \int_{\tau_s}^{\tau_{s0}} G_{\Sigma} dt. \tag{2.38}$$

In order to get an idea of the change of pressure in the chamber with time, it is necessary to solve an equation in the form of (2.28) or (2.34). For this it is necessary to assign $G_\Sigma = G_\Sigma(t)$ or additionally draw on equations of the feed system, derivation of which will be given below.

Let us examine the approximate method of construction of graph $p_к(t)$ during engine starting. Without taking into account the delay, the equation of the chamber is written so:

$$\varepsilon \dot{p}_к + p_к - \frac{\beta}{F_{кр}} G_\Sigma = 0. \tag{2.39}$$

If we consider a rather general case, when $\varepsilon(t)$ and $G_\Sigma(t)$, then the solution of linear differential equation (2.39) takes the form

$$p_к = C\exp\left[-\int\frac{dt}{\varepsilon(t)}\right] + \exp\left[-\int\frac{dt}{\varepsilon(t)}\right]\frac{p_{ном}}{G_\Sigma}\int\frac{G_\Sigma(t)}{\varepsilon(t)}\exp\left[\int\frac{dt}{\varepsilon(t)}\right]dt, \tag{2.40}$$

where C – integration constant; $p_{ном}$ – nominal pressure in the chamber.

Assuming that the flow rate of propellant is changed in terms of the exponent, then

$$p_к = C\exp\left[-\int\frac{dt}{\varepsilon(t)}\right] + \exp\left[-\int\frac{dt}{\varepsilon(t)}\right] p_{ном} \times \int\frac{1-\exp\left(-\frac{t}{\varepsilon}\right)}{\varepsilon(t)}\exp\left[\int\frac{dt}{\varepsilon(t)}\right]dt. \tag{2.41}$$

If we additionally assume ε = const, then

$$p_{\text{к}}=C\exp\left(-\frac{t}{\varepsilon}\right)+\exp\left(-\frac{t}{\varepsilon}\right)\frac{p_{\text{ном}}}{\varepsilon}\left[\varepsilon\exp\left(\frac{t}{\varepsilon}\right)-\frac{\exp\left(\frac{t}{\varepsilon}\right)\exp\left(-\frac{t}{\alpha}\right)}{\frac{1}{\varepsilon}-\frac{1}{\alpha}}\right]. \tag{2.42}$$

or

$$p_{\text{к}}=C\exp\left(-\frac{t}{\varepsilon}\right)+p_{\text{ном}}\left[1-\frac{\exp\left(-\frac{t}{\alpha}\right)}{1-\frac{\varepsilon}{\alpha}}\right]. \tag{2.43}$$

For determination of integration constant let us assume that when $t = 0$ the initial pressure $p_{\text{к}0} = 0$. By equation (2.43) we find

$$C=p_{\text{ном}}\frac{\varepsilon}{\alpha-\varepsilon}. \tag{2.44}$$

Having substituted the obtained value of integration constant in expression (2.43), after conversions we will have

$$p_{\text{к}}=p_{\text{ном}}\left[\frac{\varepsilon}{\alpha-\varepsilon}\exp\left(-\frac{t}{\varepsilon}\right)+1-\frac{\alpha}{\alpha-\varepsilon}\exp\left(-\frac{t}{\alpha}\right)\right]. \tag{2.45}$$

To constant flow rate corresponds $\alpha = 0$. In this case from expression (2.45) we find

$$p_{\text{к}}=p_{\text{ном}}\left[1-\exp\left(-\frac{t}{\varepsilon}\right)\right]. \tag{2.46}$$

Let us examine one more approximate solution, which is sometimes used. Let us assume that propellant flow rate

$$G=A\sqrt{p_1-p_к}, \qquad (2.47)$$

where A – constant, obtained by calculation or from processing experimental data; p_1 – characteristic pressure, as which it is possible to take the pressure before the injectors or after the pump, or pressure in the tank.

Generally $p_1 = f(t)$ and therefore it is better to calculate with respect to intervals of time. Having taken p_1 = const, T_1 = = const for each interval and having assumed $\tau_в = 0$, we arrive at the equation of the chamber in the form

$$\frac{V}{RT_к}\dot{p}_к+\frac{aF_{кр}}{\sqrt{RT_к}}p_к-A\sqrt{p_1-p_к}=0. \qquad (2.48)$$

After separation of variables for interval of time Δt we find

$$\Delta t=\frac{V}{RT_к}\int\limits_{p_{к1}}^{p_{к2}}\frac{dp_к}{A\sqrt{p_1-p_к}-\frac{aF_{кр}}{\sqrt{RT_к}}p_к}, \qquad (2.49)$$

where $p_{к2} - p_{к1}$ is the change of pressure in interval of time Δt.

If we assume

$$p_1-p_к=X, \qquad (2.50)$$

then expression (2.49) is reduced to a tabular integral.

If the propellant flow rate changes according to a more complex law than was accepted, one should use equation (2.40).

During calculation of steady state in the equation of the combustion chamber all derivatives equal zero. Furthermore, it is no longer necessary to consider the delay; the calculation equation, called the equation of statics of the combustion chamber, assumes the form

$$p_{\text{к}} - \frac{\beta}{F_{\text{кр}}} G_{\Sigma} = 0. \qquad (2.51)$$

This equation is well known. It is used for determination of the nozzle throat area, if the pressure in the chamber is selected, the specific pressure pulse is calculated and the flow rate of propellant is determined according to prescribed thrust. Equation (2.51) is used during analysis of the interconnection of small deviations of combustion chamber parameters. The expression of total differential from equation (2.51) will be written so:

$$dp_{\text{к}} = \frac{\partial p_{\text{к}}}{\partial \beta} d\beta + \frac{\partial p_{\text{к}}}{\partial F_{\text{кр}}} dF_{\text{кр}} + \frac{\partial p_{\text{к}}}{\partial G_{\Sigma}} dG_{\Sigma}. \qquad (2.52)$$

By changing to small finite deviations, i.e., by replacing infinitely small deviations $dp_{\text{к}}$, $d\beta$, ..., by small finite deviations $\Delta p_{\text{к}}$, $\Delta\beta$, ..., we obtain

$$\Delta p_{\text{к}} = \frac{\partial p_{\text{к}}}{\partial \beta} \Delta\beta + \frac{\partial p_{\text{к}}}{\partial F_{\text{кр}}} \Delta F_{\text{кр}} + \frac{\partial p_{\text{к}}}{\partial G_{\Sigma}} \Delta G_{\Sigma}. \qquad (2.53)$$

Small deviations are computed from nominal (or calculated) value of parameters, therefore partial derivatives, standing before them in the form of factors, are constants. Symbol $\partial p_{\text{к}}/\partial\beta$ is conveniently supplemented by the condition, which is written so:

$$\left(\frac{\partial p_{\text{к}}}{\partial \beta}\right)_{\substack{p_{\text{к}}=p_{\text{к.ном}};\\ \beta=\beta_{\text{ном}};\\ F_{\text{кр}}=F_{\text{кр.ном}};\\ G_{\Sigma}=G_{\Sigma\,\text{ном}}.}}$$

The above means that derivative $p_{\text{к}}$ with respect to β should be calculated in the region of nominal or calculated values of parameters $p_{\text{к.ном}}$, $\beta_{\text{ном}}$, etc. The complex multiline subscript for the sign of partial derivative is replaced for the sake of simplicity of writing by the mark $_*$.

For further simplification of writing we will designate the numerical values of partial derivative through a_i. For the considered case the calculation equation will be written so:

$$\Delta p_{\text{к}}=a_1\Delta\beta+a_2\Delta F_{\text{кр}}+a_3\Delta G_{\Sigma}, \tag{2.54}$$

moreover

$$a_1=\left(\frac{\partial p_{\text{к}}}{\partial \beta}\right)_*=\left(\frac{G_{\Sigma}}{F_{\text{кр}}}\right)_*, \tag{2.55}$$

$$a_2=\left(\frac{\partial p_{\text{к}}}{\partial F_{\text{кр}}}\right)_*=-\left(\frac{p_{\text{к}}}{F_{\text{кр}}}\right)_*, \tag{2.56}$$

$$a_3=\left(\frac{\partial p_{\text{к}}}{\partial G_{\Sigma}}\right)_*=\left(\frac{p_{\text{к}}}{G_{\Sigma}}\right)_*. \tag{2.57}$$

Inasmuch as deviations of pressure in the chamber and the flow rate of propellant from their nominal values under conditions of low-frequency oscillations are small, during research of stability equations (2.28) and (2.34) are sometimes written in linearized form. Let us examine equation (2.28), having accepted $f_1(p_{\text{к}}) = f_2(p_{\text{к}}) = 1$; the new initial equation is written so:

$$\tau\dot{p}_{\text{к}}+p_{\text{к}}-\frac{\beta}{F_{\text{кр}}}(G_{\Sigma})_{t-\tau_s}=0. \tag{2.58}$$

Let us designate

$$\Delta p_\kappa = p_\kappa - p_{\kappa.\text{ном}}; \qquad (2.59)$$

$$\Delta (G_\Sigma)_{-\tau_s} = (G_\Sigma)_{-\tau_s} - G_{\Sigma\,\text{ном}}. \qquad (2.60)$$

Consequently,

$$\Delta \dot{p}_\kappa = \dot{p}_\kappa. \qquad (2.61)$$

Inasmuch as $G_{\Sigma_{\text{ном}}}$ = const, then $(G_{\Sigma_{\text{ном}}})_{-\tau_s} = G_{\Sigma_{\text{ном}}}$. Now under the condition that ε = const and β = const and that they are calculated at nominal values of $p_{\kappa.\text{ном}}$ and $G_{\Sigma_{\text{ном}}}$, we obtain

$$\varepsilon \Delta \dot{p}_\kappa + \Delta p_\kappa - \frac{\beta}{F_{\kappa p}} \Delta (G_\Sigma)_{-\tau_s} = 0. \qquad (2.62)$$

Sometimes it is convenient to calculate in relative quantities

$$\widetilde{\Delta p}_\kappa = \frac{p_\kappa - p_{\kappa.\text{ном}}}{p_{\kappa.\text{ном}}}; \qquad (2.63)$$

$$\widetilde{\Delta (G_\Sigma)}_{-\tau_s} = \frac{(G_\Sigma)_{-\tau_s} - G_{\Sigma\,\text{ном}}}{G_{\Sigma\,\text{ном}}}. \qquad (2.64)$$

The equation of the chamber is written in the following manner:

$$\varepsilon \Delta \dot{\tilde{p}}_\kappa + \Delta \tilde{p}_\kappa - \Delta (\tilde{G}_\Sigma)_{-\tau_s} = 0. \qquad (2.65)$$

In such form the equation is used during the study of questions of combustion chamber stability.

2.2. The Equation of Continuity for a Two-Phase Burning Flow

For derivation of an equation let us use the law of conservation of mass.

Considering the contemporary shapes of chambers, it is convenient to write the equation in a cylindrical coordinate system.

The area of element *abcd* (see Fig. 2.1) is equal to

$$dF_x = dr\,r\,d\varphi.$$

The cross section, which coincides with area *abcd*, is intersected by the burning flow, consisting of gases and liquid particles, moreover a certain total area

$$F_x = F_{\text{ж}x} + F_{\text{г}x}, \tag{2.66}$$

where $F_{\text{ж}x}$ – the area intersected by the flow of liquid; $F_{\text{г}x}$ – the area intersected by the flow of gas. By differentiating expression (2.66), for the element we find

$$dF_x = dF_{\text{ж}x} + dF_{\text{г}x}.$$

The relative area, occupied by liquid

$$S_x = \frac{dF_{\text{ж}x}}{dF_x}. \tag{2.67}$$

Consequently, the area, occupied by liquid in section *abcd*, and the area, intersected by gas in the same section, will be respectively

$$dF_{\text{ж}x}=S_x dr\cdot rd\varphi;\ dF_{\text{г}x}=(1-S_x)\,dr\cdot rd\varphi. \qquad (2.68)$$

With the absence of liquid $S_x = 0$; if the entire area of the section is intersected only by the flow of liquid, then $S_x = 1$.

During the examination of the real process, in which there occurs motion of the burning flow, containing drops of finite size, one should be guided by the finite area of an element, equal to $\Delta r\cdot r\Delta\varphi$. However, the transition to calculation in terms of elementary areas provides increase of accuracy of calculation and, naturally, does not introduce errors; by designating the integration limits of equations, it is necessary to consider the sizes of particles of burning flow.

The time (per second) mass flow rate of gas through the area of the element will be determined by equation of continuity:

$$d\dot{m}_x=\varrho W_x dF_{\text{г}x}=\varrho W_x(1-S_x)\,dr\cdot rd\varphi. \qquad (2.69)$$

On the element of path dx the change of flow rate of gas, moving through the element in the direction of axis x, will be

$$d\dot{m}_x-d\dot{m}_{x+dx}=\frac{\partial}{\partial x}(d\dot{m}_x)\,dx=$$
$$=\frac{\partial}{\partial x}[\varrho W_x(1-S_x)\,dr\cdot rd\varphi]\,dx. \qquad (2.70)$$

Quantity $dr\cdot rd\varphi$ is not a function of x. Area $S_x dr\cdot rd\varphi$, occupied by liquid, as a result of vaporization of liquid is decreased in the direction of axis x, consequently, S_x is a variable quantity, and therefore equation (2.70) can be rewritten so:

$$d\dot{m}_x-d\dot{m}_{x+dx}=\left[\frac{\partial}{\partial x}(\varrho W_x)-\frac{\partial}{\partial x}(\varrho W_x S_x)\right]dx\cdot dr\cdot rd\varphi. \qquad (2.71)$$

The first term of the right side of equation (2.71) characterizes the change of mass flow rate of gas, which flows through the element, the cross sections of which are not constrained by the presence of liquid. The second – decrease of the considered difference of mass flow rate of gas, caused by constraint of the flow passage cross-sectional area of the element; because of the vaporization of liquid components the constraint of flows by liquid is unequal along the length of the element. In section *abcd* it is determined by quantity $dF_{жx}$ [see formula (2.67)], and in section *efjh* – by quantity

$$dF_{ж(x+dx)}=\left(S_x=\frac{\partial S_x}{\partial x}dx\right)dr\cdot rd\varphi. \qquad (2.72)$$

In the zone of preparation of propellant for burning, where there is still no vaporization of components, S_x = const and instead of the second item in the right side of (2.72) we will have

$$d\dot{m}_{x+dx}=S_x\frac{\partial}{\partial x}(\varrho W_x)\,dxdr\cdot rd\varphi. \qquad (2.73)$$

If we examine the motion of gas only in the direction of the axis of the chamber, i.e., along axis x, then the problem is called one-dimensional.

The radial component of velocity W_r appears under the action of geometric factors, and also radial components of the velocity of liquid at the injector exit, as a result of collisions between drops, because of flow turbulence and the presence of countercurrents.

The elementary mass flow rate of gas through side *aehd* will be equal to

$$d\dot{m}_r=\varrho W_r(1-S_r)\,dx\cdot rd\varphi. \qquad (2.74)$$

On the element of path dr the change of flow rate of gas, moving in the direction of axis r, will be

$$d\dot{m}_r-d\dot{m}_{r+dr}=\left[\frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r r)-\frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r S_r r)\right]dxdr\cdot rd\varphi. \qquad (2.75)$$

The tangential component of velocity W_ϕ is caused basically by the vortex motion of propellant components at the injector exit.

On the element of path $rd\phi$ the change of flow rate of gas, moving in direction W_ϕ, will be equal to

$$d\dot{m}_\varphi - d\dot{m}_{\varphi+d\varphi} = \left[\frac{\partial}{r\partial\varphi}(\varrho W_\varphi) - \frac{\partial}{r\partial\varphi}(\varrho W_\varphi S_\varphi)\right] dx dr \cdot r d\varphi. \qquad (2.76)$$

Let us note that product

$$dx dr \cdot r d\varphi = dV, \qquad (2.77)$$

i.e., equal to the volume of the examined element.

The total change of flow rate of gas in the element will comprise

$$\left[\frac{\partial}{\partial x}(\varrho W_x) + \frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r r) + \frac{\partial}{r\partial\varphi}(\varrho W_\varphi)\right] dV - \left[\frac{\partial}{\partial x}(\varrho W_x S_x) + \frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r S_r r) + \frac{1}{r}\frac{\partial}{\partial\varphi}(\varrho W_\varphi S_\varphi)\right] dV = [\operatorname{div}(\varrho\overline{W}) - \operatorname{div}(\varrho\overline{W}S)]\, dV. \qquad (2.78)$$

In the considerations given above we did not consider the feed of gas to the element as a result of vaporization (or diffusion) of liquid.

Let us examine the relationship, which characterize the transition of liquid phase into gaseous inside the element.

The propellant flow through a ring with width dr, chosen mentally inside the chamber, located perpendicular to axis x, will be

$$dG_x = \frac{\partial}{\partial r} G_x dr, \qquad (2.79)$$

where G_x - mass flow rate per second through the section, which coincides with side $abcd$.

If on the considered ring with angle $d\phi$ we separate the elementary sector, then the flow rate through this elementary sector will be

$$d^2G_x=\frac{\partial^2}{\partial r\cdot r\partial\varphi}G_x dr\cdot rd\varphi, \tag{2.80}$$

The feed of gaseous products on path dx will comprise

$$d^3G_x=\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial r\cdot r\partial\varphi}G_x dr\cdot rd\varphi\right)dx=\frac{\partial^3 G_x}{\partial x\partial r\cdot r\partial\varphi}dV. \tag{2.81}$$

Let us note that as a result of the uneven distribution of propellant along the cross section of the chamber, the change of cross-sectional area of the chamber along the length and burnout of liquid components the flow rate G_x depends on coordinates r and ϕ, i.e., $G_x(r, \phi)$.

If we similarly examine the flows, directed along other axes, then

$$d^3G_r=\frac{1}{r}\frac{\partial}{\partial r}\frac{r\partial^2 G_r}{\partial x\cdot r\partial\varphi}dV, \tag{2.82}$$

$$d^3G_\varphi=\frac{1}{r}\frac{\partial}{\partial\varphi}\frac{\partial^2 G_\varphi}{\partial x\partial r}dV. \tag{2.83}$$

In formulas (2.81), (2.82) and (2.83) some derivatives determine the elementary flow rate, and others characterize the vaporization of liquid propellant. Derivatives with respect to x in expression (2.81) characterize vaporization, and derivatives with respect to r and ϕ - elementary flow rate. In equations (2.82) and (2.83) vaporization is characterized by derivative for r and derivative for ϕ respectively.

The vaporization process of liquid, which is in the element, can be local, i.e., proceed with time. The volume of liquid in the element

$$dv = \chi dV, \tag{2.84}$$

where χ - relative volume of liquid.

The local change of liquid mass in the element

$$\frac{\partial}{\partial t}\varrho_{\text{ж}} dv = \frac{\partial}{\partial t}(\varrho_{\text{ж}}\chi)\, dV. \tag{2.85}$$

The total feed of gas in the element, caused by propellant evaporation, will be

$$\Omega dV = \left[\frac{\partial}{\partial t}(\varrho_{\text{ж}}\chi) - \frac{\partial}{\partial x}\,\frac{\partial^2 G_x}{\partial r\cdot r\partial\varphi} + \frac{1}{r}\,\frac{\partial}{\partial r}\,\frac{r\partial^2 G_r}{\partial x\cdot r\partial\varphi} + \right.$$

$$\left. + \frac{1}{r}\,\frac{\partial}{\partial\varphi}\,\frac{\partial^2 G_\varphi}{\partial x\,\partial r}\right] dV. \tag{2.86}$$

By using the derived relationships, let us write the equation of burnout curve, which is the ratio of formed gaseous products to the initial quantity of liquid propellant. Being guided by the flow rate per second in a one-dimensional formulation, let us write the flow rate per second of liquid, passing along the whole section:

$$G_{x\text{ж}} = \int_0^{r_{\text{к}}}\int_0^{2\pi} \frac{\partial^2}{\partial r\cdot r\partial\varphi}\, G_x dr\cdot r d\varphi; \tag{2.87}$$

where $r_{\text{к}}$ - the radius of the chamber. The quantity of gaseous products formed on path x, will be equal to

$$G_{x\text{г}} = \int_0^{r_{\text{к}}}\int_0^{2\pi}\int_0^{x} \frac{\partial}{\partial x}\,\frac{\partial^2}{\partial r\cdot r\partial\varphi}\, G_x dr\cdot r d\varphi dx. \tag{2.88}$$

Thus, the one-dimensional equation of the burnout curve takes the form

$$\varphi(x) = \frac{\int_0^{2\pi}\int_0^{r_{\text{к}}}\int_0^{x} \frac{\partial}{\partial x}\,\frac{\partial^2}{\partial r\cdot r\partial\varphi}\, G_x dr\cdot r d\varphi dx}{\int_0^{2\pi}\int_0^{r_{\text{к}}} \frac{\partial^2}{\partial r\cdot r\partial\varphi}\, G_x dr\cdot r d\varphi}. \tag{2.89}$$

If we take into account the local change of mass of liquid and its burnout in all directions, then

$$\varphi_2(V)=\frac{\int\limits_V \Omega dV}{\int\limits_0^{2\pi}\int\limits_0^{r_\kappa}\frac{\partial^2}{\partial r\cdot r\partial\varphi}G_x dr\cdot rd\varphi+\int\limits_0^{2\pi}\int\limits_0^{r_\kappa}\frac{\partial^2}{\partial x\cdot r\partial\varphi}G_r dx\cdot rd\varphi+\int\limits_0^{r_\kappa}\int\limits_0^{x}\frac{\partial^2}{\partial r\partial x}G_\varphi dr dx}. \quad (2.90)$$

Equation (2.90) characterizes the burnout of mass. Sometimes it proves to be convenient to write the equation of burnout curve, while considering the relative energy, which is released during combustion, in this case we are guided by the equation of law of conservation of energy of the burning propellant. During experimental construction of the burnout curve as the initial parameter we frequently take the pressure in the chamber. In order to obtain the burnout curve, into the operating combustion chamber there is fed a portion of propellant in steps. In the beginning this portion is preheated and is vaporized which leads to some lowering of pressure. At the moment of combustion of the injected portion of propellant the pressure in the chamber increases, and then it lowered to the previous value.

Let us find the expressions for determination of the relative volume of liquid in the element. By the equation of continuity we determine the value of relative area, occupied by liquid in the direction of axis x:

$$S_x=\frac{1}{\varrho_{\kappa} C_x}\cdot\frac{\partial^2 G_x}{\partial r\cdot r\partial\varphi}, \quad (2.91)$$

where C_x - the projection of the velocity of motion of liquid particles to axis x. For simplicity of writing we will subsequently designate

$$\frac{\partial^2 G_x}{\partial r\cdot r\partial\varphi}=q_x. \quad (2.92)$$

The expressions, analogous to equalities (2.91) and (2.92), are easy to write for other directions.

If in the direction of each of the axes the values of relative areas are not changed, then for determination of the relative volume of liquid we find

$$\chi=(S_x S_r S_\varphi)^{0.5}, \tag{2.93}$$

or

$$\chi=\frac{\left(\frac{q_x}{C_x}\,\frac{q_r}{C_r}\,\frac{q_\varphi}{C_\varphi}\right)^{0.5}}{\varrho_ж^{3/2}}. \tag{2.94}$$

With change of relative area its average value in the direction of axis x for any selected volume will comprise

$$S_{x\,cp}=\iint_F S_x dr\cdot rd\varphi-\frac{1}{2}\iiint_V \frac{\partial S_x}{\partial x}\,dx\,dr\cdot rd\varphi. \tag{2.95}$$

In this instance the relative volume

$$\chi=(S_{x\,cp}S_{r\,cp}S_{\varphi\,cp})^{0.5}. \tag{2.96}$$

Let us continue the derivation of equation of continuity. Let us write an expression for determination of the change of quantity of gas in the element with time. According to the law of conservation of mass the time change of density and relative volume per unit of time will be

$$d\dot{m}_t-d\dot{m}_{t+dt}=\frac{\partial}{\partial t}[\varrho(1-\chi)]\,dV. \tag{2.97}$$

By using expressions (2.78), (2.86) and (2.97), we arrive at the the equation of continuity for the element:

$$\frac{\partial}{\partial t}[\varrho(1-\chi)]+\operatorname{div}(\varrho\overline{W})-\operatorname{div}(\varrho\overline{W}S)-\Omega=0. \tag{2.98}$$

For the whole volume of the chamber we have

$$\iiint\limits_V \left\{\frac{\partial}{\partial t}[\varrho(1-\chi)] + \operatorname{div}(\varrho\overline{W}) - \operatorname{div}(\varrho\overline{W}S) - \Omega\right\} dV = 0. \qquad (2.99)$$

Equations of type (2.98) and (2.99) are convenient to use during calculation of gas-liquid versions of combustion chambers, when one of the components is fed in gaseous, and the other – in liquid form.

Thus, the change of density and volume of gas in the element per unit of time plus the change of mass of gas, which flows through the element, are equal to the mass of gases, which are formed during the same time as a result of vaporization of liquid propellant, which is in the element.

If it is necessary to take into account the specific character of bipropellant, then for each component S_i, χ_i and Ω_i must be known. In this case equation (2.98) takes the form

$$\frac{\partial}{\partial t}[\varrho(1-\chi_1-\chi_2)] + \operatorname{div}(\varrho\overline{W}) - \operatorname{div}[\varrho W(S_1+S_2)] - \Omega_1 - \Omega_2 = 0. \qquad (2.100)$$

A peculiarity of equations of the law of conservation of mass for a burning flow is the presence of functions, which characterize and reflect the constraint of gas flow by liquid, and functions, which determine the feed of gas due to vaporization of components.

Let us examine two extreme cases.

Let us assume that in the area adjacent to the injector assembly, the vaporization of liquid can be disregarded. If there only the preparation of propellant for vaporization occurs, then the relative volume of liquid phase χ is constant. Inasmuch as there is no vaporization (or diffusion), then S = const. Inasmuch as, finally, there is no mass transfer between liquid and gas, then $\Omega = 0$.

After the termination of burning, in the area where liquid phase is absent $\chi = 0$, $S = 0$ and $\Omega = 0$.

Thus, the equation of continuity assumes the form

$$\frac{\partial}{\partial t}\varrho + \operatorname{div}(\varrho\overline{W}) = 0. \qquad (2.101)$$

2.3. Equations of Motion of Burning Flow

Let us examine the previous element of burning flow and the conditions of motion of gaseous and liquid component. The mass of gaseous phase in the element

$$m_{\text{г}} = \int\limits_V \varrho(1-\chi)\,dV. \qquad (2.102)$$

For this mass the law of change of the quantity of motion in the projection to axis x will give

$$P_x = \frac{d}{dt}\int\limits_V \varrho(1-\chi)W_x\,dV. \qquad (2.103)$$

Equation (2.103) considers the change of mass $\varrho(1-\chi)$ and the change of velocity W_x. Integration is performed depending on the formulation of the problem either within the element, or within the selected volume of the chamber. For the entire chamber the integral in expression (2.103) will be written so:

$$\int\limits_{V_0} \varrho(1-\chi)W_x\,dV;$$

for a certain volume, limited by dimensions r and l,

$$\int\limits_0^{2\pi}\int\limits_0^{r}\int\limits_0^{l} \varrho(1-\chi)W_x\,dV;$$

for boundary layer with thickness $r_0 - r$ on section $l_0 - l$

$$\int_0^{2\pi}\int_r^{r_0}\int_l^{l_0} \varrho(1-\chi)W_x dV.$$

The forces, which move the gas flow, are made up of mass (volumetric) and surface. The mass forces appear as a result of the action of external forces. If a rocket is in flight, then for determination of the projection of mass force to axis x we will have

$$F_x=\int_V \varrho(1-\chi)(j_x+g_x)dV, \qquad (2.104)$$

where g - acceleration of gravity; j - acceleration of rocket flight; g_x; j_x - their projections to axis x.

As a result of the occurrence of the process of gas formation and expansion of the products of burning there appears surface force, the projection of which to axis x will be

$$P_{px}=\int_V (1-S_x)\frac{\partial p}{\partial x} dV. \qquad (2.105)$$

The projection of friction forces to the same axis will be

$$\tau_x=\int_V \varrho(1-\chi)\nu\left(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\right)dV, \qquad (2.106)$$

where ν - the coefficient of kinematic viscosity;

$$\nabla^2W_x=\frac{\partial^2W_x}{\partial x^2}+\frac{\partial^2W_x}{\partial r^2}+\frac{1}{r}\frac{\partial W_x}{\partial r}+\frac{1}{r^2}\frac{\partial^2W_x}{\partial\varphi^2}.$$

Item $\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}$ considers the compressibility of liquid, moreover

$$\operatorname{div}\overline{W}=\frac{\partial W_x}{\partial x}+\frac{\partial W_r}{\partial r}+\frac{1}{r}\frac{\partial W_\varphi}{\partial\varphi}. \qquad (2.107)$$

As a result of interaction between the drops of liquid and gas aerodynamic force appears. Its projection to axis x in the examination of one drop will be equal to

$$f_{1x}=\frac{1}{2}\varrho c_x \sigma_{1x}(W_x-C_x)^2. \qquad (2.108)$$

Coefficient c_x is determined by conditions of motion, it depends basically on the form of the drop, velocities W_x and C_x, parameters of propellant components and gas; for prescribed burning conditions it is considered depending on the Reynolds number.

The area of the profile (frontal section) of a flying drop

$$\sigma_{1x}=\pi r_{M\,x}^2, \qquad (2.109)$$

where $r_{M\,x}$ - the value of the radius of the maximum cross section of a drop. If in the element there are n identical drops, then

$$f_{nx}=nf_{1x}. \qquad (2.110)$$

If there are n drops of various size, then

$$f_{nx}=\frac{1}{2}\int_0^n \varrho c_{x\,\mathrm{cp}}\sigma_x(n)[W_x-C_x(n)]^2\,dn. \qquad (2.111)$$

Function $\sigma_x(n)$ should consider the distribution of dimensions σ_x in the element.

If we consider direction x, then the balance of projections of forces will be written so:

$$P_x=F_x-P_{px}+\tau_x-f_{nx}. \qquad (2.112)$$

By substituting the obtained expressions in equation (2.112), we arrive at the integral expression for volume V, which characterizes the motion of burning flow in the direction of axis x:

$$\frac{d}{dt}\int\limits_V \varrho(1-\chi)W_x dV=\int\limits_V \varrho(j_x+g_x)(1-\chi)\,dV-$$

$$-\int\limits_V (1-S_x)\frac{\partial p}{\partial x}dV+\int\limits_V \varrho(1-\chi)\nu\left(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\right)dV-$$

$$-\frac{1}{2}\int\limits_V \varrho c_{x\,\mathrm{cp}}\sigma_x(n)\cdot[W_x-C_x(n)]\frac{dn}{dV}\,dV. \qquad (2.113)$$

It is simple to write the equation in differential form. Let us note that

$$\frac{d}{dt}[\varrho(1-\chi)W_x]=\varrho(1-\chi)\dot{W}_x-\varrho W_x\frac{d\chi}{dt}. \qquad (2.114)$$

Formally in the right side of equation (2.114) there should be one additional item

$$(1-\chi)W_x\frac{d}{dt}\varrho.$$

However, with integration we obtain

$$\int\limits_V (1-\chi)W_x\frac{d}{dt}(\varrho dV),$$

which becomes zero, inasmuch as according to the law of conservation of elementary mass [47]

$$\frac{d}{dt}(dm)=\frac{d}{dt}(\varrho dV)=0.$$

The addend of the right side of equation (2.114) characterizes the force, equal to the product of the current value of mass of gas in element $\varrho(1-\chi)$ by the component of derived velocity in time. The augend characterizes the force, caused by the change of mass of gas in the element. In a cylindrical system of coordinates

$$\dot{W}=\frac{\partial W_x}{\partial t}+\operatorname{grad}\overline{W}_x=\frac{\partial W_x}{\partial t}+\frac{\partial W_x}{\partial x}W_x+\frac{\partial W_x}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}. \qquad (2.115)$$

Thus, the equations of motion in cylindrical coordinates take the form

$$\frac{\partial W_x}{\partial t}+\frac{\partial W_x}{\partial x}W_x+\frac{\partial W_x}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}-\frac{W_x}{1-\chi}\dot{\chi}=$$
$$=(j_x+g_x)-\frac{1}{\varrho}\frac{1-S_x}{1-\chi}\frac{\partial p}{\partial x}+\nu\left(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\right)-$$
$$-\frac{1}{2}\frac{c_x\sigma_x}{1-\chi}(W_x-C_x)^2\frac{dn}{dV}, \qquad (2.116)$$

where dn/dV - the density of distribution of drops along the volume.

In projections to other axes we will have

$$\frac{\partial W_r}{\partial t}+\frac{\partial W_r}{\partial x}W_x+\frac{\partial W_r}{\partial r}W_r+\frac{\partial W_r}{\partial \varphi}\frac{W_\varphi}{r}-\frac{1}{r}W_\varphi^2-\frac{W_r}{1-\chi}\dot{\chi}=(j_r+g_r)-$$
$$-\frac{1}{\varrho}\frac{1-S_r}{1-\chi}\frac{\partial p}{\partial r}+\nu\left(\frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\overline{W}+\nabla^2 W_r\right)-$$
$$-\frac{1}{2}\frac{c_r\sigma_r}{1-\chi}(W_r-C_r)^2\frac{dn}{dV}; \qquad (2.117)$$

$$\frac{\partial W_\varphi}{\partial t}+\frac{\partial W_\varphi}{\partial x}W_x+\frac{\partial W_\varphi}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}+\frac{1}{r}W_rW_\varphi-\frac{W_\varphi}{(1-\chi)}\dot{\chi}=$$
$$=(j_\varphi+g_\varphi)-\frac{1}{\varrho}\frac{1-S_\varphi}{1-\chi}\frac{\partial p}{r\partial \varphi}+\nu\left(\frac{1}{3}\frac{\partial}{r\partial \varphi}\operatorname{div}\overline{W}+\nabla^2 W_\varphi\right)-$$
$$-\frac{1}{2}\frac{c_\varphi\sigma_\varphi}{1-\chi}(W_\varphi-C_\varphi)^2\frac{dn}{dV}. \qquad (2.118)$$

If the area of preparation of propellant for vaporization is considered, then in the equations of motion we take S_i and χ as constants. In the third zone, where there are no liquid particles as yet, i.e., $S_i = 0$ and $\chi = 0$, equations for gas will take a well-known form:

$$\dot{W}_x=(j_x+g_x)-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\nu\left(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\right); \qquad (2.119)$$

$$\dot{W}_r=(j_r+g_r)-\frac{1}{\varrho}\frac{\partial p}{\partial r}+\nu\left(\frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\overline{W}+\nabla^2 W_r\right); \qquad (2.120)$$

$$\dot{W}_\varphi=(j_\varphi+g_\varphi)-\frac{1}{\varrho}\frac{\partial p}{r\partial \varphi}+\nu\left(\frac{1}{3}\frac{\partial}{r\partial \varphi}\operatorname{div}\overline{W}+\nabla^2 W_\varphi\right). \qquad (2.121)$$

Let us recall that in the cylindrical coordinate system:

$$\nabla^2 W_x = \frac{\partial^2 W_x}{\partial x^2} + \frac{\partial^2 W_x}{\partial r^2} + \frac{1}{r}\frac{\partial W_x}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W_x}{\partial \varphi^2};$$

$$\nabla^2 W_r = \frac{\partial^2 W_r}{\partial x^2} + \frac{\partial^2 W_r}{\partial r^2} + \frac{1}{r}\frac{\partial W_r}{\partial r} - \frac{W_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 W_r}{\partial \varphi^2} - \frac{2}{r^2}\frac{\partial W_\varphi}{\partial \varphi};$$

$$\nabla^2 W_\varphi = \frac{\partial^2 W_\varphi}{\partial x^2} + \frac{\partial^2 W_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial W_\varphi}{\partial r} - \frac{W_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 W_\varphi}{\partial \varphi^2} + \frac{2}{r^2}\frac{\partial W_r}{\partial \varphi}.$$

In order to write the equations for liquid phase, instead of projections W_i it is necessary to take the projections C_i of velocities of liquid particles. Quantities $(1 - S_i)$ and $(1 - \chi)$ should be replaced by S_i and χ respectively. Ballistic forces will enter with opposite sign in comparison with (2.116), (2.117) and (2.118), and friction forces, acting in liquid particles, are not considered. Thus, we obtain equations

$$\dot{C}_x - \frac{C_x}{\chi}\dot{\chi} = (j_x + g_x) - \frac{1}{\varrho}\frac{S_x}{\chi}\frac{\partial p}{\partial x} + \frac{1}{2}\frac{c_x\sigma_x}{\chi}(W_x - C_x)^2\frac{dn}{dV}; \quad (2.122)$$

$$\dot{C}_r - \frac{C_r}{\chi}\dot{\chi} = (j_r + g_r) - \frac{1}{\varrho}\frac{S_r}{\chi}\frac{\partial p}{\partial r} + \frac{1}{2}\frac{c_r\sigma_r}{\chi}(W_r - C_r)^2\frac{dn}{dV}; \quad (2.123)$$

$$\dot{C}_\varphi - \frac{C_\varphi}{r\chi}\dot{\chi} = (j_\varphi + g_\varphi) - \frac{1}{\varrho}\frac{S_\varphi}{\chi}\frac{\partial p}{r\partial \varphi} + \frac{1}{2}\frac{c_\varphi\sigma_\varphi}{\chi}(W_\varphi - C_\varphi)^2\frac{dn}{dV}. \quad (2.124)$$

2.4. The Equation of Law of Conservation of Mechanical Energy

The general equation of the law of conservation of energy contains terms, which characterize both mechanical energy and the energy of thermal processes. Let us examine the equation of the law of conservation of mechanical energy for a gas flow, not containing liquid phase. Let us exclude from the examination the forces of viscous friction, since with their presence part of the mechanical energy will become heat - a phenomenon called dissipation of mechanical energy.

As initial equations, apparently, we should take

$$\left.\begin{aligned}\dot{W}_x&=(j_x+g_x)-\frac{1}{\rho}\frac{\partial p}{\partial x};\\ \dot{W}_r&=(j_r+g_r)-\frac{1}{\rho}\frac{\partial p}{\partial r};\\ \dot{W}_\varphi&=(j_\varphi+g_\varphi)-\frac{1}{\rho}\frac{\partial p}{r\partial\varphi}.\end{aligned}\right\} \tag{2.125}$$

By multiplying the right and left sides of the equations by the appropriate velocity components, we obtain

$$\left.\begin{aligned}\dot{W}_x W_x&=(j_x+g_x)W_x-\frac{1}{\rho}\frac{\partial p}{\partial x}W_x;\\ \dot{W}_r W_r&=(j_r+g_r)W_r-\frac{1}{\rho}\frac{\partial p}{\partial r}\dot{W}_r;\\ \dot{W}_\varphi W_\varphi&=(j_\varphi+g_\varphi)W_\varphi-\frac{1}{\rho}\frac{\partial p}{r\partial\varphi}W_\varphi.\end{aligned}\right\} \tag{2.126}$$

After term-by-term summation we find

$$\begin{gathered}\dot{W}_x W_x+\dot{W}_r W_r+\dot{W}_\varphi W_\varphi=\sum(j_i+g_i)W_i-\\ -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial\varphi}\frac{W_\varphi}{r}\right).\end{gathered} \tag{2.127}$$

It is easy to check that

$$\dot{W}_x W_x+\dot{W}_r W_r+\dot{W}_\varphi W_\varphi=\frac{1}{2}\frac{d}{dt}(W^2). \tag{2.128}$$

By using the rules of differentiation of derivative, we find

$$\begin{gathered}\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial\varphi}\frac{W_\varphi}{r}=\frac{\partial}{\partial x}(pW_x)+\frac{1}{r}\frac{\partial}{\partial r}(pW_r r)+\\ +\frac{\partial}{\partial\varphi}\frac{(pW_\varphi)}{r}-p\left(\frac{\partial W_x}{\partial x}+\frac{\partial W_r}{\partial r}+\frac{\partial W_\varphi}{r\partial\varphi}+\frac{W_r}{r}\right)=\\ =\operatorname{div}(p\overline{W})-p\operatorname{div}\overline{W}.\end{gathered} \tag{2.129}$$

Thus, the equation of law of conservation of mechanical energy takes the following from:

$$\frac{1}{2}\frac{d}{dt}(W^2)=\sum(j_i+g_i)W_i-\frac{1}{\varrho}[\operatorname{div}(p\overline{W})-p\operatorname{div}\overline{W}]. \qquad (2.130)$$

Sometimes it is convenient to have equation (2.130) in another form. Inasmuch as

$$\dot{p}=\frac{\partial p}{\partial t}+\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial \varphi}\frac{W_\varphi}{r}, \qquad (2.131)$$

then

$$\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial \varphi}\frac{W_\varphi}{r}=\dot{p}-\frac{\partial p}{\partial t}. \qquad (2.132)$$

Therefore,

$$\frac{1}{2}\frac{d}{dt}(W^2)=\sum(j_i+g_i)W_i-\frac{1}{\varrho}\left(\dot{p}-\frac{\partial p}{\partial t}\right). \qquad (2.133)$$

If we do not consider the action of external mass force, then instead of expression (2.133) we will have

$$\frac{1}{2}\frac{d}{dt}(W^2)=-\frac{1}{\varrho}\left(\dot{p}-\frac{\partial p}{\partial t}\right). \qquad (2.134)$$

Consequently, the change of kinetic energy of flow is equivalent to the change of convective terms

$$\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial \varphi}\frac{W_\varphi}{r}.$$

If pressure does not change with time, then $\partial p/\partial t=0$.

If we consider the problem in one-dimensional formulation, then

$$\frac{1}{2}d(W^2)=(j+g)dx-\frac{dp}{\varrho}. \qquad (2.135)$$

By integrating, we find

$$\frac{W_2^2 - W_1^2}{2} = (j+g)l - \int_{p_1}^{p_2} \frac{dp}{\varrho}. \tag{2.136}$$

For an incompressible medium we will have

$$\frac{W_2^2 - W_1^2}{2} = (j+g)l - \frac{p_2 - p_1}{\varrho}. \tag{2.137}$$

The relationship between the velocity and pressure for a compressible medium will be obtained below [see equation (2.182)].

Let us return to equation (2.130). After multiplication by ϱdV and integration we find

$$\frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV = \int_V \varrho \sum (j_i + g_i) W_i dV - \int \operatorname{div}(p\overline{W})\, dV + \int_V p \operatorname{div} \overline{W} dV. \tag{2.138}$$

According to Gauss formula [50]:

$$\int_V \operatorname{div}(p\overline{W}) dV = \int_F p W_n dF, \tag{2.139}$$

where F - the surface, limiting volume V; W_n - normal velocity component.

Thus, integral (2.139) characterizes the work of pressure forces, applied to surface F, pertaining to a unit of time. The last integral (2.138) characterizes the work of pressure forces, pertaining to a unit of time and spent on the change of volume V.

The equation of law of conservation of mechanical energy of a flow, containing liquid particles, will be written in the following form:

$$\frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi)W^2dV+\frac{1}{2}\frac{d}{dt}\int_V \varrho_{\text{ж}}\chi C^2dV=$$

$$=\int_V \varrho(1-\chi)\sum(j_i+g_i)W_i dV+\int_V \varrho_{\text{ж}}\chi\sum(j_i+g_i)W_i dV-$$

$$-\int_V (1-\chi)\operatorname{div}(p\overline{W})\,dV-\int_V \chi\operatorname{div}(p\overline{C})dV+$$

$$+\int_V (1-\chi)p\operatorname{div}\overline{W}dV+\int_V \chi p\operatorname{div}C dV. \qquad (2.140)$$

The two terms of the left side of the equation characterize the change of kinetic energy of gas and liquid with time. The first two terms of the right side characterize the action of external mass forces on gas and on liquid respectively. The value of subsequent integrals was examined above. If liquid can be considered imcompressible, then the density of liquid is taken as the integral sign, but inasmuch as when $\varrho_{\text{ж}}$ = const the volume does not change, the last integral of (2.140) becomes zero.

2.5. Equation of the Law of Conservation of Energy of a Burning Flow

The total amount of heat, obtained by the element of burning flow in a unit of time, is equal to the change of internal energy of gas, to change of heat content of liquid and to work of gas expansion, pertaining to the same unit of time.

The derivative from the quantity of heat, obtained by the element, is equal to

$$\frac{d}{dt}\int_V Q dV=\frac{d}{dt}\left[\int_V (Q_G+Q_H+Q_R)\,dV\right], \qquad (2.141)$$

where Q_G - the quantity of heat, which is liberated during propellant burning; Q_H - the quantity of heat, applied to the element by thermal conductivity, by flow of radiant energy or due to convective heat exchange; Q_R - the quantity of heat, which was liberated as a result of dissipation of mechanical energy.

With combustion of a unit of mass of propellant in the combustion chamber there is liberated Q_0 heat; the transition of liquid phase into gaseous, as was shown above, is characterized by function Ω. Consequently,

$$\frac{d}{dt}\int_V Q_G dV = \frac{d}{dt}\int_V Q_0 \Omega dV. \qquad (2.142)$$

The supply of heat due to thermal conductivity

$$\frac{d}{dt}\int_V Q_\lambda dV = \int_V \left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{1}{r\partial\varphi}\left(\lambda\frac{\partial T}{r\partial\varphi}\right) + \frac{1}{r}\lambda\frac{\partial T}{\partial r}\right] dV. \qquad (2.143)$$

If it is more convenient to describe heat exchange in a spherical coordinate system, then

$$\frac{d}{dt}\int_V Q_\lambda dV = \int_V \left[\frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{2}{r}\lambda\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial\psi}\left(\lambda\frac{\partial T}{\partial\psi}\right) + \frac{\cos\psi}{r^2\sin\psi}\lambda\frac{\partial T}{\partial\psi} + \frac{1}{r^2\sin^2\psi}\frac{\partial}{\partial\varphi}\left(\lambda\frac{\partial T}{\partial\varphi}\right)\right] dV. \qquad (2.144)$$

With other types of heat exchange between gas and liquid equations (2.143) and (2.144) should be appropriately exchanged.

Dissipation - the process which characterizes the transition of mechanical energy into heat. In a burning flow three types of dissipation are distinguished. First, there is considered the part of the work, which normal and tangential components of forces of friction perform in a gas flow. Secondly, supply of heat takes place as a result of friction, which appears with deformation of liquid particles. The third type of dissipation considers heat, which is liberated as a result of friction between gas and liquid particles.

Thus

$$\frac{d}{dt}\int_V Q_R dV = \frac{d}{dt}\int [\mu_г(1-\gamma)\Phi_г + \mu_ж\gamma\Phi_ж + \mu\Phi] dV, \qquad (2.145)$$

where $\mu_г$ - the friction coefficient for gas; $\mu_ж$ - the friction coefficient for liquid; μ - the coefficient which characterizes the friction between gas and liquid particles; Φ_i - dissipative functions, moreover

$$\Phi_i=\frac{Q_i}{\mu_i}. \tag{2.146}$$

For determination of the dissipative function $\Phi_г$ for gas one should use the equation, which in Cartesian coordinates is written so [94]:

$$\Phi_г=2\left[\left(\frac{\partial W_x}{\partial x}\right)^2+\left(\frac{\partial W_y}{\partial y}\right)^2+\left(\frac{\partial W_z}{\partial z}\right)^2\right]+\left(\frac{\partial W_y}{\partial x}+\frac{\partial W_x}{\partial y}\right)^2+$$
$$+\left(\frac{\partial W_z}{\partial y}+\frac{\partial W_y}{\partial z}\right)^2+\left(\frac{\partial W_x}{\partial z}+\frac{\partial W_z}{\partial x}\right)^2-\frac{2}{3}\left(\frac{\partial W_x}{\partial x}+\frac{\partial W_y}{\partial y}+\frac{\partial W_z}{\partial z}\right)^2. \tag{2.147}$$

The second dissipative function can be written by analogy with the first, having substituted components W_i by the appropriate velocity components.

Let us determine the heat, which is liberated with friction between liquid and gas, i.e., with mixing of flows. The equation of the law of conservation of energy for a certain volume V under the assumption of complete alignment of velocities will be written so:

$$\int_V \varrho(1-\chi)W^2\,dV+\int_V \varrho_ж\chi C^2 dV=\int_V \varrho_0 U^2 dV+2Q. \tag{2.148}$$

According to the law of conservation of momentum

$$\varrho(1-\chi)W+\varrho_ж\chi C=\varrho_0 U. \tag{2.149}$$

Having substituted the values of total velocity U from equation (2.149) in equation (2.148), after conversions we obtain

$$Q=\frac{1}{2}\int_V\left[\varrho(1-\chi)W^2+\varrho_ж\chi C^2-\varrho_0\left(\frac{\varrho}{\varrho_0}(1-\chi)W+\frac{\varrho_ж}{\varrho_0}\chi C\right)^2\right]dV. \tag{2.150}$$

The value of the dissipative function is determined from equation (2.146). Equation (2.150) shows that in the initial period of burning, when $\chi = 1$, just as at the end of the process, when $\chi = 0$, dissipation is absent, which is entirely evident. The greatest thermal effect will be when $\chi = 0.5$. If we are guided by the average value of relative velocity, then

$$Q_{max} = 0{,}125\varrho(W-C)^2_{cp}V. \tag{2.151}$$

The change of internal energy of gas and liquid will be

$$\frac{d}{dt}\int_m (c_V T)\,dm = \frac{d}{dt}\int_V \varrho(1-\chi)c_V T\,dV + \frac{d}{dt}\int_V \varrho_{\text{ж}}\chi c_{\text{ж}} T_{\text{ж}}\,dV. \tag{2.152}$$

The work, being spent on change of the volume of gas and liquid, in the case of consideration of the compressibility of liquid will be

$$L_V = \int_V (1-\chi)p\,\mathrm{div}\,\overline{W}\,dV + \int_V \chi p\,\mathrm{div}\,\overline{C}\,dV. \tag{2.153}$$

By equating the heat applied to the element to the change of internal energy and to work being spent on deformation of the burning flow, we obtain a general equation of the law of conservation of energy:

$$\begin{aligned}
&\frac{d}{dt}\int_V Q_0\,dV + \frac{d}{dt}\int_V Q_n\,dV + \frac{d}{dt}\Big[\int_V \mu_r(1-\chi)\phi_r\,dV + \\
&+\int_V \mu_{\text{ж}}\chi\phi_{\text{ж}}\,dV + \int_V \mu\phi\,dV\Big] = \frac{d}{dt}\int_V \varrho(1-\chi)c_V T\,dV + \\
&+\frac{d}{dt}\int_V \varrho_{\text{ж}}\chi c_{\text{ж}} T_{\text{ж}}\,dV + \int_V (1-\chi)\,p\,\mathrm{div}\,\overline{W}\,dV + \int_V \chi p\,\mathrm{div}\,\overline{C}\,dV.
\end{aligned} \tag{2.154}$$

Here all quantities, which characterize the supply or removal of heat for the burning flow, are calculated for a unit of volume.

If in this equation we convert the last two terms, using equation (2.130) of the law of convervation of mechanical energy, then we obtain

$$\frac{d}{dt}\int_V Q_G dV + \frac{d}{dt}\int_V Q_H dV + \frac{d}{dt}\Big[\int_V \mu_r (1-\chi)\phi_r dV +$$
$$+ \int_V \mu_ж \chi \phi_ж dV + \int_V \mu\phi dV\Big] + \int_V \varrho(1-\chi)\sum(j_l+g_l)\,W_l dV +$$
$$+ \int_V \varrho_ж \chi \sum(j_l+g_l)\,C_l dV = \frac{d}{dt}\int_V \varrho(1-\chi)\,c_V T dV +$$
$$+ \frac{d}{dt}\int_V \varrho_ж \chi c_ж T_ж dV + \frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi)W^2 dV +$$
$$+ \frac{1}{2}\frac{d}{dt}\int_V \varrho_ж \chi C^2 dV + \int_V (1-\chi)\operatorname{div}(p\overline{W})dV + \int_V \chi \operatorname{div}(p\overline{C})dV. \qquad (2.155)$$

Let us examine the integrands of the last two terms of equation (2.155). As was shown

$$\operatorname{div}(p\overline{W}) = \dot{p} - \frac{\partial p}{\partial t} + p\operatorname{div}\overline{W}. \qquad (2.156)$$

By using the equation of continuity in the form

$$\dot{\varrho} + \varrho\operatorname{div}\overline{W} = 0, \qquad (2.157)$$

we find

$$p\operatorname{div}\overline{W} = -\frac{p}{\varrho}\dot{\varrho}. \qquad (2.158)$$

Let us add to the right side of expression (2.158) and subtract from it

$$\varrho\frac{d}{dt}\left(\frac{p}{\varrho}\right).$$

With respect to the rule of differentiation of fraction

$$\varrho\frac{d}{dt}\left(\frac{p}{\varrho}\right) = \dot{p} - \frac{p}{\varrho}\dot{\varrho}. \qquad (2.159)$$

Consequently,

$$p\,\mathrm{div}\,\overline{W}=-\dot{p}+\varrho\frac{p}{dt}\left(\frac{p}{\varrho}\right) \qquad (2.160)$$

and further

$$\mathrm{div}\,(p\overline{W})=\varrho\frac{d}{dt}\left(\frac{p}{\varrho}\right)-\frac{\partial p}{\partial t}. \qquad (2.161)$$

Let us use the equation of state

$$p=\varrho RT. \qquad (2.162)$$

Consequently,

$$\mathrm{div}\,(p\overline{W})=\varrho\frac{d}{dt}(RT)-\frac{\partial p}{\partial t}. \qquad (2.163)$$

As is known,

$$c_V+R=c_p. \qquad (2.164)$$

Thus

$$\begin{aligned}&\frac{d}{dt}\int\limits_V\varrho(1-\chi)\,c_VT\,dV+\int\limits_V(1-\chi)\,\mathrm{div}\,(p\overline{W})\,dV=\\&=\frac{d}{dt}\int\limits_V\varrho(1-\chi)\,c_pT\,dV+\int\limits_V(1-\chi)\,\frac{\partial p}{\partial t}\,dV.\end{aligned} \qquad (2.165)$$

For the case of compressible liquid analogically we find

$$\begin{aligned}&\frac{d}{dt}\int\limits_V\varrho_*\chi c_*T_*\,dV+\int\limits_V\chi\,\mathrm{div}\,(p\overline{C})\,dV=\\&=\frac{d}{dt}\int\limits_V\varrho_*\chi c_*T_*\,dV+\int\limits_V\chi\,\frac{\partial p}{\partial t}\,dV.\end{aligned} \qquad (2.166)$$

By using the obtained relationships, we find

$$\frac{d}{dt}\int\limits_V Q_G dV+\frac{d}{dt}\int\limits_V Q_H dV+\frac{d}{dt}\left[\int\limits_V \mu_{\text{г}}(1-\chi)\phi_{\text{г}} dV+\right.$$
$$\left.+\int\limits_V \mu_{\text{ж}}\chi\phi_{\text{ж}} dV+\int\limits_V \mu\phi dV\right]+\int\limits_V \varrho(1-\chi)\sum (j_i+g_i)W_i dV+$$
$$+\int\limits_V \varrho_{\text{ж}}\chi\sum(j_i+g_i)C_i dV=\frac{d}{dt}\int\limits_V \varrho(1-\chi)c_p T dV+$$
$$+\frac{d}{dt}\int\limits_V \varrho_{\text{ж}}\chi c_{\text{ж}}T_{\text{ж}} dV+\frac{1}{2}\frac{d}{dt}\int\limits_V \varrho(1-\chi)W^2 dV+$$
$$+\frac{1}{2}\frac{d}{dt}\int\limits_V \varrho_{\text{ж}}\chi C^2 dV-\int\limits_V \frac{\partial p}{\partial t} dV. \quad (2.167)$$

The integrands in equation (2.167) depend on coordinates, i.e., they can be different for various points of intrachamber space.

Thus, for instance, the gas velocity is a function of x, r and ϕ, i.e., $W(x, r, \phi)$.

The left side of equation (2.167) characterizes the supply of energy to the burning flow, contained in selected volume V. The first integral determines the supply of heat as a result of propellant burning.

Where the preparation of propellant for vaporization occurs, $\Omega = 0$. Inasmuch as burning is also absent, then $Q_G = 0$. In the area of vaporization and burning Q_G reaches the greatest value. It is closer to the nozzle, where only the combustion products are found, the thermal effect is caused by chemical transformations in gaseous phase.

The second term characterizes the supply of heat to propellant due to heat exchange. In the area of preparation of propellant for vaporization the heat exchange is the determining process. Further it substantially affects the intensity of vaporization and burning.

The last two terms of the left side of the equation consider the supply of energy, caused by the action of external mass forces.

The first two terms of the right side of the equation show the change of gas and liquid enthalpy with consideration of the change of quantity of available liquid. The following two integrals characterize the energy, being spent on change of the velocity of gas and liquid particles. The last term of the equation determines the local (time) change of pressure in the combustion chamber.

By using equations of continuity (2.100), motion (2.116), (2.117), (2.118), (2.122), (2.123), (2.124), law of conservation of energy (2.167) and equation of state, we determine W_i, C_i, p and ρ. The remaining parameters, entering these equations, are assigned in the form of numbers or with the aid of additional equations.

Let us examine particular cases of utilization of the equation of energy.

With the absence heat-mass transfer and not allowing for external effects the equation of energy of gas flow takes the form

$$\frac{d}{dt}\int\limits_V \rho c_V T\, dV + \int\limits_V p \operatorname{div} \overline{W}\, dV = 0. \tag{2.168}$$

By using equations (2.160), (2.162) and (2.164), we find

$$\frac{d}{dt}\int\limits_V \rho c_p T\, dV - \int\limits_V p\, dV = 0. \tag{2.169}$$

Let us divide integrands by ρ. When the integrands do not depend on coordinates or they are taken in the form of average values, we obtain

$$d(c_p T) = \frac{dp}{\rho}. \tag{2.170}$$

By using equation of state, we find

$$\frac{d(c_pT)}{RT}=\frac{dp}{p}. \tag{2.171}$$

Consequently,

$$\frac{c_p}{R}\frac{dT}{T}+\frac{dc_p}{R}=\frac{dp}{p}. \tag{2.172}$$

Inasmuch as the adiabatic index

$$k=\frac{c_p}{c_V},$$

we obtain

$$\frac{k}{k-1}\frac{dT}{T}+d\left(\frac{k}{k-1}\right)=\frac{dp}{p}. \tag{2.173}$$

This equation describes the process with consideration of the change of adiabatic index. If we are guided by the average and constant values of adiabatic index, then after integration and conversions of equation (2.173) we arrive at the adiabatic equation in the form

$$\frac{T}{T_0}=\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}. \tag{2.174}$$

Let us write an equation for a zone, where vaporization and burning of propellant were finished. While not taking into account heat exchange and the action of external mass forces we find

$$\frac{1}{2}\frac{d}{dt}\int\limits_V \rho W^2 dV+\frac{d}{dt}\int\limits_V \rho c_p T\,dV-\int\limits_V \frac{\partial p}{\partial t}dV=0. \tag{2.175}$$

If in any section the pressure is not changed with time, then, by substituting the average values of parameters of integrands, we obtain

$$\frac{1}{2}d(W^2)+d(c_pT)=0. \tag{2.176}$$

By using equation (2.176), let us examine two sections of a gas line of arbitrary shape. According to the law of conservation of energy for these two sections

$$\frac{W_1^2}{2}+c_pT_1=\frac{W_2^2}{2}+c_pT_2. \tag{2.177}$$

By using the equation of state for ideal gas

$$p=\rho RT \tag{2.178}$$

and known relationship

$$c_p-c_V=R, \tag{2.179}$$

we obtain

$$c_p=c_p\frac{c_V}{c_V}\frac{R}{R}=\frac{c_p}{c_V}\frac{Rc_V}{c_p-c_V}=\frac{k}{k-1}R. \tag{2.180}$$

Consequently,

$$c_pT=\frac{k}{k-1}\frac{p}{\rho}. \tag{2.181}$$

Thus, the relationship between pressure, density and the velocity of the moving deformable medium takes the form

$$\frac{W_1^2}{2}+\frac{k}{k-1}\frac{p_1}{\rho_1}=\frac{W_2^2}{2}+\frac{k}{k-1}\frac{p_2}{\rho_2} \tag{2.182}$$

or

$$\frac{W^2}{2}+\frac{k}{k-1}\frac{p}{\rho}=\text{const}. \tag{2.183}$$

For determination of the efflux velocity of gas from the nozzle let us integrate relationship (2.176) and we find

$$W_a=\sqrt{2(c_pT)_k-2(c_pT)_a+W_k^2}. \tag{2.184}$$

Let us designate the relationship of heat capacities through

$$\zeta=\frac{c_{pa}}{c_{p\text{к}}}. \qquad (2.185)$$

For determination of the efflux velocity of gas we have

$$W_a=\sqrt{2c_{p\text{к}}(T_\text{к}-\zeta T_a)+W_\text{к}^2}. \qquad (2.186)$$

Thermal efficiency

$$\eta_t=1-\zeta\frac{T_a}{T_\text{к}}. \qquad (2.187)$$

By using adiabatic equation (2.174), after conversions we obtain

$$W_a=\sqrt{2c_{p\text{к}}T_\text{к}\eta_t+W_\text{к}^2}, \qquad (2.188)$$

where

$$\eta_t=1-\zeta\left(\frac{p_a}{p_\text{к}}\right)^{\frac{k-1}{k}}; \qquad (2.189)$$

p_a - nozzle exit pressure. The product of $c_{p\text{к}}T_\text{к}$ is equal to the heat content of gases in the chamber $i_\text{к}$. If we do not consider heat losses, then the heat content $i_\text{к}$ is equal to the quantity of heat, which is liberated during the combustion of propellant. Sometimes we use relationship

$$c_{p\text{к}}=\frac{k}{k-1}R. \qquad (2.190)$$

In this case

$$W_a=\sqrt{2\frac{k}{k-1}RT_\text{к}\eta_t+W_\text{к}^2}. \qquad (2.191)$$

In engineering calculations we consider the deviation of acutal efflux conditions from calculated by the velocity coefficient ϕ or by replacement of the adiabatic index by politropic index. Calculation formulas have the form

$$W_a=\varphi\sqrt{2\frac{k}{k-1}RT_K\eta_t+W_K^2} \qquad (2.192)$$

or

$$W_a=\sqrt{2\frac{n}{n-1}RT_K\eta_t+W_K^2}, \qquad (2.193)$$

where n - the average value of politropic index.

In actuality the politropic index is changed along the length of the combustion chamber and nozzle, so that n is a variable, depending on the distance from the injector edge to the nozzle exit section, i.e., on x. The average value of politropic index

$$n=\frac{1}{x_a}\int_0^{x_a} n(x)\,dx. \qquad (2.194)$$

When performing engineering calculations the second term in equality (2.193) is disregarded, inasmuch as $W_a^2 \gg W_K^2$.

Then

$$W_a=\sqrt{2\frac{n}{n-1}RT_K\eta_t}. \qquad (2.195)$$

The adiabatic and politropic indexes are connected together by relationship

$$\varphi^2\frac{k}{k-1}=\frac{n}{n-1}. \qquad (2.196)$$

Hence it follows that

$$n=\frac{\varphi^2}{\varphi^2-1+\frac{1}{k}}. \qquad (2.197)$$

With the absence of losses $\phi = 1$ and then according to relationship (2.197) $n = k$. In the isothermal process $k = 1$ and $n = 1$ at any values of ϕ.

Sometimes it is convenient to determine the efflux velocity through quantity of heat Q, which is liberated in the chamber during propellant combustion. With the absence of losses

$$Q=i, \tag{2.198}$$

where $i=c_{p\,\text{к}}T_{\text{к}}$ - the heat content of combustion products.

In this case

$$i=\frac{k}{k-1}RT. \tag{2.199}$$

For determination of the efflux velocity of losses we have

$$W_a=\varphi\sqrt{2Q\eta_t}. \tag{2.200}$$

Thus the efflux velocity of combustion products from the nozzle depends on three factors.

For increase of the quantity of heat Q, which is liberated during propellant combustion, one should select propellant and such a ratio of components k_1, in order to obtain the highest value of complex $\frac{k}{k-1}RT$ with prescribed pressure in the chamber.

In this case it is expedient to have the lowest possible temperature of combustion products and the highest possible value of gas constant, i.e., low apparent molecular weight.

2.6. Reactive Force and Thrust

Reactive force R appears as a result of the interaction between burning flow (gas flow) and the interior surface of combustion chamber and nozzle walls. Reactive force is made up of elementary forces, normal to the surface of the chamber and nozzle, and the resultant of these forces is directed along the length of the axis opposite the direction of motion of gases. If the pressure of the surrounding medium is nonzero, then the external surface of the chamber is affected by elementary forces of external pressure, normal to the

external surface of the combustion chamber and nozzle. The resultant of these forces $P_{н}$ is directed along the axis of the chamber and coincides with the direction of motion of gases. The total force effect on the interior and external surface of the combustion chamber and nozzle is thrust

$$P=R-P_{н}. \qquad (2.201)$$

The thrust vector

$$\overline{P}=\overline{R}+\overline{P}_{н}. \qquad (2.202)$$

Let us determine the reactive force. The vector of surface forces, applied to elementary particles of flow on the interior surface, limited by the walls of combustion chamber and nozzle

$$\int_{F}\overline{p}\,dF=\int_{F_{к}}\overline{p}_{к}\,dF+\int_{F_{a}}\overline{p}_{a}\,dF, \qquad (2.203)$$

where

$$F=F_{к}+F_{a};$$

$F_{к}$ - the interior surface of the combustion chamber and nozzle;
F_{a} - the area of the nozzle exit section;

$$\overline{p}=p\overline{n};$$

p - modulus of pressure; $\overline{n}$ - unit vector, normal to the surface; $p_{к}$ - the current value of pressure in the combustion chamber with nozzle; p_{a} - the pressure of outflowing gases in the nozzle exit section.

The first integral of the right side of equality (2.203) characterizes the action on the gas flow on the part of walls. The equal in value, but oppositely directed vector is the reactive force. Therefore,

$$\overline{R} = -\int_{F_\kappa} \overline{p}_\kappa dF. \qquad (2.204)$$

The work of the reactive force, pertaining to a unit of time,

$$L_R = -\int_{F_\kappa} p_\kappa W_a dF. \qquad (2.205)$$

By using formula (2.139), we find

$$L_R = -\int_V \operatorname{div}(p\overline{W}) dV. \qquad (2.206)$$

Now, returning to equation (2.138), we note that the work of expansion of gases taking into account the action of mass forces is equal to the kinetic energy of outflowing gases and to the work being performed by the reactive force.

For a closed internal contour of combustion chamber and nozzle the effective force on the gas flow is the integral, standing in the left side of equation (2.203). In order to obtain the expression of total internal force, it is necessary to take into account the external mass forces, affecting the gaseous and liquid products, moving along the internal cavity of the combustion chamber and nozzle. The elementary pulse of total internal force

$$dI = \left[\int_F \overline{p} dF + \int_V \varrho(1-\chi)(\overline{J}+\overline{g}) dV + \int_V \varrho_{ж}\chi(\overline{J}+\overline{g}) dV\right] dt. \qquad (2.207)$$

Gas and liquid particles at every point of the internal volume V can have different density $\rho(x, r, \phi)$ and $\rho_{ж}(x, r, \phi)$ and various velocities $\overline{W}$, $\overline{C}$. The relative quality of liquid particles in a unit of volume also depends on coordinates, i.e., $\chi(x, r, \phi)$. Therefore, the change of momentum of the gas flow, which fills volume V, will be

$$dQ = d\left[\int_V \varrho(1-\chi)\overline{W} dV + \int_V \varrho_{ж}\chi C dV\right]. \qquad (2.208)$$

By equating the pulse of force dI to the change of momentum $d\theta$ and considering equations (2.203) and (2.204), we find

$$\bar{R} = -\frac{d}{dt}\left[\int_V \rho(1-\chi)\bar{W}\,dV + \int_V \rho_{ж}\chi\bar{C}\,dV\right] + \int_V \rho(1-\chi)(\bar{j}+\bar{g})\,dV + \int_V \rho_{ж}\chi(\bar{j}+\bar{g})\,dV + \int_{F_a} \bar{p}_a\,dF. \quad (2.209)$$

If we disregard the effect of liquid particles, then in expression (2.209) one should take $\chi = 0$. In this case for determination of reactive force we will have

$$\bar{R} = -\frac{d}{dt}\int_V \rho\bar{W}\,dV + \int_V \rho(\bar{j}+\bar{g})\,dV + \int_{F_a} \bar{p}_a\,dF. \quad (2.210)$$

The first integral of the right side of equation (2.210) can be presented by the sum of [57]:

$$\frac{d}{dt}\int_V \rho\bar{W}\,dV = \int_F \rho\bar{W}W_n\,dF + \frac{\partial}{\partial t}\int_V \rho\bar{W}\,dV. \quad (2.211)$$

During examination of stationary conditions the augend of the right side of equation (2.211) becomes zero. The addend of the right side of the same equation corresponds to internal surface F, it can be represented by the sum

$$\int_F \rho\bar{W}W_n\,dF = \int_{F_{nx}} \rho\bar{W}W_n\,dF + \int_{F_n} \rho\bar{W}W_n\,dF + \int_{F_a} \rho\bar{W}W_a\,dF, \quad (2.212)$$

moreover

$$F = F_{nx} + F_n + F_a.$$

The second integral of the right side of equation (2.212) is nonzero with the presence of outflow or inflow of mass, for example, from a system of film protection of the chamber wall. In the flow, directed only along the axis of the chamber, in the absence of liquid feed through openings or slots in the chamber wall, instead of equation (2.212) we will have

$$\int_F \varrho W^2 dF = \int_{F_{\text{вх}}} \varrho W_{\text{вх}}^2 dF - \int_{F_a} \varrho W_a^2 dF. \tag{2.213}$$

The minus sign for the augend of equation (2.213) is used because the direction of velocity W_a is opposite the direction of the action of reactive force. Integral

$$\int_{F_{\text{вх}}} \varrho W_{\text{вх}} dF = G_{\text{вх}} \tag{2.214}$$

is equal to the inflow of working medium into the combustion chamber through inlet openings, whose total area of useful cross section, measured in the plane perpendicular to the axis of the chamber, is equal to $F_{\text{вх}}$.

The flow rate of gases through the nozzle exit section F_a is equal to

$$\int_{F_a} \varrho W_a dF = G_a. \tag{2.215}$$

Velocities $W_{\text{вх}}$, W_a and density ρ generally depend on the distance from the axis of the chamber. If we are guided by average values of density and velocity, then

$$\varrho W_{\text{вх}} F_{\text{вх}} = G_{\text{вх}}; \tag{2.216}$$

$$\varrho W_a F_a = G_a. \tag{2.217}$$

In this case, if $G_{вх} = G_a$, then

$$\int_F \varrho W^2 dF = -G_a(W_a - W_{вх}). \tag{2.218}$$

If we consider the intake velocities of liquid or gaseous working medium into the combustion chamber $W_{вх} \ll W_a$ or if we determine the magnitude of reactive force not for the chamber alone, but for the entire power plant, then

$$\int_F \varrho W^2 dF = -G_a W_a. \tag{2.219}$$

The second integral of the right side of equation (2.211) reflects the peculiarities of nonstationary operating conditions of the combustion chamber and nozzle. In fact,

$$\int_V \varrho dV = m_к \tag{2.220}$$

is equal to the mass of products, filling the internal volume of the chamber. If all this mass as one whole would move with variable speed $W_{ср}$, then

$$\frac{\partial}{\partial t}\int_V \varrho \bar{W} dV = m_к \dot{W}_{ср}. \tag{2.221}$$

In actuality $\rho(t, x, r, \phi)$ and $W(t, x, r, \phi)$. In one-dimensional flow $\rho(t, x)$ and $W(t, x)$. In this case

$$\frac{\partial}{\partial t}\int_V \varrho W dV = \frac{\partial}{\partial t}\int_0^{x_a}\left(\int_{F(x)} \varrho W dF\right)dx, \tag{2.222}$$

where x_a - the distance from the injector assembly of the chamber to the nozzle exit section.

Integral

$$\int_{F(x)} \varrho W \, dF = G(t, x) \qquad (2.223)$$

represents mass flow rate, which under nonstationary conditions is changed with time and is different for various cross sections of the chamber and nozzle. Thus,

$$\frac{\partial}{\partial t} \int_V \varrho W \, dV = \frac{\partial}{\partial t} \int_0^{x_a} G(t, x) \, dx. \qquad (2.224)$$

Let us determine the resultant of elementary forces of external pressure. Let us write an expression for the determination of the total vector of surface forces, applied to elementary particles of the external medium on the part of the external contour of the combustion chamber and nozzle:

$$\int_F \bar{p}_{\text{н}} \, dF = \int_{F_{\text{к}}} \bar{p}_{\text{н}} \, dF + \int_{F_a} \bar{p}_{\text{н}} \, dF, \qquad (2.225)$$

where

$$\bar{p}_{\text{н}} = p_{\text{н}} \bar{n};$$

p_H - modulus of external pressure. The vector of surface forces, directed from the external medium to the outer surface F_K of the combustion chamber and nozzle, is equal in magnitude to the first integral of the right side of equality (2.225), but is opposite in sign, i.e.,

$$\bar{P}_{\text{к}} = -\int_{F_{\text{к}}} \bar{p}_{\text{н}} \, dF. \qquad (2.226)$$

The total force pulse of surface forces

$$dI_{\text{н}} = \left(\int_F \bar{p}_{\text{н}} \, dF \right) dt \qquad (2.227)$$

should be equal to the change of the momentum of external medium, i.e.,

$$d\theta_H = d\int_{m_H} W_H\, dm_H, \tag{2.228}$$

where W_H - the flow velocity of external medium relative to the combustion chamber; m_H - the mass of flow of external medium, acting on the engine. If the combustion chamber is protected from the action of external medium, then $dI_H = 0$ and $d\theta_H = 0$, and consequently,

$$\int_F \bar{p}_H\, dF = 0. \tag{2.229}$$

Thus, for the combustion chamber and nozzle

$$\bar{P}_H = \int_{F_a} \bar{p}_H\, dF. \tag{2.230}$$

Now, using equations (2.202) and (2.230), we arrive at the equation for determination of the thrust vector:

$$\bar{P} = -\frac{d}{dt}\left[\int_V \rho(1-\chi)\bar{W}\, dV + \int_V \rho_{ж}\chi\bar{C}\, dV\right] + \int_V \rho(1-\chi)(\bar{j}+\bar{g})\, dV + \int_V \rho_{ж}\chi(\bar{j}+\bar{g})\, dV + \int_{F_a}(\bar{p}_a - \bar{p}_H)\, dF. \tag{2.231}$$

By discarding liquid particles from examination and using equation (2.211), we find

$$\bar{P} = -\int_F \rho\bar{W}W_x\, dF - \frac{\partial}{\partial t}\int_V \rho\bar{W}\, dV + \int_V \rho(\bar{j}+\bar{g})\, dV + \int_{F_a}(\bar{p}_a - \bar{p}_H)\, dF. \tag{2.232}$$

For pressures p_a and p_H let us take their average values along the entire surface F_a and let us designate the projections of accelerations in the direction of action of thrust force through j and g, then we obtain

$$P = G_a W_a - \frac{\partial}{\partial t}\int_0^{x_a}\left(\int_{F(x)} \varrho W \, dF\right) dx + \\ + \int_V \varrho (j + g) \, dV + F_a (p_a - p_H). \tag{2.233}$$

For steady state and not taking into account the action of external mass forces

$$P = G_a W_a + F_a (p_a - p_H). \tag{2.234}$$

During engine operation at high altitudes $p_H \approx 0$; consequently, thrust force in a void

$$P_\infty = G_a W_a + F_a p_a. \tag{2.235}$$

The calculation condition, at which the greatest thrust corresponds to the assigned pressure of surrounding medium, is satisfied by condition $p_a = p_H$. In this case

$$P_p = G_a W_a. \tag{2.236}$$

Specific thrust

The quantity of Newtons of thrust, obtained with combustion of a unit of mass of propellant in one second, or the ratio of thrust to the mass flow rate of propellant per second, is called specific thrust:

$$P_{\text{уд}} = \frac{P}{G} = W_a + \frac{F_a}{G}(p_a - p_H). \tag{2.237}$$

During operation at calculated conditions, when $p_a = p_н$,

$$P_{уд.р} = W_a. \tag{2.238}$$

If the flow rate of propellant is not changed with time, then the total propellant consumption

$$m_т = G t_0,$$

where t_0 - the time of operation of engine. Consequently,

$$P_{уд.р} = \frac{P_р}{m_т} t_0 = \frac{I_р}{m_т}. \tag{2.239}$$

Equations (2.238) and (2.239) show that specific thrust $P_{уд.р}$ is the efflux velocity (in m/s), which 1 kg of combustion products acquires under the action of the thrust impulse $I_р$, equal to 1 N·s. By multiplying and dividing the right side of equation (2.239) by $m_{рак}+m_т$ after simple conversions we obtain

$$P_{уд.р} = \frac{P_р}{m_{рак.н}}\left(1 + \frac{m_{рак}}{m_т}\right) t_0, \tag{2.240}$$

where $m_{рак}$ - the mass of a rocket not taking into account the mass of propellant being fueled; $m_{рак.н}$ - the launching (initial) mass of the rocket, moreover

$$m_{рак.н} = m_{рак} + m_т.$$

When designing a rocket and engine we provide optimum values of ratios

$$\mu = \frac{P_р}{m_{рак.н}} \text{ and } \bar{m} = \frac{m_{рак}}{m_т}. \tag{2.241}$$

For the realization of launch it is necessary that μ be equal to or exceed the projection of acceleration of gravity to the direction of the rocket axis, i.e., $\bar{\mu} \geq \bar{g}$. Under conditions (2.241) we obtain

$$t_0=\frac{P_{\text{уд.р}}}{\mu(1+\bar{m})}. \qquad (2.242)$$

Thus, specific thrust characterizes the maximum possible time of operation of the engine, corresponding to actually being attained values of μ and m.

2.7 Equation of Entropy

During the calculation of motion of combustion products along the chamber and through the nozzle it is convenient to use the equation of entropy. As is known,

$$dS=\frac{dQ}{T}. \qquad (2.243)$$

By using expression (2.154), but without considering external mass forces, for one-dimensional flow we find

$$\frac{d}{dt}(\varrho Q)=\frac{d}{dt}(\varrho c_V T)+p\operatorname{div}\overline{W}. \qquad (2.244)$$

By equations (2.129) and (2.131) we find

$$p\operatorname{div}\overline{W}=\operatorname{div}(p\overline{W})-\dot{p}+\frac{\partial p}{\partial t}. \qquad (2.245)$$

According to equation (2.161) we have

$$\operatorname{div}(p\overline{W})=\dot{p}-\frac{\dot{\varrho}}{\varrho}p-\frac{\partial p}{\partial t}. \qquad (2.246)$$

Consequently,

$$p\operatorname{div}\overline{W}=-\frac{\dot{\varrho}}{\varrho}p. \qquad (2.247)$$

Now the equation of energy (2.244) takes the form

$$\frac{d}{dt}(\varrho Q)=\frac{d}{dt}(\varrho c_V T)-\frac{\dot{\varrho}}{\varrho}p. \qquad (2.248)$$

By using the equation of state and having divided the expressions after the sign of derivative by ρ, we obtain

$$\frac{dQ}{dt}=\frac{d}{dt}(c_V T)-RT\,\frac{\dot{\rho}}{\rho}. \tag{2.249}$$

Having divided both sides of the equalities by T, we find

$$\frac{dS}{dt}=\frac{dc_V}{dt}+c_V\,\frac{\dot{T}}{T}-R\,\frac{\dot{\rho}}{\rho}. \tag{2.250}$$

Being guided by the average value of heat capacity, after integration we arrive at the equation of entropy

$$\Delta S=S_1-S_2=c_V\ln\frac{T_1}{T_2}-R\ln\frac{\rho_1}{\rho_2}. \tag{2.251}$$

By replacing in equation (2.251) the densities through pressures by the equation of state and taking into account that $c_p=c_V+R$, we obtain

$$\Delta S=S_1-S_2=c_p\ln\frac{T_1}{T_2}-R\ln\frac{p_1}{p_2}. \tag{2.252}$$

2.8. Equations of State of Ideal and Real Gases

Equation (2.162) is valid for an ideal gas. For real gases when performing engineering calculations it is applicable at relatively low pressures, with higher possible temperatures and under conditions far from the area of saturation.

In modern liquid-propellant rocket engines the pressures are rather high. The gases move in separate ducts at very low temperature. Inasmuch as the equation of ideal gas does not consider intermolecular forces and the volume of molecules, in contemporary calculations we are sometimes guided by equations of state of real gases.

For the mathematical description of the state of real gas a large quantity of working formulas is suggested. In most cases these are empirical expressions, in which the specific features of the considered gas or gas mixture are considered correction factors.

The widest distribution was received by the equations of van der Walls, Berthelot, Dieterici, Betti-Bridgmen, Bol, Key and others.

Many scientists considered that the specific constants can be excluded, if in the equations of state instead of absolute we substitute the reduced values of parameters, found from experiment or on the basis of some theoretical examinations.

The van der Walls equation, for example, has the form

$$p=\frac{RT}{v-b}-\frac{a}{v^2}, \tag{2.253}$$

where a, b - constants, which characterize the peculiarities of the considered gas. Here the specific volume is corrected by quantity b, considering the effect of the natural volume of molecules of gas on pressure, due to which the free space is decreased. Therefore, in real gas the collision of molecules together and their impacts against walls of the shell will be more frequent than in ideal gas. This correction is approximately equal to quadruple the natural volume of molecules.

The cohesive forces between molecules attract the molecules, located closer to the surface of the shell, to the center of the capacity. The decrease in this case of the force of impacts of molecules against the shell, and consequently, pressure are considered by quantity a/v^2.

The values of constants a and b can be found in reference books and in specialized literature.

The van der Walls equation in the given form is written in the following manner:

$$p_r = \frac{R'T_r}{v_r - b'} - \frac{a'}{v_r^2}, \tag{2.254}$$

where a', b' and R' - universal constants, which do not depend on the properties of gas and are connected with primary constants a and b by equations

$$a' = \frac{a}{p_s v_s^2}; \quad b' = \frac{b}{v_s}; \quad R' = \frac{RT_s}{p_s v_s}, \tag{2.255}$$

where p_s, v_s, T_s - critical parameters.

The determination of constants, entering equation (2.253), is based on the fact that the isotherm at a critical point has inflection. By using equation (2.254), in courses of thermodynamics are found expressions for determination of critical temperatures and pressures in the form

$$T_s = \frac{8a}{27Rb}; \quad p_s = \frac{a}{27b}, \tag{2.256}$$

whence, by knowing T_s, p_s and R, we determine the desired constants.

The amount of error, being obtained during calculation by the given formulas, depends on the pressure for each gas of it. As shown by comparison of experimental data with results of calculation by formulas of Clapeyron and van der Walls, for nitrogen, for example, at normal temperature and pressure up to 10 MN/m^2 the cohesive force and the volume of molecules do not noticeably affect the results of calculation, and they cannot be taken into account, thus considering nitrogen an ideal gas up to the shown pressure. Carbon dioxide, for example, deviates from the laws of ideal gas considerably earlier than nitrogen, which is attested to by values of specific volumes at normal temperature in m^3/mole, given in Table 2.1.

Table 2.1.

p bar	According to experiments	According to equation	
		Clapeyron	van der Waal
1.0	25.57	25.7	25.6
10.0	2.449	2.57	2.471
50.0	0.380	0.513	0.395
100.0	0.069	0.257	0.089

In the technology of computation of parameters of real gas rather wide application was received by equation

$$v=Z\frac{RT}{p}, \tag{2.257}$$

where Z - the compressibility factor, equal to one for an ideal gas. The compressibility factor Z is dimensionless and for real gases is changed from 0.3 to a quantity somewhat exceeding one, if we do not consider the region of very high pressures. There are many methods of of determination of the compressibility factor. In the first approximation we consider

$$Z=f(p_r, T_r), \tag{2.258}$$

where

$$p_r=\frac{p}{p_s};\quad T_r=\frac{T}{T_s}. \tag{2.259}$$

For determination of Z by formula (2.258) in [78] there are provided charts Nelson and Oberth.

CHAPTER III

SOME QUESTIONS OF INTRACHAMBER PROCESSES

To the research of intrachamber processes are devoted many works: [10], [14], [20], [33], [48], [75] and others. Research is conducted usually in accordance with a particular engine layout and prescribed operating conditions of the combustion chamber. In this case there are considered low-frequency and high-frequency (acoustic) oscillations, the motion of burning flow and combustion products, the interconnection between the basic flow and boundary layer under conditions of starting, operating, shutdown.

In this chapter are examined only some separate questions, which characterize the specific character of intrachamber processes in the first approximation.

3.1. Simplified Method of Construction of the Boundary of Low-Frequency Stability

The character of intrachamber processes is judged by their external manifestations. The basic source of information is the oscillogram, on which there is reflected a change of many parameters with time, including thrust, the flow rates of propellant components, pressure in the central part of the chamber. The oscillogram or the complete set of oscillograms, where the character of change of basic parameters during the entire period of operation of the combustion chamber is recorded, can be called the external characteristic of the chamber. Figure 3.1 shows oscillograms of

pressures before the injectors and in the combustion chamber, measured during the test of a training model of a [ZhRD] (ЖРД) liquid-propellant rocket engine.

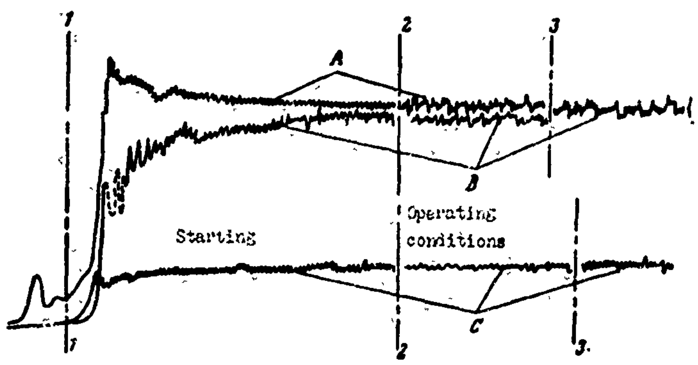


Fig. 3.1. Oscillogram of change of pressures before injectors and in the combustion chamber when starting and during operating conditions: *A* – before the oxidizer injector; *B* – before the fuel injectors; *C* – in the combustion chamber.

In the examination of oscillograms there are observed changes, various in character, of instantaneous values of parameters during starting, during prolonged continuous operation of the engine and during shutdown.

The character of change of parameters with time, especially of pressure in the chamber, is exceptionally complex.

As can be seen from Fig. 3.1, even before the ignition of propellant (to the left of line *1-1*) there was recorded a temporary increase of pressure before the oxidizer injectors, caused by the motion of liquid through hydraulic ducts.

Before starting an abrupt increase of pressure before the oxidizer injectors is noticeable. After this there are observed some decrease of pressure in the chamber and high-amplitude, gradually damped oscillations of pressure before the fuel injectors. It is usually difficult to establish a clear boundary of the termination of starting or the beginning of operation at nominal rating.

On the examined oscillogram after line *1-1* the pressure in the chamber is slowly increased to nominal value; as line *2-2* is approached the calculated value of pressure is gradually established. Curves, enclosed between lines *2-2* and *3-3*, characterize the operation of the engine in the first seconds after starting, and curves to the right of line *3-3* correspond already to approximately a hundredth of a second of operation of the engine under operating conditions.

Figure 3.2 for an example shows oscillograms of pressure in the combustion chamber, obtained during the test of a small experimental ZhRD. Depending on the launching conditions various curves (А, Б, В, Г). At operating conditions (curves Д, Е, Ж, З) there was observed superposition of high-frequency oscillations. flowing with small amplitude, on low-frequency. There are recorded sawtooth oscillations (И, К) and oscillations with amplitude periodically changing with time (Л, М). Recording with the aid of more precise instruments allowed refining the character of oscillations.

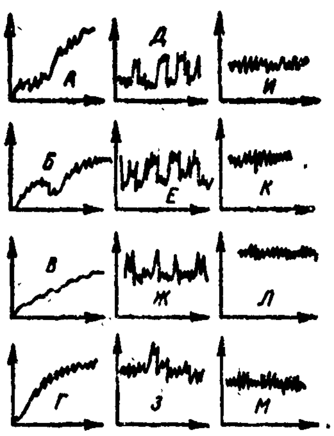


Fig. 3.2. Pressure fluctuation in the combustion chambers.

After the examination of many oscillograms it is possible to distinguish the characteristic types of oscillations.

Frequently we encounter "impact" low-frequency oscillations (Fig. 3.3, curve A), in certain cases high-frequency oscillations are superimposed on them. Low-frequency oscillations can be "sawtooth" (B) or close to sinusoidal (Д). On these types of oscillations there also can be superimposed high-frequency oscillations (Г), (E).

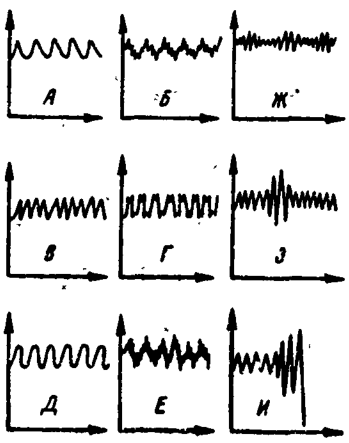


Fig. 3.3. The characteristic types of pressure fluctuations in the combustion chamber.

There are also encountered more complex types of oscillations; there is revealed nonsimilarity of type, nonrepetition of figures, there are encountered continuously following pulsations (curve Ж), caused by resonance phenomena, proceeding both in the chamber and in units of the feed system. Sometimes pulsations appear a total of one time during the entire period of engine testing. If the amplitude of oscillations is relatively small (З), then the previous mode is again established. Too powerful pulsations, caused by resonance И, lead to engine failure.

Low-frequency oscillations are of several types. Along with the oscillations, which appear as a result of connection of the chamber with hydraulic circuits, there are observed pressure fluctuations, caused only by intrachamber processes. The character of the fluctuations depends on the sensitivity of combustion delay to pressure, inertness of hydraulic circuits, the change of component ratio with time, the absolute value of pressure in the chamber and on other factors.

Let us examine the oscillations, which appear as a result of the presence of connection between the chamber and the feed system. Let us take the simplest case of connection of the chamber with the feed system, when the component ratio is constant and the combustion delay is equal to zero. Under the effect of a randomly appearing external influence the pressure in the chamber was increased from nominal value p_0 to a certain new value p_1 (Fig. 3.4). Since in this case the flow rate of gases from the nozzle, characterized by line *1* will be greater than the inflow of propellant into the chamber, determined by curve *2*, pressure in the chamber begins to decrease.

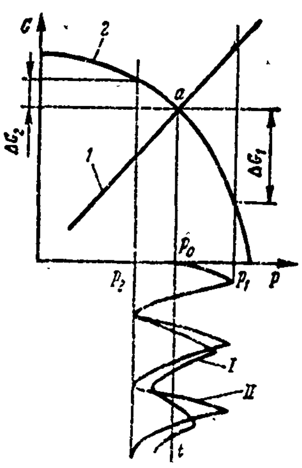


Fig. 3.4. Diagram of the appearance of oscillations, caused by the connection between parameters of the chamber and the feed system.

After the influence of a disturbing factor the change of pressure in the chamber with time can be exponential or oscillatory. If the factors, damping the system, are rather intensive, then motion will be aperiodic with exponential change of parameters, otherwise oscillations will appear. Inasmuch as the moving liquid and combustion products are inertial, in the course of lowering of pressure the point *a*, which corresponds to steady state, will be "gone," and pressure is decreased to a certain value p_2 (curve *I*). Now the inflow of propellant will be greater than the flow rate of gases. This will lead to a new increase of pressure. Thus, low-frequency damped oscillations will be observed in the system.

In actuality the propellant burns with delay. Let us consider the moment when pressure in the chamber was decreased to quantity p_2.

The inflow of propellant into the chamber is now greater by quantity ΔG_2 than nominal, determined by point a. The new, additional portion of propellant burns up not at the moment, which corresponds to pressure p_2, but with delay. The system will receive additional excitation, leading to increase of the amplitude (curve II) in comparison with what it would be in the case of damped oscillations. Upon achievement of pressure, close to p_1, the inflow of propellant into the chamber will be less than nominal. However, with consideration of delay the effect, which from combustion is obtained understated by approximately ΔG_1 of the quantity of propellant, appears with delay already when the pressure will be less than p_1, this will facilitate maintaining the onset oscillations. If the deviations of flow rates ΔG_1 and ΔG_2 are not equal to each other, then the character of pressure rise in the oscillatory process will be distinguished from the character of lowering of pressure.

It is known that the period of delay is decreased with increase of pressure. This connection affects the character of oscillations. Ultimately, under the effect of all the examined factors for the time of each oscillation period the inflow of mass into the chamber proves to be equal to the flow rate of mass just as the inflow of energy is equal to its flow rate, due to which the amplitude of low-frequency oscillations during operation of the engine is kept constant.

Low-frequency oscillations can proceed even without the presence of disturbing and damping factors on the part of the feed system. Let us assume that the feed system supplies components to the combustion chamber uniformly and with constant ratio between them. However, in the presence of intrachamber oscillations the disturbances are partially transferred into the hydraulic circuit. They are revealed in the flow area of injectors, sometimes in the injector assembly, before the injector inlet. The intrachamber instability

appears as a result of periodically repeating additional feedings of the chamber, caused by the action of pressure on the amount of delay.

Let us assume that the chamber operates at constant pressure p_0, which some constant value of delay corresponds to. Consequently, in the chamber there is a constant quantity of liquid propellant, which is prepared for combustion. With increase of pressure period τ_s is decreased, which leads to decrease of the amount of liquid propellant which is in the chamber and, consequently, to increase of the inflow of gaseous products. As before, the transition of liquid phase into gaseous proceeds with delay. With decrease of pressure the reverse phenomenon is observed — growth of τ_s, increase of the amount of liquid phase.

In the previous cases the additional feeding of the chamber by propellant occurred under the action of mass forces of moving liquid and as a result of the effect of pressure on delay. Intrachamber instability is explained only by the one last factor. Therefore, the power of intrachamber oscillations is distinguished from the power of oscillations caused by the connection of the chamber with hydraulic circuits of the feed system.

In the forming of low-frequency oscillations the whole mass of gases, located in the chamber takes part. During research we consider that the gas parameters do not change along coordinates of the chamber. With such an assumption for the study of low-frequency oscillations there are attracted differential equations, considering the changes of parameters only with time. In actuality, the gas velocity and pressure are changed along the length, and the cross section of the chamber, and in the region of location of injectors even countercurrents are observed. Preparation of propellant for combustion and propellant combustion proceed so that the intensity of gas formation is changed with time and is different at various points of the chamber volume. Combustion products, forming at various places of the chamber, intersect the nozzle throat only through a certain time interval. From the moment of

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the beginning of burning of a certain portion of propellant to the moment of detection of the greatest pressure increase a certain time passes. The moment of the beginning of burning does not coincide with the moment of discharge of those gases, which are formed during combustion of the portions of propellant, being additionally fed to the chamber in the oscillatory process. The indicated displacements are considered by the introduction of time of delay of intrachamber processes in comparison with the moment of propellant combustion. Besides this, as a result of the deformation of manifolds and the compressibility of liquid there is observed a shift of the moment of injection with time.

The presence of the examined factors indicates the need for research of low-frequency oscillations with the aid of equations, considering the change of parameters along the length of the chamber.

The behavior of the component ratio of propellant k_1 in time has a definite effect on the character of oscillations.

With throttling of the engine, i.e., with lowering of the pressure in the chamber, there is observed a so-called threshold of stability, which is manifested in the fact that with lowering of pressure in the chamber at a certain moment of time there occurs a rather rapid and sharp increase of the amplitude of oscillations. The appearance of the threshold of stability is facilitated by at least two factors. With engine throttling there occurs decrease of the hydraulic losses on injectors as a result of decrease of the flow rate of propellant components. If even period $\tau_s = 0$, then, as will be shown further, the exponential character of pressure attenuation, which increased under the action of an external disturbance, becomes periodic. With decrease of hydraulic losses the quality of atomization is impaired and, consequently, the value of combustion delay is increased. Thus, increase of τ_s and decrease of the stability of engine operation on the whole lead to the abrupt increase of the amplitude of engine oscillations.

In order to give an initial idea of low-frequency instability, let us examine a simplified method of its calculation. Let flow rate

$$G=\mu F\sqrt{2(p_1-p_\kappa)\rho_ж}. \tag{3.1}$$

In the examination of the vicinity of steady state, it can be considered that the feed pressure of propellant p_1 is little changed; let us take p_1 = const. For small deviations instead of expression (3.1) let us write the linear approximation; if $\mu F\sqrt{\rho_ж}=\text{const}$, then with consideration of delay

$$\frac{\Delta G_{-\tau}}{G_0}=\Delta\tilde{G}_{-\tau_0}=\frac{(\Delta\tilde{p}_\kappa)_{-\tau_s}}{\Delta\tilde{p}_1}, \tag{3.2}$$

where relative change of pressure

$$\Delta\tilde{p}_1=2\frac{p_1-p_{\kappa 0}}{p_{\kappa 0}};\ \Delta\tilde{p}_\kappa=\frac{\Delta p_\kappa}{p_{\kappa 0}}. \tag{3.3}$$

Here $p_{\kappa 0}$ – nominal pressure in the chamber.

The equation of the chamber is now written so:

$$\varepsilon\Delta\dot{\tilde{p}}_\kappa+\Delta\tilde{p}_\kappa=-\frac{(\Delta\tilde{p}_\kappa)_{-\tau_s}}{\Delta\tilde{p}_1}. \tag{3.4}$$

Let us examine the solution of (3.4) in the following form [64]:

$$\Delta\tilde{p}_\kappa=C\exp(\lambda t). \tag{3.5}$$

By substituting the solution of (3.5) in equation (3.4), we arrive at characteristic equation

$$\varepsilon\lambda+1+\frac{1}{\Delta\tilde{p}_1}\exp(-\lambda\tau)=0. \tag{3.6}$$

The value of the roots of the characteristic equation depends on ε, τ_s and Δp_1. If the real part of the roots of equation (3.6) is negative, then amplitude $\Delta\tilde{p}_{\text{н}}$ will be decreased with time.

The equality of the imaginary part of roots to zero corresponds to the stability limit. Therefore, to get the equation of the stability limit it is necessary to substitute $\lambda = i\omega$ in equation (3.6). Replacing t by τ_s and assuming then

$$\exp(-i\omega\tau_s) = \cos(\omega\tau_s) - i\sin(\omega\tau_s), \tag{3.7}$$

we arrive at equation

$$\Delta\tilde{p}_1 + \cos(\omega\tau_s) + i[(\varepsilon\Delta\tilde{p}_1\omega - \sin(\omega\tau_s)] = 0. \tag{3.8}$$

After conversions of equation (3.8) we obtain

$$\Delta\tilde{p}_1 - \tau_s = \frac{1}{\sqrt{1+(\omega\varepsilon)^2}} + \frac{\operatorname{arctg}(\omega\varepsilon) - k\pi}{\omega}. \tag{3.9}$$

When conducting research in the first approximation instead of an infinite sequence of whole and positive numbers we take $k = 1$. Further for the considered chamber we find ε and, by changing ω from 0 to ∞, we construct the relationship of τ_s to Δp_1; the line obtained on the graph is the stability limit. The area, which corresponds to large values of τ_s, refers to unstable operation, and the area of large values of $\Delta\tilde{p}_1$ characterizes stable operation of the engine. Therefore, by knowing τ_s, according to the graph we determine the smallest permissible value of $\Delta\tilde{p}_1$ then by equation (3.3) we find the difference of $p_1 - p_{\text{к}0}$, providing steady operation of the engine.

Fundamental research of low-frequency instability is discussed in a number of works (see, for example, [48]).

3.2. High-Frequency Oscillations in the Starting Period

Theoretical and experimental research allowed partially presenting a qualitative picture of the onset of high-frequency oscillations during starting [62], [77], Sometimes they are manifested immediately, in the starting period of the chamber, but there are cases when the high-frequency is developed as if with delay, in the first quarter of a second of engine operation (Fig. 3.5). The impression is created that the engine normally started and that hf appears already at operating conditions.

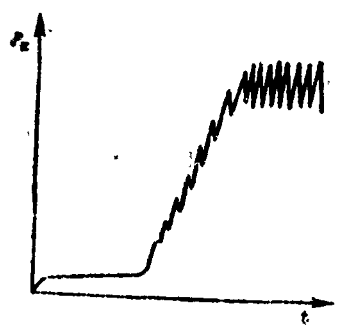

Fig. 3.5. The development of high-frequency oscillation during engine starting.

The study of oscillograms shows that hf is generated apparently in the starting period of the engine, moreover sometimes high-frequency oscillations precede low-frequency (Fig. 3.6). The amplitude of the generated low-frequency oscillations is kept constant. However, there are cases of sharp increase of the amplitude, terminating in failure of the combustion chamber (Fig. 3.7).

The overall picture of emergence of hf during starting can be represented as the development of randomly formed and rather rapidly amplified disturbance, which is accompanied by a change in the state.

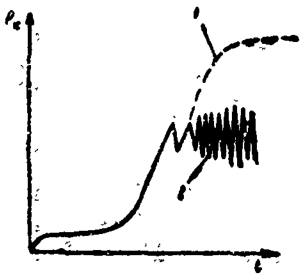

Fig. 3.6. The appearance of high-frequency oscillations after the appearance of low-frequency: 1 – expected values; 2 – actual values.

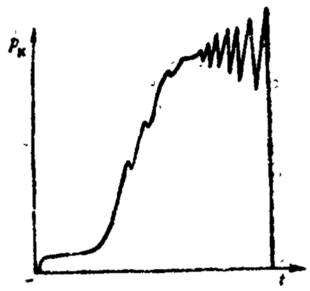

Fig. 3.7. High-frequency oscillations, which lead to failure of the chamber.

The hf appears in the presence of a shock wave – sharp abrupt increase of pressure, which is accompanied by compression, heating and by change of the velocity of motion of burning flow. The possibility of the appearance of knocking is not excluded – the explosive propagation of chemical transformations, which are accompanied by heat liberation.

The shock wave, which appeared and was reflected from chamber walls or the nozzle, in transit through the burning zone is intensified, if its intensity is higher than a certain level. The intensification of the wave process also depends on the time the wave is located in the burning zone.

Longitudinal and transverse waves are observed, moreover the longitudinal shock wave can excite transverse waves, inasmuch as with passage of longitudinal wave through burning zone there can be formed waves, which are propagated in all directions [15].

The generalization of calculation and experimental data allows determining some conditions, which facilitate the emergence of hf during starting.

It was revealed that the probability of emergence of hf is increased with increase of thrust and pressure in the combustion chamber. This is apparently explained by the fact that with increase of the combustion chamber volume and the pressure in it the power of the shock wave is increased. With a number of simplifying assumptions it is obtained that for a cylindrical chamber the average length of the chain [43]

$$\nu=\frac{d^2}{10ql_{\text{пр}}^2}, \tag{3.10}$$

and for a spherical chamber

$$\nu=\frac{d^2}{20ql_{\text{пр}}^2}, \tag{3.11}$$

where $l_{\text{пр}}$ – the average length of the path; q – probability factor.

Thus, a spherical chamber should operate more stably than cylindrical. It is established that other conditions being equal the engines with a pressurized system are steadier than engines with a [TNA] (THA) turbopump assembly. It is possible that here during starting there is an effect of sharp lowering of pressure in the manifolds, which connect the tanks with pumps; in this case cavitation appears in the pumps, and vaporous products penetrate the combustion chamber.

It is established that upon the introduction of a small portion of gaseous oxygen into the chamber, which operates in steady state, hf sometimes appears [67]. Here, apparently, active centers are developed – atomic oxygen or radical OH. As a result a chain reaction appears, the rate of which

$$U=U_0\nu\left[1-\exp\left(-\frac{t}{t_0}\right)\right], \qquad (3.12)$$

where ν — average length of the chain; t_0 — the lifetime of active center.

In the kinetics of chain reactions catalysts and the initiating action of surfaces play a large role. Chain reactions are very sensitive to the smallest quantities of catalysts, which can accelerate or impede the process. By experiments it is established that with small additions of certain products — antiknock agents to the propellant, the stability of engine operation is sharply increased. The active center, which was initiated in the region of the injector assembly, can be moved to the main volume of the chamber. For decrease of the probability of the initiation of chain reactions we recommend before starting to cover the surface of the chamber with a thin layer of the appropriate substance, which burns up or is removed from the chamber after starting.

The sequence of ignition and the amount of advance of feed of components have a large effect on the probability of appearance of hf. The closer the initial component ratio to stoichiometric, the greater the quantity of heat that is liberated at the initial moment, and the more favorable are the conditions for initiation of a shock wave. On the other hand, depending on the starting conditions there can be such a combination of parameters of the chamber, at which excitation of the system is very probable. It is also revealed that with advance of fuel feed the high-frequency appears more frequently than with advance of oxidizer feed.

For anergolic propellants the power of the ignition source has a substantial effect, with increase of which the probability of the appearance of hf is increased.

Preliminary supercooling of components leads to smoother starting.

Depending on the starting conditions, the layout of the propulsion system and the parameters of its units there is developed a certain character of pressure buildup in the chamber during firing. The appearance of hf facilitates the appearance of regions of sharp changes of pressure on curve $p_{\text{к}}(t)$. In certain cases they are manifested in the form of a serrated discontinuity, so for a very short time interval the sign of derivative $\dot{p}_{\text{к}}$ changes two times (Fig. 3.8). Such a character of change of pressure to a certain degree depends on the total amount of propellant, accumulated in the chamber at the moment of starting, on the starting conditions and on parameters of the feed system. After serrated discontinuity sometimes, to be sure with small probability, somewhat low-frequency pulses appear, changing into high-frequency.

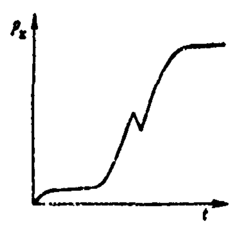

Fig. 3.8. An example of the change of pressure in a chamber during starting.

It is noticed that the appearance of hf is facilitated by high values of product $k_1 \frac{\partial (RT)}{\partial k_1}$.

There were observed cases of appearance of hf above permissible with failure during actuation of the generator and of combustion chamber.

The intensity of shock waves in the region of the injector assembly is decreased with increase of the frequency of oscillations, i.e., with increase of the frequency of their affect on the burning zone.

During research of the engine there is revealed a field of values $p_{\text{к}}$ and k_1, where the appearance of hf is most probable. Rational methods of decrease of the probability of appearance of hf are bypass of the zones of instability in coordinates $p_{\text{к}}k_1$ during starting or the accelerated passage of these zones. Depending on the relationship of parameters $k_1(t), \dot{k}_1(t)$ and $p_{\text{к}}(t)$ it is possible to accomplish accident-free starting or, on the contrary, to excite hf.

Inasmuch as the intensification of shock wave occurs in the burning zone, then one of the effective methods of lowering the probability of appearance of hf or the amplitude of oscillations is the distribution energy, released during burning along the length, of the chamber, i.e., extension of the burning front or the organization of several fronts.

For decrease of the intensity of reflection of acoustic waves the chamber wall is made "elastic." This is attained by the installation of acoustic dampers. In one of the training engines as a damper there was used an additional interior wall with openings. If it is required to eliminate hf during starting, then it is possible to apply dampers, which burn out or are ejected after starting. In well used engines the acoustic oscillations are hunting, moreover the amplitudes and frequencies of oscillations are barely changed during the entire period of operation of the engine. Nevertheless we find cases when in separate (rare) specimens of the same batch of engines developing acoustic oscillations are observed.

Acoustic oscillations represent a system of random waves and are caused by compression of medium.

In the presence of acoustic oscillations the velocity of motion of the medium can be as low as desired in comparison with the speed of propagation of acoustic waves, equal to the speed of propagation of sound. In the region of the nozzle throat during calculation of wave processes it is necessary to reckon with the speed of motion of combustion products. An acoustic wave does not move a material

medium, but only swings it around constant or slowly changing values of parameters in cruise conditions or around values of parameters which are changed specifiably and rather rapidly during engine starting and during shutdown. Acoustic oscillations are characterized by discrete distribution of energies of pulsations with respect to frequencies. In the gas flow there is a basic frequency of acoustic oscillations, which depends on the chamber geometry. Besides this, there is observed a number of other, weaker expressed harmonics, multiple or almost multiple of the basic.

During the study of acoustic oscillations it is necessary to consider the effect of inlet gas ducts and the nozzle; for this a model of the complex is created – a conditional combustion chamber. The oscillation period of gas in such a chamber (pipe) is equal to the time of run of the wave along the gas flow and back. In the simplest case the oscillation period

$$t=2\int_{0}^{l_0}\frac{C\,dx}{C^2-w^2},$$

where C – local speed of sound; w – the velocity of motion of gas; l_0 – characteristic dimension. The frequency of the first harmonic

$$\omega_1=\frac{2\pi}{t}.$$

If acoustic oscillations are not caused by external disturbing influences and are hunting, then supply and loss of acoustic energy are equal to each other. The supply of energy is caused by intensification of propellant burning at the moment of intersection of the burning zone by an acoustic wave. Loss of energy is determined mainly by conditions of the passage of the wave in a gas medium. The interaction between the burning zone and acoustic wave is determined by conditions of reflection of the wave from walls In contemporary chambers the transverse (radial) oscillations proceed with incommensurably greater activity than longitudinal oscillations.

During the measurement of engine parameters with the aid of fast-response instruments, besides acoustic oscillations there are revealed turbulent fluctuations, which represent chaotic turbulent motion of the medium. The vortex scale is relatively small. The motion of the medium is characterized by the speed of rotation of vortices and by their movement along the chamber volume; the latter depends on the coefficient of turbulent diffusion.

Turbulent fluctuations are caused by the existence, mainly, of fluidity of liquid and are little connected with its compressibility; in any case, the theory of turbulence is based on the condition of incompressibility of liquid. For turbulent fluctuations the continuous distribution of energy of fluctuation with respect to frequencies is characteristic. However, the compressibility of liquid affects the character of the turbulent field and therefore turbulence generates acoustic oscillations.

For the determination of turbulent fluctuations we frequently use the mean-square value of fluctuations

$$\sqrt{\overline{U'}}.$$

Because of turbulent and acoustic fluctuations the burning conditions acquire a vibratory character with noticable periodicity of change of the parameters of burning flow. Many works are devoted to the study of vibratory burning, the works of B. V. Raushenbakh [75] occupy an important place in these investigations.

3.3. Wave Equation

For now we are not able to solve the complete system of equations, which describe intrachamber processes, even with utilization of contemporary computer technology. It is necessary to be limited to the examination of simplified problems. One of them is the problem of research of the character of occurre[illegible] wave processes.

Let us examine the derivation of a wave equation for the case when a chamber is filled with gas. This equation is solved relative to the flow velocity, velocity and density potential or sonic pressure. The velocity potential ψ is introduced so that the following relationships occur [50]:

$$\left.\begin{aligned} W_x &= -\frac{\partial\psi}{\partial x}; \\ W_r &= -\frac{\partial\psi}{\partial y}; \\ W_\varphi &= -\frac{\partial\psi}{\partial z}. \end{aligned}\right\} \tag{3.13}$$

Let us give the derivation of wave equation, using the Cartesiar coordinate system.

Let us use three equations of motion, the equation of state and the equation of continuity. The equations of motion not allowing for the action of external forces and viscous friction forces are written so:

$$\left.\begin{aligned} \varrho\dot{W}_x &= -\frac{\partial p}{\partial x}; \\ \varrho\dot{W}_y &= -\frac{\partial p}{\partial y}; \\ \varrho\dot{W}_z &= -\frac{\partial p}{\partial z}. \end{aligned}\right\} \tag{3.14}$$

During the study of small deviations of parameters from their nominal values we replace ϱ by constant quantity $\varrho_0=\text{const.}$ The total derivatives contain local accelerations and transfer accelerations, in other words

$$\dot{W}_x = \frac{\partial W_x}{\partial t} + \overline{W}_x \operatorname{grad} W_x. \tag{3.15}$$

Local acceleration is created with field of parameters variable in time, and transfer acceleration is caused by the fact that the element of gas during time dt changes its attitude. For small amplitudes the transfer acceleration in comparison with local can be disregarded, and then the equation of motion assumes the form

$$\varrho_0 \frac{\partial W_x}{\partial t} + \frac{\partial p}{\partial x} = 0. \tag{3.16}$$

We use the equation of state in such a form:

$$\frac{\partial p}{\partial \varrho} = C^2, \tag{3.17}$$

where C – the speed of sound.

The equation of continuity has the form

$$\frac{1}{\varrho_0} \frac{\partial \varrho}{\partial t} + \operatorname{div} \overline{W} = 0. \tag{3.18}$$

By replacing the pressure in three equations of motion by density from the equation of state, we arrive at system:

$$\left.\begin{aligned} \varrho_0 \frac{\partial W_x}{\partial t} + C^2 \frac{\partial \varrho}{\partial x} &= 0; \\ \varrho_0 \frac{\partial W_y}{\partial t} + C^2 \frac{\partial \varrho}{\partial y} &= 0; \\ \varrho_0 \frac{\partial W_z}{\partial t} + C^2 \frac{\partial \varrho}{\partial z} &= 0; \\ \frac{\partial \varrho}{\partial t} + \varrho_0 \operatorname{div} \overline{W} &= 0. \end{aligned}\right\} \tag{3.19}$$

For further transformations we use operators: for the first equation operator $\partial/\partial x$, for the second – $\partial/\partial y$, for the third – $\partial/\partial z$ and for the fourth – $\partial/\partial t$.

Now we obtain system:

$$\left.\begin{aligned}
&\varrho_0\frac{\partial^2 W_x}{\partial t\,\partial x}+C^2\frac{\partial^2\varrho}{\partial x^2}=0;\\
&\varrho_0\frac{\partial^2 W_y}{\partial t\,\partial y}+C^2\frac{\partial^2\varrho}{\partial y^2}=0;\\
&\varrho_0\frac{\partial^2 W_z}{\partial t\,\partial z}+C^2\frac{\partial^2\varrho}{\partial z^2}=0;\\
&\frac{\partial^2\varrho}{\partial t^2}+\varrho_0\left(\frac{\partial^2 W_x}{\partial t\,\partial x}+\frac{\partial^2 W_y}{\partial t\,\partial y}+\frac{\partial^2 W_z}{\partial t\,\partial z}\right)=0.
\end{aligned}\right\} \qquad (3.20)$$

By adding term by term the first three equation and with the aid of the fourth by excluding expression

$$\varrho_0\left(\frac{\partial^2 W_x}{\partial t\,\partial x}+\frac{\partial^2 W_y}{\partial t\,\partial y}+\frac{\partial^2 W_z}{\partial t\,\partial z}\right)=\varrho_0\,\frac{\partial}{\partial t}(\operatorname{div}\overline{W}), \qquad (3.21)$$

we arrive at the wave equation for density:

$$\frac{1}{C^2}\,\frac{\partial^2\varrho}{\partial t^2}-\nabla^2\varrho=0, \qquad (3.22)$$

where operator

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}. \qquad (3.23)$$

In a cylindrical coordinate system the equation for density takes the form

$$\frac{1}{C^2}\,\frac{\partial^2\varrho}{\partial t^2}-\frac{\partial^2\varrho}{\partial x^2}-\frac{\partial^2\varrho}{\partial r^2}-\frac{1}{r}\,\frac{\partial\varrho}{\partial r}-\frac{1}{r^2}\,\frac{\partial^2\varrho}{\partial\varphi^2}=0. \qquad (3.24)$$

Analogically there is written an equation for sound pressure, in this case in equation (3.24) the density should be expressed through pressure.

With the presence of external mass (volumetric) forces ($j_i + g_i$) and with supply of gas into the element due to vaporization of liquid the basic equations with the assumption that

$$S_l \to 0;$$
$$\chi \to 0,$$

will take the form

$$\left.\begin{aligned}
&\rho_0\frac{\partial W_x}{\partial t}+\frac{\partial p}{\partial x}=\rho_0(j_x+g_x);\\
&\rho_0\frac{\partial W_y}{\partial t}+\frac{\partial p}{\partial y}=\rho_0(j_y+g_y);\\
&\rho_0\frac{\partial W_z}{\partial t}+\frac{\partial p}{\partial z}=\rho_0(j_z+g_z);\\
&\frac{\partial \rho}{\partial t}+\rho_0\left(\frac{\partial W_x}{\partial x}+\frac{\partial W_y}{\partial y}+\frac{\partial W_z}{\partial z}\right)=\Omega.
\end{aligned}\right\} \tag{3.25}$$

After conversions, analogous to those performed above, by using the equation of state in the form [84]

$$p=C^2\rho+\mathrm{const} \tag{3.26}$$

and using replacements

$$\left.\begin{aligned}
&\frac{\partial^2 p}{\partial x^2}=C^2\frac{\partial^2 \rho}{\partial x^2};\\
&\frac{\partial^2 p}{\partial y^2}=C^2\frac{\partial^2 \rho}{\partial y^2};\\
&\frac{\partial^2 p}{\partial z^2}=C^2\frac{\partial^2 \rho}{\partial z^2};
\end{aligned}\right\} \tag{3.27}$$

we obtain a wave equation in the form

$$\frac{1}{C^2}\frac{\partial^2 \rho}{\partial t^2}-\nabla^2\rho=\frac{\partial}{\partial t}\Omega-\rho_0\,\mathrm{div}\,(\overline{j+g}). \tag{3.28}$$

In the right side of sonic equation (3.28) there is considered the supply of mass $\frac{\partial}{\partial t}\Omega$ and the effect of external volumetric forces $\rho_0\,\mathrm{div}\,(\overline{j+g})$.

3.4. High-Frequency Oscillations at Operating Conditions

A peculiarity of calculation is the fact that the amplitude of oscillations is small in comparison with nominal values of appropriate parameters. Therefore, calculation can be performed in small deviations, i.e., linearization of equations is possible. We will consider that disturbances are propagated with the speed of sound, constant along the whole volume of the chamber. Let us examine the area of the chamber after the burning zone, for which the equation of the law of conservation of mass and the equation of motion will be written so:

$$\left.\begin{aligned}&\frac{\partial\varrho}{\partial t}+\operatorname{div}(\varrho\overline{W})=0;\\&\varrho\frac{\partial W_x}{\partial t}+\varrho\frac{\partial W_x}{\partial x}W_x+\frac{\partial p}{\partial x}=0;\\&\varrho\frac{\partial W_y}{\partial t}+\varrho\frac{\partial W_y}{\partial y}W_y+\frac{\partial p}{\partial y}=0;\\&\varrho\frac{\partial W_z}{\partial t}+\varrho\frac{\partial W_z}{\partial z}W_z+\frac{\partial p}{\partial z}=0.\end{aligned}\right\}\qquad(3.29)$$

In expanded form the first equation of the system will be written so:

$$\frac{\partial\varrho}{\partial t}+\frac{\partial}{\partial x}(\varrho W_x)+\frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r r)+\frac{\partial}{r\partial\varphi}(\varrho W_\varphi)=0\qquad(3.30)$$

or

$$\frac{\partial\varrho}{\partial t}+\varrho\frac{\partial W_x}{\partial x}+\frac{\partial\varrho}{\partial x}W_x+\varrho\frac{W_r}{r}+\frac{\partial\varrho}{\partial r}W_r+\varrho\frac{\partial W_r}{\partial r}+$$
$$+\varrho\frac{\partial W_\varphi}{r\partial\varphi}+\frac{\partial\varrho}{\partial\varphi}\frac{W_\varphi}{r}=0.\qquad(3.31)$$

Let us introduce small deviations so that along axis x the total velocity will be determined by the sum of main velocity W_{x0} and disturbance δW_x. Along other axes there will be considered only disturbances δW_r and δW_φ, since flows here are absent. Thus

$$\left.\begin{aligned} W_x &= W_{x0} + \delta W_x; \\ W_r &= \delta W_r; \\ W_\varphi &= \delta W_\varphi; \\ p &= p_0 + \delta p; \\ \varrho &= \varrho_0 + \delta\varrho. \end{aligned}\right\} \qquad (3.32)$$

With substitution of equalities (3.32) in equation (3.31) some terms, containing the product of small deviations, are excluded. Ultimately we arrive at system

$$\left.\begin{aligned} &\frac{\partial}{\partial t}\delta\varrho + \varrho_0\frac{\partial}{\partial x}\delta W_x + W_{x0}\frac{\partial}{\partial x}\delta\varrho + \frac{\varrho_0\delta W_r}{r} + \\ &\quad + \varrho_0\frac{\partial}{\partial r}\delta W_r + \varrho_0\frac{\partial}{r\partial\varphi}\delta W_\varphi = 0; \\ &\varrho_0\frac{\partial}{\partial t}\delta W_x + \varrho_0\frac{\partial}{\partial x}\delta W_x W_{x0} + \frac{\partial}{\partial x}\delta p = 0; \\ &\varrho_0\frac{\partial}{\partial t}\delta W_r + \varrho_0\frac{\partial}{\partial x}\delta W_r W_{x0} + \frac{\partial}{\partial r}\delta p = 0; \\ &\varrho_0\frac{\partial}{\partial t}\delta W_\varphi + \varrho_0\frac{\partial}{\partial x}\delta W_\varphi W_{x0} + \frac{\partial}{r\partial\varphi}\delta p = 0. \end{aligned}\right\} \qquad (3.33)$$

Let us introduce dimensionless parameters

$$W'_x = \frac{\delta W_x}{C}; \quad W'_r = \frac{\delta W_r}{C}; \quad W'_\varphi = \frac{\delta W_\varphi}{C};$$

$$\varrho' = \frac{\delta\varrho}{\varrho_0}; \quad p' = \frac{\delta p}{p_0}; \quad x' = \frac{x}{L}; \quad r' = \frac{r}{L},$$

where L – characteristic dimension of the chamber. Since Mach number

$$M = \frac{W_{x0}}{C}, \qquad (3.34)$$

the system of equations will be written so [17]:

$$\frac{\partial\varrho'}{\partial t} + \frac{\partial W'_x}{\partial x'} + M\frac{\partial\varrho'}{\partial x'} + \frac{\partial W'_r}{\partial r'} + \frac{W'_r}{r'} + \frac{\partial W'_\varphi}{r'\partial\varphi} = 0; \qquad (3.35)$$

$$\frac{\partial W'_x}{\partial t} + M\frac{\partial W'_x}{\partial x'} + \frac{\partial\varrho'}{\partial x'} = 0; \qquad (3.36)$$

$$\frac{\partial W'_r}{\partial t}+M\frac{\partial W'_r}{\partial x'}+\frac{\partial \varrho'}{\partial r'}=0; \qquad (3.37)$$

$$\frac{\partial W'_\varphi}{\partial t}=M\frac{\partial W'_\varphi}{\partial x'}+\frac{\partial \varrho'}{r'\partial \varphi}=0. \qquad (3.38)$$

Having the taken partial derivative during x' from function (3.35), we obtain

$$\frac{\partial^2 \varrho'}{\partial t\partial x'}+\frac{\partial^2 W'_x}{\partial (x')^2}+M\frac{\partial^2 \varrho'}{\partial (x')^2}+\frac{\partial^2 W'_r}{\partial x'\partial r'}+\frac{1}{r'}\frac{\partial W'_r}{\partial x'}+\frac{\partial^2 W'_\varphi}{\partial x'\cdot r'\partial \varphi}=0, \qquad (3.39)$$

whence we find

$$\frac{\partial^2 W'_x}{\partial (x')^2}+\frac{\partial^2 W'_r}{\partial x'\partial r'}+\frac{\partial^2 W'_\varphi}{\partial x'\cdot r'\partial \varphi}=$$
$$=-\frac{\partial^2 \varrho'}{\partial t\,\partial x'}-M\frac{\partial^2 \varrho'}{\partial (x')^2}-\frac{1}{r'}\frac{\partial W'_r}{\partial x'}. \qquad (3.40)$$

From the same function (3.35) let us take the partial derivative during t and we obtain

$$\frac{\partial^2 \varrho'}{\partial t^2}+\frac{\partial^2 W'_x}{\partial t\,\partial x'}+M\frac{\partial^2 \varrho'}{\partial t\partial x'}+\frac{\partial^2 W'_r}{\partial t\,\partial r'}+\frac{1}{r'}\frac{\partial W'_r}{\partial t}+$$
$$+\frac{\partial^2 W'_\varphi}{\partial t\cdot r'\partial \varphi}=0. \qquad (3.41)$$

Let us take partial derivatives from functions (3.36), (3.37), (3.38) during x', r' and ϕ respectively:

$$\frac{\partial^2 W'_x}{\partial t\,\partial x'}+M\frac{\partial^2 W'_x}{\partial (x')^2}+\frac{\partial^2 \varrho'}{\partial (x')^2}=0; \qquad (3.42)$$

$$\frac{\partial^2 W'_r}{\partial t\,\partial r'}+M\frac{\partial^2 W'_r}{\partial x'\,\partial r'}+\frac{\partial^2 \varrho'}{\partial (r')^2}=0; \qquad (3.43)$$

$$\frac{\partial^2 W'_\varphi}{\partial t\cdot r'\,\partial \varphi}+M\frac{\partial^2 W'_\varphi}{\partial x'\cdot r'\partial \varphi}+\frac{\partial^2 \varrho'}{(r')^2\partial \varphi^2}=0. \qquad (3.44)$$

Having added term by term the last three equations and performed replacement with the aid of equation (3.40), we find

$$\frac{\partial^2 W'_x}{\partial t\,\partial x'}+\frac{\partial^2 W'_r}{\partial t\,\partial r'}+\frac{\partial^2 W'_\varphi}{\partial t\cdot r'\,\partial\varphi}+\frac{\partial^2\varrho'}{\partial(x')^2}+\frac{\partial^2\varrho'}{\partial(r')^2}+$$
$$+\frac{\partial^2\varrho'}{(r')^2\,\partial\varphi^2}-M\frac{\partial^2\varrho'}{\partial t\,\partial x'}-M^2\frac{\partial^2\varrho'}{\partial(x')^2}-M\frac{1}{r'}\frac{\partial W'_r}{\partial x'}=0. \qquad (3.45)$$

Let us replace the first three terms, using equation (3.41). Then, with the aid of equation (3.37) after conversions we finally find

$$(1-M^2)\frac{\partial^2\varrho'}{\partial(x')^2}+\frac{\partial^2\varrho'}{\partial(r')^2}+\frac{\partial^2\varrho'}{(r')^2\,\partial\varphi^2}+\frac{1}{r'}\frac{\partial\varrho'}{\partial r'}=$$
$$=\frac{\partial^2\varrho'}{\partial t^2}+2M\frac{\partial^2\varrho'}{\partial t\,\partial x'}. \qquad (3.46)$$

To investigate only axial or only radial oscillations we use equations

$$\frac{\partial^2\varrho'}{\partial t^2}+2M\frac{\partial^2\varrho'}{\partial t\,\partial x'}=(1+M^2)\frac{\partial^2\varrho'}{\partial(x')^2};$$
$$\frac{\partial^2\varrho'}{\partial t^2}=\frac{\partial^2\varrho'}{\partial(r')^2}+\frac{1}{r'}\frac{\partial\varrho'}{\partial r'},$$

which have been considered by other authors [48]. Equation (3.46) does not consider the supply of energy and the supply of mass into the burning flow. If it is necessary to take into account the supply of mass and energy, then with derivation of equation (3.46) one should take into account the considerations, analogous to those discussed during derivation of equation (3.28). Inasmuch as (3.46) is written in linearized form, then it can be used for the study of wave processes, proceeding in the vicinity of steady state.

Let us examine the solution of equation (3.46), being guided by harmonic oscillations in the chamber. Let us take

$$\varrho'=A_\varrho(x', r', \varphi)\exp(i\omega' t)=A_\varrho\exp(i\omega' t), \qquad (3.47)$$

where A_ϱ — some function of density, depending on space coordinates; let us take also

$$\exp(i\omega' t)=\cos\omega' t+i\sin\omega' t. \qquad (3.48)$$

It is obvious that

$$\frac{\partial\varrho'}{\partial x'}=\frac{d}{dx'}A_\varrho\exp(i\omega' t), \qquad (3.49)$$

since $\exp(i\omega' t)$ does not depend on x'. Derivative

$$\frac{\partial\varrho'}{\partial t}=-i\omega' A_\varrho\exp(i\omega' t), \qquad (3.50)$$

since A_ϱ does not depend on t.

Now let us write equation (3.46) relative to function A_ϱ, using expression (3.47):

$$(1-M^2)\frac{\partial^2 A_\varrho}{\partial(x')^2}-2Mi\omega'\frac{\partial A_\varrho}{\partial x'}+(\omega')^2 A_\varrho+\frac{\partial^2 A_\varrho}{\partial(r')^2}+$$
$$+\frac{1}{r'}\frac{\partial A_\varrho}{\partial r'}+\frac{1}{(r')^2}\frac{\partial^2 A_\varrho}{\partial(\varphi')^2}=0. \qquad (3.51)$$

By using the Fourier method, let us substitute the sought function A_ϱ by the product of three functions, each of which depends only on one space coordinate [84]:

$$A_\varrho(x', r', \varphi)=X(x')R(r')\Phi(\varphi). \qquad (3.52)$$

Each new function has a derivative, nonzero only when the derivative is taken during the space coordinate, on which the function depends. Thus, by substituting equality (3.52) in equation (3.51), after conversions we obtain

$$(1-M^2)\frac{1}{X}\frac{d^2X}{dx'^2}-2\frac{Mi\omega'}{X}\frac{dX}{dx'}+(\omega')^2=$$
$$=-\frac{1}{R}\frac{d^2R}{d(r')^2}-\frac{1}{r'R}\frac{\partial R}{\partial r'}-\frac{1}{(r')^2\Phi}\frac{d^2\Phi}{d\varphi^2}. \quad (3.53)$$

The left side depends only on x', and the right – only on r' and ϕ. Consequently, for providing equality between them, each should be equal to the same parameter, which we designate n^2, and at assigned values of terms of equation (3.53) – to the same number. The conversion, which was finished by the obtaining of equation (3.53), eliminated the need to use partial derivatives, which considerably simplifies calculation. Now equation (3.53) is decomposed into two:

$$(1-M^2)\frac{1}{X}\frac{d^2X}{d(x')^2}-2\frac{Mi\omega'}{X}\frac{dX}{dx'}+(\omega')^2=-n^2; \quad (3.54)$$
$$\frac{1}{R}\frac{d^2R}{d(r')^2}+\frac{1}{r'R}\frac{dR}{dr'}+\frac{1}{(r')^2\Phi}\frac{d^2\Phi}{d\varphi^2}=n^2. \quad (3.55)$$

Let us rewrite equation (3.55) so:

$$\frac{(r')^2}{R}\frac{d^2R}{d(r')^2}+\frac{r'}{R}\frac{dR}{dr'}-n^2(r')^2=-\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}. \quad (3.56)$$

Now we can equate both sides of equation (3.56) to the same number, for example m^2, inasmuch as its left side depends on r', and the right only on, i.e.,

$$\frac{(r')^2}{R}\frac{d^2R}{d(r')^2}+\frac{r'}{R}\frac{dR}{dr'}-n^2(r')^2=m^2, \quad (3.57)$$

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}=m^2. \quad (3.58)$$

Thus, for each of the three functions separate equations are obtained: (3.54), (3.57) and (3.58). Constants n and m are determined from boundary conditions.

Equation (3.54) is homogeneous with constant factors. Its solution has the form

$$X(x')=C_1\exp\left(-i\frac{\sqrt{(\omega')^2-n^2(1-M^2)}-M\omega'}{1-M^2}x'\right)+$$
$$+C_2\exp\left(i\frac{\sqrt{(\omega')^2-n^2(1-M^2)}-M\omega'}{1-M^2}x'\right); \tag{3.59}$$

where C_1 and C_2 – integration constants.

The solution of equation (3.57) has the form

$$R(r')=BI_m(nr')+DN_m(nr'), \tag{3.60}$$

and equation (3.58) -

$$\Phi(\varphi)=E_1\exp(-im\varphi)+E_2\exp(im\varphi), \tag{3.61}$$

where B, D and E – integration constants; $I_m(nr')$ — Bessel function; $N_m(nr')$ — Neumann function.

By multiplying the right parts of solutions (3.59), (3.60) and (3.61), we obtain $A'_\varrho(x', r', \varphi)$.

Analogous conversions can be used for determination of components

$$W'_x,\ W'_r,\ W'_\varphi.$$

3.5. Investigation of Intrachamber Process by the Method of Small Deviations

The solution of a total system of equations in partial derivatives, obtained in the second chapter, with the contemporary state of methods of mathematical analysis presents considerable difficulties. It is necessary to simplify equations, to apply approximate methods of solution, one of which is the method of

small deviations. Let us assume that as a result of integration of differential equation in partial derivatives with assigned boundary conditions a continuous solution is obtained. If so, then around any fixed value of coordinates it is possible to trace the effect of small deviation of one parameter (or a series of parameters) to any other parameter.

In the beginning let us examine an equation of continuity in the form

$$\frac{\partial \varrho}{\partial t}=\varrho_0\left(\frac{\partial W_x}{\partial x}+\frac{\partial W_r}{\partial r}+\frac{W_r}{r}+\frac{\partial W_\varphi}{r\partial \varphi}\right). \tag{3.62}$$

Here any parameters and derivatives can obtain small deviations, therefore

$$\Delta\frac{\partial \varrho}{\partial t}=\left(\frac{\partial W_x}{\partial x}+\frac{\partial W}{\partial r}+\frac{W_r}{r}+\frac{\partial W_\varphi}{r\partial \varphi}\right)_*\Delta\varrho_0+$$
$$+\varrho_{0*}\left(\Delta\frac{\partial W_x}{\partial x}+\Delta\frac{\partial W_r}{\partial r}+\Delta\frac{W_r}{r}+\Delta\frac{\partial W_\varphi}{r\partial \varphi}\right), \tag{3.63}$$

where Δ means small deviation, and * indicates that the given parameter (or parameters) is calculated for the selected fixed value of coordinates.

Let us assume that in section x_* at moment of time t_* there occurred change of derivative $\partial W_x/\partial x$ to quantity $\Delta\frac{\partial W_x}{\partial x}$. It is necessary to determine how derivative $\partial\varrho/\partial t$ will change, if all the remaining parameters and their derivatives remained constant.

By using equation (3.63), we immediately we find

$$\Delta\frac{\partial \varrho}{\partial t}=\varrho_{0*}\Delta\frac{\partial W_x}{\partial x}. \tag{3.64}$$

For one-dimensional unsteady flow let us write the system, which includes the equation of the law of conservation of mass, equations of motion of gas and liquid phases, equations of energy and equation of state. While not allowing for forces of viscous friction for condition $\varrho_ж = \mathrm{const}$ we will have

$$(1-\chi)\frac{\partial\varrho}{\partial t}-\varrho\frac{\partial\chi}{\partial t}+\varrho\frac{\partial W}{\partial x}+W\frac{\partial\varrho}{\partial x}-\varrho_ж\frac{\partial\chi}{\partial t}-\frac{\partial}{\partial x}\left(\frac{\partial^2 G_x}{\partial r\cdot r\partial\varphi}\right)=0; \tag{3.65}$$

$$\frac{\partial W}{\partial t}+\frac{\partial W}{\partial x}W-\frac{W}{1-\chi}\frac{\partial\chi}{\partial t}-\frac{W^2}{1-\chi}\frac{\partial\chi}{\partial x}-(j_x+g_x)+$$

$$+\frac{1}{\varrho}\frac{1-S_x}{1-\chi}\frac{\partial p}{\partial x}+\frac{1}{2}\frac{c_{x^\circ}}{1-\chi}(W-C)^2\frac{dn}{dV}=0; \tag{3.66}$$

$$\frac{\partial C}{\partial t}+\frac{\partial C}{\partial x}C-\frac{C}{\chi}\frac{\partial\chi}{\partial t}-\frac{C^2}{\chi}\frac{\partial\chi}{\partial x}-(j_x+g_x)+$$

$$+\frac{1}{\varrho}\frac{S_x}{\chi}\frac{\partial p}{\partial x}-\frac{1}{2}\frac{c_{x^\circ}}{\chi}(W-C)^2\frac{\partial n}{\partial V}=0; \tag{3.67}$$

$$Q_0\varrho_ж\frac{\partial\chi}{\partial t}+Q_0\frac{\partial}{\partial x}\left(\frac{\partial^2 G_x}{\partial r\cdot r\partial\varphi}\right)+\varrho(1-\chi)(j_x+g_x)W+$$

$$+\varrho_ж\chi(j_x+g_x)C-\varrho(1-\chi)\frac{\partial}{\partial t}(c_pT)+\varrho(c_pT)\frac{\partial\chi}{\partial t}-$$

$$-\varrho(1-\chi)\frac{\partial}{\partial x}(c_pT)W+\varrho(c_pT)W\frac{\partial\chi}{\partial x}-\varrho_ж(c_ж T_ж)\frac{\partial\chi}{\partial t}-$$

$$-\varrho_ж\chi\frac{\partial}{\partial t}(c_ж T_ж)-\varrho_ж(c_ж T_ж)C\frac{\partial\chi}{\partial x}-$$

$$-\varrho_ж\chi C\frac{\partial}{\partial x}(c_ж T_ж)-\varrho(1-\chi)W\frac{\partial W}{\partial t}+\varrho\frac{W^2}{2}\frac{\partial\chi}{\partial t}-$$

$$-\varrho(1-\chi)W^2\frac{\partial W}{\partial x}+\varrho\frac{W^3}{2}\frac{\partial\chi}{\partial x}-\varrho_ж\chi C\frac{\partial C}{\partial t}-\varrho_ж\frac{C^2}{2}\frac{\partial\chi}{\partial t}-$$

$$-\varrho_ж\chi C^2\frac{\partial C}{\partial x}-\varrho_ж\frac{C^3}{2}\frac{\partial\chi}{\partial t}-\frac{\partial p}{\partial t}=0; \tag{3.68}$$

$$\frac{\partial p}{\partial t}+\frac{\partial p}{\partial x}W-(RT)\frac{\partial\varrho}{\partial t}-(RT)W\frac{\partial\varrho}{\partial x}-\varrho\frac{\partial}{\partial t}(RT)-$$

$$-\varrho W\frac{\partial}{\partial x}(RT)=0. \tag{3.69}$$

Let us examine the process, proceeding along axis x with fixation of the moment of time, or the process, proceeding with time, but at fixed value of x. During solution of the last problem all partial derivatives during x are equated to zero. The system of equations will take the form:

$$(1-\chi)\frac{\partial\varrho}{\partial t}-\varrho\frac{\partial\chi}{\partial t}-\varrho_{ж}\frac{\partial\chi}{\partial t}=0; \tag{3.70}$$

$$\frac{\partial W}{\partial t}-\frac{W}{1-\chi}\frac{\partial\chi}{\partial t}-(j_x+g_x)+\frac{1}{2}\frac{c_x\sigma}{1-\chi}(W-C)^2\frac{dn}{dV}=0; \tag{3.71}$$

$$\frac{\partial C}{\partial t}-\frac{C}{\chi}\frac{\partial\chi}{\partial t}-(j_x+g_x)-\frac{1}{2}\frac{c_x\sigma}{\chi}(W-C)^2\frac{dn}{dV}=0; \tag{3.72}$$

$$\begin{gathered}Q_0\varrho_{ж}\frac{\partial\chi}{\partial t}+\varrho(1-\chi)(j_x+g_x)W+\varrho_{ж}\chi(j_x+g_x)C-\\ -\varrho(1-\chi)c_p\frac{\partial}{\partial t}T+\varrho c_pT\frac{\partial\chi}{\partial t}+\varrho\frac{W^2}{2}\frac{\partial\chi}{\partial t}-\varrho_{ж}c_{ж}T_{ж}\frac{\partial\chi}{\partial t}-\\ -\varrho_{ж}\chi c_{ж}\frac{\partial}{\partial t}T_{ж}-\varrho(1-\chi)W\frac{\partial W}{\partial t}-\varrho_{ж}\chi C\frac{\partial C}{\partial t}-\\ -\varrho_{ж}\frac{C^2}{2}\frac{\partial\chi}{\partial t}-\frac{\partial p}{\partial t}=0;\end{gathered} \tag{3.73}$$

$$\frac{\partial p}{\partial t}-(RT)\frac{\partial\varrho}{\partial t}-\varrho\frac{\partial}{\partial t}(RT)=0. \tag{3.74}$$

During derivation of equations heat exchange was not considered and we accepted $c_p=\mathrm{const}$ and $c_{ж}=\mathrm{const}$.

As the sought unknowns let us take the following small deviations, which for simplicity of writing we designate so:

$$\left.\begin{aligned}&\Delta\left(\frac{\partial\varrho}{\partial t}\right)=\varrho_t;\ \Delta\left(\frac{\partial p}{\partial t}\right)=p_t;\ \Delta\left(\frac{\partial T}{\partial t}\right)=T_t;\\ &\Delta\left(\frac{\partial W}{\partial t}\right)=W_t;\quad \Delta\left(\frac{\partial C}{\partial t}\right)=C_t.\end{aligned}\right\} \tag{3.75}$$

We will consider the disturbing factors deviations

$$\Delta(j_x+g_x) \text{ and } \Delta\varrho.$$

Under conditions of our problem the remaining parameters should not receive any deviations from their nominal values.

By performing linearization of equations (3.70), (3.71), (3.72), (3.73) and (3.74) and converting to small finite deviations, we obtain system:

$$\left.\begin{aligned}
&a_{11}\varrho_t=\Delta V_1;\\
&a_{24}W_t=\Delta V_2;\\
&a_{35}C_t=\Delta V_3;\\
&a_{42}p_t+a_{43}T_t+a_{44}W_t+a_{45}C_t=\Delta V_4;\\
&a_{51}\varrho_t+a_{52}p_t+a_{53}T_t=\Delta V_5,
\end{aligned}\right\} \tag{3.76}$$

where constant coefficients

$$\left.\begin{aligned}
&a_{11}=(1-\chi_*);\quad a_{24}=1;\quad a_{35}=1;\quad a_{42}=-1;\\
&a_{43}=-[\varrho(1-\chi)c_p]_*;\\
&a_{44}=-[\varrho(1-\chi)W]_*;\\
&a_{45}=-(\varrho_ж\chi C)_*;\ a_{51}=-(RT)_*;\\
&a_{52}=1;\quad a_{53}=-(\varrho R)_*.
\end{aligned}\right\} \tag{3.77}$$

Disturbances are determined by formulas

$$\Delta V_1=\left(\frac{\partial\chi}{\partial t}\right)_*\Delta\varrho; \tag{3.78}$$

$$\Delta V_2=\Delta(j_x+g_x); \tag{3.79}$$

$$\Delta V_3=\Delta(j_x+g_x); \tag{3.80}$$

$$\begin{aligned}\Delta V_4=&-[\varrho(1-\chi)W+\varrho_ж\chi C]_*\Delta(j_x+g_x)-\\&-\left(c_pT\frac{\partial\chi}{\partial t}+\frac{W^2}{2}\frac{\partial\chi}{\partial t}\right)_*\Delta\varrho;\end{aligned} \tag{3.81}$$

$$\Delta V_5=\left[\frac{\partial}{\partial t}(RT)\right]_*\Delta\varrho. \tag{3.82}$$

Mark $*$ as before means that the numerical values of parameters, enclosed in brackets, are calculated for accepted values of x_0 and t_0. During computation of disturbances V_i is assigned by small deviations of only those quantities, the effect of which is being studied; in this case the remaining small deviations, which enter V_i, are taken equal to zero.

Solutions have the form

$$\varrho_t=\frac{D_\varrho}{D};\quad p_t=\frac{D_p}{D};\quad T_t=\frac{D_T}{D};\quad W_t=\frac{D_W}{D};\quad C_t=\frac{D_C}{D}, \tag{3.83}$$

where D – principal determinant; D_i – complementary determinant. Principal determinant

$$D=\begin{vmatrix} a_{11} & & & & \\ & & & 1 & \\ & & & & 1 \\ & -1 & a_{43} & a_{44} & a_{45} \\ a_{51} & 1 & a_{53} & & \end{vmatrix}=-a_{11}(a_{53}+a_{43}). \tag{3.84}$$

By substituting the values of a_{ik}, we find

$$D=(1-\chi)_*[\varrho(1-\chi)c_p+\varrho R]_*. \tag{3.85}$$

In the region, where vaporization of liquid components was finished, and also when using gaseous propellant $\chi = 0$. In this case principal determinant

$$D=\varrho_*(c_p+R)_*. \tag{3.86}$$

Let us write, for example, complementary determinant

$$D_\varrho=\begin{vmatrix} V_1 & & & & \\ V_2 & & & 1 & \\ V_3 & & & & 1 \\ V_4 & -1 & a_{43} & a_{44} & a_{45} \\ V_5 & 1 & a_{53} & & \end{vmatrix}=-(a_{53}+a_{43})V_1. \tag{3.87}$$

By substituting V_1 from formula (3.78), we obtain

$$D_\varrho=\left(\frac{\partial\chi}{\partial t}\right)_* \varrho_*[(1-\chi)c_p+R]_*\Delta\varrho. \tag{3.88}$$

Now according to the first equation (3.83) we find solution

$$\varrho_t=\Delta\left(\frac{\partial\varrho}{\partial t}\right)=\frac{\left(\frac{\partial\chi}{\partial t}\right)_*}{(1-\chi)_*}\Delta\varrho. \tag{3.89}$$

In the zone of vaporization preparation χ = const and $\partial\chi/\partial t=0$; therefore, here the change of density will not affect derivative $\partial\varrho/\partial t$. Past burning zone $\chi = 0$ and $\frac{\partial\chi}{\partial t}=0$. Consequently, here the change of density does not lead to deviation of derivative $\partial\varrho/\partial t$. Thus, according to formula (3.89) the change of gas density leads to change of derivative $\partial\varrho/\partial t$ only in the vaporization zone of liquid propellant, where $\partial\chi/\partial t$ is nonzero.

3.6. Engine Shutdown[1]

During organization of the shutdown process it is necessary for the designer to solve a number of complex and various problems, which include the following:

- decrease or elimination of hydraulic shock in lines;

- decrease of acceleration of rocket flight at the moment of transmission of the command for cessation of propellant feed into the combustion chamber;

- decrease of propellant residue in tanks toward the end of engine operation;

- providing precision and speed of engine cutoff;

- decrease of the aftereffect pulse;

- decrease of scattering of the aftereffect pulse.

The general solution to the problem of hydraulic shock was given by N. Ye. Zhukovskiy in 1898 in his classic work "Hydraulic Shock in Water Pipes" [31].

[1]Worked out together with V. A. Orlov.

By using the theorem about the change of momentum and the principle of conservation of mass for one-dimensional elementary tube of flow, disregarding the effect of inertia of walls of the pipeline, friction forces of liquid and the flow velocity of liquid by comparison with the speed of propagation of disturbance, we have two differential equations, connecting the change of pressure along the length of the line with time:

$$\frac{\partial U}{\partial t} = \frac{1}{\varrho_{ж}} \frac{\partial p}{\partial x}; \tag{3.90}$$

$$C \frac{\partial U}{\partial x} = \frac{1}{\varrho_{ж}} \frac{\partial p}{\partial t}; \tag{3.91}$$

where C — the rate of propagation of shock wave; U — the velocity of motion of liquid.

For a thin-walled, round and uniform tube, i.e., for a case having practically the most important value, the quantity of the rate of propagation of shock wave is computed by using formula

$$C = \sqrt{\frac{\frac{E_{ж}}{\varrho_{ж}}}{1 + 2\frac{rE_{ж}}{\delta_{ст}E_{ст}}}}, \tag{3.92}$$

where $E_{ж}$ — modulus of elasticity of liquid; $E_{ст}$ — modulus of elasticity of wall material; $\delta_{ст}$ — wall thickness.

On the average for the remaining lines during flow of water through them the magnitude of velocity C is of order 1000 m/s.

By analysing the solutions of equations (3.90) and (3.91), obtained in the form

$$p - p_0 = \varrho_{ж} C [\varphi(x - Ct) + \psi(x + Ct)]; \tag{3.93}$$

$$U - U_0 = -\varphi(x - Ct) + \psi(x + Ct). \tag{3.94}$$

for the case of instantaneous closing of cutoff valve and for the case of propagation of direct wave against the flow of liquid, we have relationship

$$p-p_0=\varrho_{ж}CU_0. \qquad (3.95)$$

The hydraulic shock, with which the increase of head is determined by expression (3.95), is called direct shock, and the given formula bears the name Zhukovskiy formula. In the case of slow closing of cutoff valve, when the reflected wave manages to penetrate the considered section of the line, pressure of hydraulic shock will be considerably less. In this instance for a direct wave the solution of equations of hydraulic shock has the form

$$p-p_0=\varrho_{ж}C(U_0-U). \qquad (3.96)$$

As can be seen from equations (3.95) and (3.96), the magnitude of hydraulic shock depends on the initial flow velocity of components, the speed of propagation of disturbances, the density of liquid and the value of velocity, up to which the initial velocity manages to drop during the time of arrival to the given point of the reflected wave.

The tensile stress in a thin-walled tube from internal pressure

$$\sigma=\frac{rp}{\delta_{ст}}. \qquad (3.97)$$

With increase of engine thrust the radius of the line is increased, inasmuch as with increase of thrust the flow rate of propellant increases, and the velocity of motion of liquid practically remains constant. Accordingly to equality (3.97) with increase of the radius of the line the amount of stress in the wall of the line will increase, therefore, with increase of thrust the wall thickness of the line should be increased, which, however, will lead to increase of weight, or direct shock should be eliminated, carrying out slow closing of cutoff valve.

Decrease of acceleration of rocket flight at the moment of transmission of the command for cessation of propellant feed into the combustion chamber is attained in various ways.

If the rocket is equipped with steering engines, then it is expedient to shut down the main engine with the steering engines operating. In certain cases thrust under shutdown conditions is created due to escape of turbogas from the generator through deflecting or steering nozzles. In this case the turbine is cutoff, and turbogas in the required quantity is fed directly to the nozzles, If there are no steering engines and steering (or deflecting) nozzles, then thrust under shutdown conditions can be created with the aid of an additional small ZhRD or [RDTT] (РДТТ) solid-propellant rocket engine.

When the features of the main power plant permit, small thrust can be obtained with stepped shutdown of the main engine. With such shutdown it sometimes proves to be expedient to change the component ratio. After shutdown in tanks there proves to be a certain quantity of components. For its decrease one should apply a [SOB] (СОБ) tank emptying system.

The precision of shutdown is determined by the design, by the degree of development and by operating conditions of shutdown controls – principal or cutoff valves, and also by the level of production technology. The rate of shutdown depends on inertness and the amount of movement of moving parts, on the force, which actuates the moving parts of valves, and on forces, which prevent their movement.

During engine shutdown in its gas and liquid cavities past the cutoff valves remains a certain quantity of components.

Usually the engine is shut down at high altitudes with low pressure of the environment, which provides almost complete subsequent escape of propellant available in engine cavities.

With escape of the propellant components reacting together to the engine, and consequently, and to the entire rocket on the whole an additional thrust pulse will be imparted, called the aftereffect thrust pulse. The time, during which decrease of thrust occurs after shutdown, is called the aftereffect period.

The aftereffect thrust pulse [ITPD] (ИТПД) of the engine is a statistical quantity and depends on many both internal and external parameters for the engine. As any statistical quantity the ITPD is characterized by a mean value (mathematical expectation) and its scattering.

Since the ITPD is the additional thrust pulse, being imparted to the rocket after feed of the command for cessation of engine operation, then it is very important to decrease the value of ITPD and its scattering as much as possible. However, this requires special structural equipment and a specific cyclogram of engine operation, which often leads to decrease of the reliability of operation of the rocket device on the whole.

Thus, for instance, simultaneous rapid actuation of the cutoff valves in the fuel and oxidizer lines allows decreasing the number of commands for shutdown and at the same time allows decreasing the quantity of some component, getting into the combustion chamber, i.e., in other words to decrease the ITPD.

On the other hand, this leads to the appearance of hydraulic shocks in the lines of the engine and rocket because of the sharp braking of components, and because of sharp decrease of engine thrust – to large overloads in all systems of the rocket on the whole.

Knowledge of the value of mean quantity of ITPD and its scattering allows introducing correction earlier to the operation of rocket systems. In this case there is used an expression for ITPD in the form of

$$I_{н}=\int_{0}^{t_{к}} P dt. \qquad (3.98)$$

Inasmuch as engine shutdown occurs at rather high altitudes, i.e., when $p_{н}\approx 0$, then

$$I_{н}=\int_{0}^{t_{к}} K_{т.п} F_{кр} p_{к} dt, \qquad (3.99)$$

where $K_{т.п}$ — thrust coefficient [15] when $p_{н}\approx 0$:

$$K_{т.п}=2\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\frac{k}{\sqrt{k^{2}-1}}\sqrt{1-\left(\frac{p_{a}}{p_{к}}\right)^{\frac{k-1}{k}}}\times$$
$$\times\left[1+\frac{k-1}{2k}\,\frac{\left(\frac{p_{a}}{p_{к}}\right)^{\frac{k-1}{k}}}{1-\left(\frac{p_{a}}{p_{к}}\right)^{\frac{k-1}{k}}}\right].$$

Below are examined some cases of calculation of change of pressure in the combustion chamber $p_{к}$ with respect to time of after effect, which is the basic calculated quantity, since in expression (3.99) the product of $K_{т.п}F_{кр}$ is a practically quantity.

The character of decrease of pressure in the combustion chamber is caused by escape of combustion products, which are in the chamber at the moment of feed of shutdown command (Fig. 3.9, section $а$—$б$), by escape of gaseous products, which are formed as a result of combustion of liquid propellant, which is in the chamber at the moment of shutdown and additionally enters the chamber after the beginning of closing of cutoff valves (section $б$—$в$). With stepped shutdown the character of the curve on the second stage corresponds to section $б$—$г$.

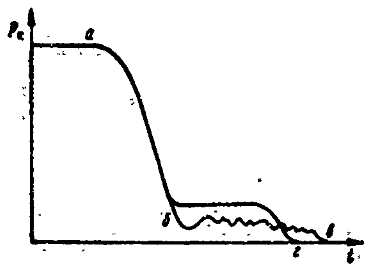

Fig. 3.9. Example of change of pressure in a chamber during engine shutdown.

In the beginning let us examine the problem about emptying of the gas volume of the combustion chamber and the condition of adiabaticity of processes in it. Considering the gas ideal, we have system of equations:

$$\frac{dY_\kappa}{dt}=G; \tag{3.100}$$

$$p_\kappa v^k=\text{const}; \tag{3.101}$$

$$p_\kappa=\frac{Y_\kappa RT_\kappa}{V_\kappa}. \tag{3.102}$$

By integrating the system of equations of material and energy balance for the chamber, we have

$$p_\kappa^{-\frac{k-1}{2k}}=p_{\kappa 0}^{-\frac{k-1}{2k}}\left(1+\frac{(k-1)F_{\kappa p}a\sqrt{RT_{\kappa 0}}}{2V_\kappa}t\right). \tag{3.103}$$

The maximum gradient of pressure drop, which can affect the efficiency of the engine, is computed by formula

$$\dot{p}_\kappa=-\frac{kp_{\kappa 0}F_{\kappa p}a\sqrt{RT_{\kappa 0}}}{V_\kappa}, \tag{3.104}$$

where k - adiabatic index.

Under the assumption of isothermicity of the outflow process the previous system of equations has the form

$$\frac{dY_\kappa}{dt}=-G_\kappa;\ p_\kappa v=\text{const};\ p_\kappa=\frac{Y_\kappa RT_\kappa}{V_\kappa}. \tag{3.105}$$

In this instance the solution of the system of equations will be expressed through function

$$p_к = p_{к0} \exp\left(-\frac{F_{кр} a R \sqrt{T_{к0}}}{V_к} t\right). \qquad (3.106)$$

Inasmuch as the quantity of aftereffect pulse is relatively small, frequently for its computation we use equation of chamber in the following form:

$$\varepsilon \dot{p}_к + p_к - \frac{\beta}{F_{кр}} [(G_1)_{-\tau_{з1}} + (G_2)_{-\tau_{з2}}] = 0. \qquad (3.107)$$

Depending on the magnitude of component ratio $k_1(t)$ certain regularity $\beta(t)$ is established, and the change of pressure in time taking into consideration the variable value of component ratio allows determining $\varepsilon(t)$.

With closing of main valves in the case of utilization of liquid components the injection of components into the chamber can take place both because of movement of working elements and as a result of deformation of lines or other elements of the hydraulic system. Time and the character of feed of components in time depends on the design of the engine and its operating conditions. The total quantity of component, entering the chamber after supply of shutdown command,

$$Y_i = \Sigma \left[\int_0^{t_1} G_i(t)\,dt + \int_0^{t_2} G_i'(t)\,dt \right], \qquad (3.108)$$

where t_1 – the time of feed, caused by movement of the working elements of a valve; t_2 – the time of feed, caused by deformation of structural elements; $G_i(t), G_i'(t)$ – functions, which characterize flow rates.

Due to combustion of propellant, which is in the chamber in liquid state, gaseous products are formed in the quantity

$$Y_{\tau_s}=\int_{\tau_{s1}}^{\tau_{s10}} G_1 dt+\int_{\tau_{s2}}^{\tau_{s20}} G_2 dt\approx\int_{\tau_s}^{\tau_{s0}} G_\Sigma dt, \tag{3.109}$$

where τ_{s0} and τ_{sf} – periods of delay, which correspond to initial and final pressure.

Let us examine a case, when from the feed system after the command for closing of the valve the propellant is not fed. Let us assume that

$$\tau_s=\frac{\tau_0}{p_\kappa}, \tag{3.110}$$

where τ_0 – delay, which corresponds to $p_\kappa = 1$ and is determined experimentally.

The equation of the chamber will be written so:

$$\varepsilon \dot{p}_\kappa p_\kappa^2+p_\kappa^3+\frac{3}{F_{\kappa p}}\tau_0 G_\Sigma p_\kappa=0. \tag{3.111}$$

After separation of variables and integration

$$\varepsilon\int_{p_{\kappa 0}}^{p_\kappa}\frac{dp_\kappa}{p_\kappa}+\frac{3}{F_{\kappa p}}\tau_0 G_\Sigma\int_{p_{\kappa 0}}^{p_\kappa}\frac{dp_\kappa}{p_\kappa^3}=-t, \tag{3.112}$$

the solution has the form

$$t=\varepsilon\ln\left(\frac{p_{\kappa 0}}{p_\kappa}\right)-\frac{3}{2F_{\kappa p}}\tau_0 G_\Sigma\left(\frac{1}{p_\kappa^2}-\frac{1}{p_{\kappa 0}^2}\right), \tag{3.113}$$

With the absence of additional input of propellant residue into the chamber the equation of the chamber will be written so:

$$\varepsilon\dot{p}_\kappa+p_\kappa=0, \tag{3.114}$$

time

$$\varepsilon = \frac{F_{\text{кр}} I}{RT_{\text{к}}}\beta. \tag{3.115}$$

If we consider that β and $RT_{\text{к}}$ do not practically depend on pressure, then for the period of decrease of pressure in the chamber it is possible to consider ε = const. In this case for construction of graph $p_{\text{к}}(t)$ we will have

$$p_{\text{к}} = p_{\text{к}0} \exp\left(-\frac{t}{\varepsilon}\right), \tag{3.116}$$

where $p_{\text{к}0}$ – pressure in the chamber up to the start of engine shutdown.

As a result of the nonuniformity of processes observed after engine shutdown, the aftereffect pulse is characterized not only by integral quantity (3.98), but by scattering $\Delta I_{\text{п}}$.

SECTION II

FEED SYSTEMS

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CHAPTER IV

TANK PRESSURIZING SYSTEM

4.1. Equation of the Law of Conservation of Energy for a Pressurized System

During calculation and research of the change with time of parameters of a tank pressurizing system, consisting of pressure accumulator and a tank, the equations of laws of conservation of energy and mass are included. The equation of law of conservation of energy can be derived in general form, and it will be suitable for the study of the process of displacement of liquid from a tank with the aid of any type of accumulator.

For the system, shown in Fig. 4.1, the law of conservation of energy is written so:

$$Q=U+L, \tag{4.1}$$

where Q - the quantity of heat, supplied to the working medium (gas) of the system in a unit of time; U - the internal energy of working medium; L - the work, being performed by gas.

The gas, entering the upper cavity of the tank from the accumulator, is expanded. If the liquid is motionless, then the gas

does not perform any work. If displacement of liquid from the tank occurs, then the work, being performed by gas in the process of expansion (the operation of expansion), will be spent on overcoming the forces of resistance at the tank exit and on increase of the kinetic energy of outflowing liquid.

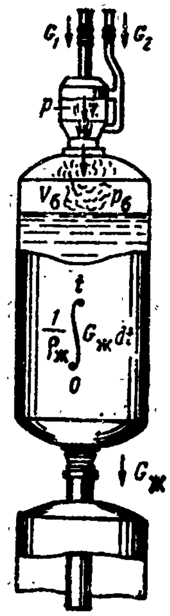

Fig. 4.1. Diagram of tank pressurizing system.

If in the internal cavity of the accumulator as a result of propellant combustion and thermal processes in gas mixtures there is liberated heat Q_0, and heat exchange with the surrounding medium is characterized by quantities Q_a for the accumulator and $Q_б$ for the tank, then

$$Q=Q_0+Q_a+Q_б. \tag{4.2}$$

If necessary there is considered the heat of chemical reactions Q_α, appearing on the surface of liquid, heat being consumed on preheating and vaporization of liquid $Q_И$, heat of dissolution of gas in liquid Q_p.

The internal energy of gas is determined by the sum

$$\dot{U} = \dot{U}_a + \dot{U}_{\text{б}}. \qquad (4.3)$$

For gas, contained in the accumulator,

$$U_a = Y_a c_{Va} T_a. \qquad (4.4)$$

In precise calculations we take $c_V \neq$ const, in this case we find

$$\dot{U}_a = c_{Va} \frac{d}{dt}(Y_a T_a) + Y_a T_a \frac{d}{dt} c_{Va} = \xi c_{Va} \frac{d}{dt}(Y_a T_a), \qquad (4.5)$$

where ξ - coefficient, considering the effect of change of specific heat. The total mass of gas, which escaped from the accumulator

$$Y_a = \int_0^t G_a dt,$$

where G_a - the flow rate of gas from the accumulator.

Let us note that in the right side of equation (4.5) under the differential sign there is found the product of $Y_a T_a$, inasmuch as in the process of operation of the system both the quantity of gas in the accumulator and the gas temperature change with time.

For the gas, located in the tank, we will have

$$\dot{U}_{\text{б}} = \xi c_{V\text{б}} \frac{d}{dt}(Y_{\text{б}} T_{\text{б}}). \qquad (4.6)$$

The total mass of gas at any moment of time

$$Y_{\text{б}} = Y_{\text{б}0} + \int_0^t G_a dt - \int_0^t G_{\text{Д}} dt + \int_0^t G_{\text{И}} dt, \qquad (4.7)$$

where $Y_{\text{б}0}$ - the initial quantity of gas in the tank; $G_{\text{Д}}$ - the diffusion flow of gas per second into liquid; $G_{\text{И}}$ - the inflow per second of vaporizing liquid into the upper cavity of the tank.

By applying the equation of state, we obtain

$$\dot{U}=\frac{1}{R}\left[c_{V\text{а}}(V_{\text{а}}\dot{p}_{\text{а}}+\dot{V}_{\text{а}}p_{\text{а}})+c_{V\text{б}}(V_{\text{б}}\dot{p}_{\text{б}}+\dot{V}_{\text{б}}p_{\text{б}})+\right.$$
$$\left.+p_{\text{а}}V_{\text{а}}\dot{c}_{V\text{а}}+p_{\text{б}}V_{\text{б}}\dot{c}_{V\text{б}}\right]. \qquad (4.8)$$

The derivative of work of expansion of gas in the tank will be expressed so:

$$\dot{L}=p_{\text{б}}\dot{V}_{\text{б}}. \qquad (4.9)$$

Let us recall that

$$c_p-c_V=R;\ \frac{c_p}{c_V}=k. \qquad (4.10)$$

The equation of the law of conservation of energy can now be written so [67]:

$$\frac{c_{V\text{а}}}{c_{V\text{б}}}(V_{\text{а}}\dot{p}_{\text{а}}+p_{\text{а}}\dot{V}_{\text{а}})+V_{\text{б}}\dot{p}_{\text{б}}+kp_{\text{б}}\dot{V}_{\text{б}}=$$
$$=\frac{R}{c_{V\text{б}}}(\dot{Q}_0+\dot{Q}_{\text{а}}+\dot{Q}_{\text{б}})-p_{\text{а}}V_{\text{а}}\frac{\dot{c}_{V\text{а}}}{c_{V\text{б}}}-p_{\text{б}}V_{\text{б}}\frac{\dot{c}_{V\text{б}}}{c_{V\text{а}}}. \qquad (4.11)$$

In the examination of current moments of time to get more precise results it is necessary to take into account the kinetic energy of gases. The kinetic energy of gas, contained in the accumulator,

$$E_{\text{а}}=\frac{1}{2}\int_0^{x_{\text{а}}}\varrho F_{\text{а}}(x)[W_{\text{а}}(x)]^2dx, \qquad (4.12)$$

where $F_a(x)$ - the current value of accumulator cross-sectional; $W_a(x)$ - the current value of gas velocity in the accumulator.

Analogically we write the equation for gas, which fills the tank,

$$E_{\text{б}}=\frac{1}{2}\int_0^{x_{\text{б}}}\varrho F_{\text{б}}(x)[W_{\text{б}}(x)]^2dx. \qquad (4.13)$$

Quantities $E_{\text{а}}$ and $E_{\text{б}}$ should be subtracted from the right side of equation (4.11).

If the change of temperature in a system is small, then when performing engineering calculations it is possible to accept $\dot{c}_{V\text{а}} = \dot{c}_{V\text{б}} = 0$, which considerably simplifies the calculation by formula (4.11).

The equation of energy can be expressed through the flow rate of liquid (component).

Free volume in the tank

$$V_{\text{б}} = V_{\text{б}0} + \frac{1}{\rho_{\text{ж}}} \int_0^t G_{\text{ж}} dt, \tag{4.14}$$

where $V_{\text{б}0}$ - initial free volume; $\rho_{\text{ж}}$ - density of liquid; $G_{\text{ж}}$ - mass flow rate of liquid.

It is obvious that

$$\dot{V}_{\text{б}} = \frac{G_{\text{ж}}}{\rho_{\text{ж}}}. \tag{4.15}$$

The equation of energy takes the following form:

$$\varepsilon (V_{\text{а}} \dot{p}_{\text{а}} + p_{\text{а}} \dot{V}_{\text{а}}) + V_{\text{б}0} \dot{p}_{\text{б}} + \dot{p}_{\text{б}} \frac{1}{\rho_{\text{ж}}} \int_0^t G_{\text{ж}} dt + k \frac{G_{\text{ж}}}{\rho_{\text{ж}}} p_{\text{б}} = \xi \frac{R}{c_{V\text{б}}} (\dot{Q}_0 + \dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}), \tag{4.16}$$

where

$$\xi = 1 - \frac{p_{\text{а}} V_{\text{а}} \frac{\dot{c}_{V\text{а}}}{c_{V\text{б}}} + p_{\text{б}} V_{\text{б}} \frac{\dot{c}_{V\text{б}}}{c_{V\text{б}}}}{\frac{R}{c_{V\text{б}}} Q}; \tag{4.17}$$

$$\varepsilon = \frac{c_{V\text{а}}}{c_{V\text{б}}}. \tag{4.18}$$

4.2. The Equation of Energy for a Gas Accumulator

Taking into account that in a system with gas accumulator the heat flows are directed from the surrounding medium into the system, we obtain

$$\xi V_a \dot{p}_a + V_{\text{б}0}\dot{p}_\text{б} + \dot{p}_\text{б}\frac{1}{\varrho_\text{ж}}\int_0^t G_\text{ж}\,dt + k\frac{G_\text{ж}}{\varrho_\text{ж}}p_\text{б} = \xi\frac{R}{c_{V\text{б}}}\left(\dot{Q}_a + \dot{Q}_\text{б}\right). \tag{4.19}$$

In certain cases it is convenient to write equation (4.19) with the use of politropic index n:

$$\xi V_a \dot{p}_a + V_{\text{б}0}\dot{p}_\text{б} + \dot{p}_\text{б}\frac{1}{\varrho_\text{ж}}\int_0^t G_\text{ж}\,dt + n\frac{G_\text{ж}}{\varrho_\text{ж}}p_\text{б} = 0, \tag{4.20}$$

moreover the politropic index

$$n = k - \xi\frac{R}{c_{V\text{б}}}\,\frac{\dot{Q}_a + \dot{Q}_\text{б}}{G_\text{ж}p_\text{б}}\varrho_\text{ж}. \tag{4.21}$$

It is obvious that $\dot{Q}_a$ and $\dot{Q}_\text{б}$ can vary with time. In engineering calculations we are frequently guided by some mean value of politropic index for the process.

If heat exchange is absent, then

$$Q_a = Q_\text{б} = 0 \tag{4.22}$$

and, as a consequence,

$$\dot{Q}_a = \dot{Q}_\text{б} = 0. \tag{4.23}$$

In this instance

$$n = k + \frac{\varrho_\text{ж}}{G_\text{ж}}\left(\frac{p_a}{p_\text{б}}V_a\frac{\dot{c}_{Va}}{c_{V\text{б}}} + V_\text{б}\frac{\dot{c}_{V\text{б}}}{c_{V\text{б}}}\right). \tag{4.24}$$

Condition (4.24) will be valid in the presence of heat exchange, but only if

$$\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}} = 0. \tag{4.25}$$

Equation (4.25) shows that total heat flow into the system should not vary with time.

With an isothermal process, when $n = 1$,

$$\xi \frac{R}{c_{V\text{б}}} \frac{\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}}{G_{\text{ж}} p_{\text{б}}} \varrho_{\text{ж}} = k - 1. \tag{4.26}$$

As is known,

$$k - 1 = \frac{R}{c_{V\text{б}}}, \tag{4.27}$$

Therefore, in an isothermal process

$$\xi (\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}) = \frac{G_{\text{ж}}}{\varrho_{\text{ж}}} p_{\text{б}}, \tag{4.28}$$

or

$$\xi (\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}) = p_{\text{б}} \dot{V}_{\text{б}}. \tag{4.29}$$

Let us determine the required volume of accumulator. Being guided by expression (4.29), let us write the initial expression so:

$$\xi V_{\text{а}} \dot{p}_{\text{а}} + V_{\text{б}} \dot{p}_{\text{б}} + k \frac{G_{\text{ж}}}{\varrho_{\text{ж}}} p_{\text{б}} = \frac{R}{c_{V\text{б}}} (\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}) - \\ - \left(p_{\text{а}} V_{\text{а}} \frac{\dot{c}_{V\text{а}}}{c_{V\text{б}}} + p_{\text{б}} V_{\text{б}} \frac{\dot{c}_{V\text{б}}}{c_{V\text{б}}} \right). \tag{4.30}$$

Let us examine a case when $p_{\text{б}}$ = const. Having assumed $\dot{c}_{Va}$ = = $\dot{c}_{V\text{б}}$ = 0, we find

$$V_{\text{а}} \dot{p}_{\text{а}} = -k \frac{G_{\text{ж}}}{\varrho_{\text{ж}}} p_{\text{б}} + \frac{R}{c_{V\text{а}}} (\dot{Q}_{\text{а}} + \dot{Q}_{\text{б}}). \tag{4.31}$$

By integrating, we obtain

$$V_a \int\limits_{p_{a0}}^{p_a} dp_a = -k \frac{p_\text{б}}{\varrho_\text{ж}} \int\limits_0^{t_\text{в}} G_\text{ж} dt + \frac{R}{c v_a} \int\limits_0^{Q_{a.\text{б}}} d(\dot{Q}_a + \dot{Q}_\text{б}). \tag{4.32}$$

The solution has the following form:

$$V_a = k \frac{V_\text{б} p_\text{б}}{p_{a0} - p_a} - \frac{R}{c v_a} \frac{\dot{Q}_a + \dot{Q}_\text{б}}{p_{a0} - p_a} \tag{4.33}$$

or

$$V_a = \frac{n V_\text{б} p_\text{б}}{p_{a0} - p_a}, \tag{4.34}$$

moreover

$$n = k - \frac{R}{c v_a} \frac{\dot{Q}_a + \dot{Q}_\text{б}}{p_\text{б} V_\text{б}} \varrho_\text{ж} \tag{4.35}$$

or

$$n \approx k - (k-1) \frac{\dot{Q}_a + \dot{Q}_\text{б}}{p_\text{б} V_\text{б}} \varrho_\text{ж}. \tag{4.36}$$

Thus, for decrease of the volume of accumulator one should not equip the accumulator and tank with heat insulation. It is necessary to raise the initial pressure in the accumulator and decrease final pressure p_a. It is obvious that p_a will always be larger than $p_\text{б}$. For approach of quantity p_a or $p_\text{б}$ one should apply reducers, which operate in subcritical conditions. It is expedient to use a system for feeding gas into the tank, consisting of a valve, controlled with the aid of a pressure relay.

The politropic index in equation (4.34) is frequently determined experimentally. In the process of displacement of liquid index n in the beginning, when $t = 0$, is equal to the adiabatic index. Then it is rather rapidly and sharply decreased, whereupon it begins to increase, asymptotically approaching (according to data of theoretical calculations) the value of isothermal index, equal to one.

4.3. The Equation of Energy for Cartridge Accumulator

According to expression (4.16), considering $G_{\text{ж}}$ = const, we find

$$\varepsilon(V_a\dot{p}_a+p_a\dot{V}_a)+V_{\text{б}}\dot{p}_{\text{б}}+k\frac{G_{\text{ж}}}{\varrho_{\text{ж}}}p_{\text{б}}=\xi\frac{R}{c_{V\text{б}}}(\dot{Q}_0-\dot{Q}_a-\dot{Q}_{\text{б}}). \qquad (4.37)$$

Here $\dot{Q}_a$ and $\dot{Q}_{\text{б}}$ have a minus sign, inasmuch as heat flows Q_a and $Q_{\text{б}}$ in the considered case are directed from the system into the surrounding medium.

The elementary quantity of heat, which was liberated per second during combustion of grain, will be equal to

$$\dot{Q}_0=c_VT_1\dot{Y}_{\text{п}}, \qquad (4.38)$$

where T_1 - the combustion temperature; $Y_{\text{п}}$ - the mass of grain, moreover

$$Y_{\text{п}}=\varrho_{\text{п}}\int_0^l\varphi(l)\,dl=\varrho_{\text{п}}\int_0^t\varphi(l)\,u\,dt, \qquad (4.39)$$

where $\rho_{\text{п}}$ - the density of grain; u - the rate of burning.

For cylindrical grain with diameter D, height $l = D$ during cigarette burning

$$\varphi(l)=\frac{\pi D^2}{4}=\text{const}; \qquad (4.40)$$

$$\dot{Y}_{\text{п}}=\varrho_{\text{п}}\frac{\pi D^2}{4}u. \qquad (4.41)$$

For the rate of burning let us take law

$$u=u_0+ap_a^{\nu}. \qquad (4.42)$$

Now instead of expressions (4.39) and (4.42) we will have

$$Y_n = \varrho_n \int \varphi(l)(u_0 + a p_a^{\nu})\,dt; \qquad (4.43)$$

$$\dot{Y}_n = \varrho_n \varphi(l)(u_0 + a p_a^{\nu}). \qquad (4.44)$$

Let us examine expression

$$\frac{R}{c_V}\dot{Q}_0 = \frac{R}{c_V} c_V T_1 \dot{Y}_n = R T_1 \dot{Y}_n. \qquad (4.45)$$

Taking into account that the force of powder, i.e., the efficiency of powder gases [81]

$$f = RT_1, \qquad (4.46)$$

instead of expression (4.45) we find

$$\frac{R}{c_V}\dot{Q}_0 = f\dot{Y}_n = k f_0 \dot{Y}_n \qquad (4.47)$$

or

$$\frac{R}{c_V}\dot{Q}_0 = k f_0 \varrho_n \varphi(l)(u_0 + a p_a^{\nu}), \qquad (4.48)$$

where f_0 - the given powder force; k - adiabatic index.

The law of the burning rate depends on the quality of solid propellant, the burning conditions and other factors. Therefore, expression (4.42) cannot be used in all cases. When performing calculations the law of burning rate is selected with consideration of a particular situation, being guided by results of special research.

Equation (4.38) can be written thus:

$$\dot{Q}_0 = c_p T_a \dot{Y}_n. \qquad (4.49)$$

Then instead of expression (4.45) we obtain

$$\frac{R}{c_V}\dot{Q}_0=kRT_a\dot{Y}_n. \tag{4.50}$$

Taking into account that

$$RT_a=f_0 \tag{4.51}$$

we obtain, as in equation (4.47),

$$\frac{R}{c_V}\dot{Q}_0=kf_0\dot{Y}_n. \tag{4.52}$$

The equation of energy takes the following form:

$$\begin{gathered}\varepsilon(V_a\dot{p}_a+p_a\dot{V}_a)+V_{\text{б}}\dot{p}_{\text{б}}+kp_{\text{б}}\dot{V}_{\text{б}}= \\ =\xi\left[kf_0\varrho_n\varphi(l)(u_0+ap_a^{\nu})-\frac{R}{c_V}(\dot{Q}_a+\dot{Q}_{\text{б}})\right].\end{gathered} \tag{4.53}$$

The given powder force f_0 is a constant reference quantity, whereas quantity RT_a can vary with time. Therefore, after the introduction of the value of powder force into calculation the equation takes a quasi-static character.

If $p_{\text{б}}$ = const, then expression (4.53) will be written so:

$$p_{\text{б}}dV_{\text{б}}=\eta\xi f_0dY_n, \tag{4.54}$$

where coefficient η considers heat losses and the change of accumulator parameters:

$$\eta=1-\frac{\frac{R}{c_V}(\dot{Q}_a+\dot{Q}_{\text{б}})+\frac{\varepsilon}{\xi}\frac{d}{dt}(p_aV_a)}{kf_0\dot{Y}_n}. \tag{4.55}$$

Integration of expression (4.54) is possible if relationship $\eta(\dot{Q}_i, Y_n, V_a, \ldots)$, is known, or if η is assigned according to results

of processing the experimental data or is calculated and its mean value taken. In the last case

$$\dot{p}_6=\eta\frac{f_0 Y_п}{V_6}. \tag{4.56}$$

Heat losses can be considerable and then coefficient η proves to be substantially less than one. As a result of heat losses being variable with time the pressure in the tank will be variable. In order to provide p_6 = const, the decrease of pressure due to lowering of gas temperature should be compensated by additional feed of gas from the accumulator into the tank. For this purpose we apply profiled restricted grains. Sometimes the tank is additionally equipped with a regulator.

4.4. The Equation of Energy for a Hot (Liquid-Propellant) Accumulator

If we accept $\dot{p}_a = 0$, $\dot{V}_a = 0$, then the equation of energy, not allowing for delay of burning, will be written so:

$$V_6\dot{p}_6+kp_6\dot{V}_6=\xi\frac{R}{c_{V6}}(\dot{Q}_0-\dot{Q}_a-\dot{Q}_6). \tag{4.57}$$

The quantity of liberated heat

$$Q_0=\int_{\tau_s}^{t}(G_1+G_2)c_pT_a\,dt, \tag{4.58}$$

where G_1 and G_2 - the mass flow rates per second of the components, which feed the generator.

If G_1 = const, G_2 = const, then c_pT_a = const.

With constant pressure in the tank, considering that heat flow is directed from the system into the surrounding medium, we will have

$$kp_6\dot V_6 \approx \xi\left[k(G_1+G_2)RT_a - \frac{R}{c_{V6}}(\dot Q_a+\dot Q_6)\right]. \qquad (4.59)$$

After conversions under the condition that $\dot V_6$ = const, we obtain

$$p_6 = \eta_t \frac{G_1+G_2}{\dot V_6} RT_a, \qquad (4.60)$$

where the coefficient, considering heat losses,

$$\eta_t = \xi\left[1 - \frac{\dot Q_a + \dot Q_6}{c_p T_a (G_1+G_2)}\right] \approx \xi\left(1 - \frac{\dot Q_a + \dot Q_6}{\dot Q_0}\right). \qquad (4.61)$$

Coefficient η_t is the thermal efficiency, inasmuch as it is the ratio of usefully utilized heat

$$\dot Q = \dot Q_0 - \dot Q_a - \dot Q_6 \qquad (4.62)$$

to the total quantity of heat, which was liberated in a hot accumulator,

$$\dot Q_0 = (G_1+G_2)c_p T_a, \qquad (4.63)$$

moreover

$$\eta_t = \varphi(t). \qquad (4.64)$$

When designing feed systems with a hot accumulator constant pressure in the tank is provided due to change of flow rates of components, which feed the generator.

4.5. The Equation of Law of Conservation of Mass for Tank Pressurizing System

The law of conservation of mass for accumulators not allowing for ignition delay is written thus:

$$Y = Y_{\text{и}} + Y_{\text{a}}, \tag{4.65}$$

where Y - the mass of propellant, which entered the accumulator at moment of time t; $Y_{\text{и}}$ - the mass of products, which escaped at the same moment of time; Y_a - the mass of products, which were accumulated in the accumulator.

The equation of the law of conservation of mass for a tank can be written so:

$$Y_{\text{a}} = Y_{\text{б}} + Y_{\text{бо}}, \tag{4.66}$$

where $Y_{\text{б}}$ - the mass of gas in the tank at moment of time t.

From expressions (4.65) and (4.66) follows the equation of law of conservation of mass for the entire pressurized system:

$$\dot{Y} = \dot{Y}_{\text{a}} + \dot{Y}_{\text{б}}. \tag{4.67}$$

Let us return to equation (4.65). For further conversions we use the equation of state and we find

$$\dot{Y}_{\text{a}} = \left[\frac{1}{R_{\text{a}} T_{\text{a}}} p_{\text{a}} \dot{V}_{\text{a}} + V_{\text{a}} \dot{p}_{\text{a}} - p_{\text{a}} V_{\text{a}} \frac{\frac{d}{dt}(R_{\text{a}} T_{\text{a}})}{R_{\text{a}} T_{\text{a}}}\right]. \tag{4.68}$$

The total mass of escaped gases

$$Y_{\text{и}} = \int_0^t G dt, \tag{4.69}$$

consequently,

$$\dot{Y}_{\text{и}} = \frac{F_{\text{кр}} a}{\sqrt{R_{\text{a}} T_{\text{a}}}} p_{\text{a}}, \tag{4.70}$$

where, as before,

$$a=\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\left(2\frac{n}{n+1}\right)^{0.5}. \qquad (4.71)$$

The derivative from total flow rate

$$\dot{Y}=G_1+G_2. \qquad (4.72)$$

By substituting the obtained values of derivatives into expression (4.65), we find

$$(G_1+G_2)-\frac{F_{кр}a}{\sqrt{R_aT_a}}p_a=\frac{\frac{d}{dt}(p_aV_a)}{R_aT_a}-\frac{p_aV_a}{R_aT_a}\,\frac{\frac{d}{dt}(R_aT_a)}{R_aT_a}. \qquad (4.73)$$

By using equation (4.66) and the equation of state of gas, contained in the tank, we find

$$\int_0^t G\,dt=\frac{p_б V_б}{R_б T_б}. \qquad (4.74)$$

If the flow rate of liquid from the tank varies with time, then

$$V_б=V_{б0}+\frac{1}{\rho_ж}\int_0^t G_ж\,dt, \qquad (4.75)$$

where V_{60} - initial free volume in the tank; $G_ж$ - the mass flow rate of liquid per second from the tank; $\rho_ж$ - liquid density.

Consequently,

$$p_б=R_б T_б\frac{\int_0^t G\,dt}{V_{б0}+\frac{1}{\rho_ж}\int_0^t G_ж\,dt}. \qquad (4.76)$$

Equation (4.76) establishes the connection between the inflow of gas to the tank and the flow rate of liquid from the tank.

4.6. The Equation of Mass for Gas Accumulator

Let us use expression (4.73). During operation of an engine the accumulator is not filled with gas, and therefore $G_1 + G_2 = 0$. The volume of accumulator V_a = const, therefore, when R_a = const

$$F_{\text{кр}} a\sqrt{R_a T_a} = -V_a\left(\frac{\dot p_a}{p_a} - \frac{\dot T_a}{T_a}\right). \tag{4.77}$$

If the process of overflow of gas is considered polytropic, then

$$\frac{\dot T_a}{T_a} = \frac{n-1}{n}\,\frac{\dot p_a}{p_a};\quad T_a = T_{a0}\left(\frac{p_a}{p_{a0}}\right)^{\frac{n-1}{n}} \tag{4.78}$$

and instead of equation (4.77) we can write

$$F_{\text{кр}} a\sqrt{R_a T_a} = -\frac{V_a}{n}\,\frac{\dot p_a}{p_a}. \tag{4.79}$$

After transformations

$$\dot p_a = -Z p_a^{\frac{3n-1}{2n}}, \tag{4.80}$$

where

$$Z = n\,\frac{F_{\text{кр}} a\sqrt{R_{a0} T_{a0}}}{V_a p_{a0}^{\frac{n-1}{2n}}}. \tag{4.81}$$

Having separated the variables and integrated, we find

$$t = 2V_a\,\frac{\left(\frac{p_{a0}}{p_a}\right)^{\frac{n-1}{2n}} - 1}{(n-1)F_{\text{кр}} a\sqrt{R_{a0} T_{a0}}}. \tag{4.82}$$

Equation (4.82) allows constructing a curve of the change of pressure in accumulator with time. Further, from equation (4.79) it is possible to find the change of temperature T_a with time, and from expression (4.70) in the case of supercritical outflow - the relationship between flow rate of gas and the time of outflow. For performing such a calculation there are necessary graphs of functions $F_{\text{кр}}(t)$ and $n(t)$, which are obtained by calculation or by processing experimental data.

4.7. The Equation of Mass for Cartridge Accumulator

Let us use equation (4.73). Taking into account that [81]

$$f_0 = R_a T_a;\; Y_n = G_1 + G_2,$$

we obtain

$$f_0 \dot{Y}_n - F_{\text{кр}} a \sqrt{f_0}\, p_a = p_a \dot{V}_a + \frac{1}{n} V_a \dot{p}_a. \tag{4.83}$$

If $\dot{p}_a = \dot{V}_a = 0$, then

$$\dot{Y}_n = \frac{F_{\text{кр}} a}{\sqrt{f_0}}\, p_a. \tag{4.84}$$

After transformations and integrations we find

$$Y_n = \frac{F_{\text{кр}} a}{\sqrt{f_0}}\, p_a t, \tag{4.85}$$

where t - the burning time of grain.

Calculation of the accumulator can be performed in the following sequence. By formula (4.56) determine the required weight of charge. By knowing the law of burning according to formula (4.42), by using equality (4.41), we find

$$Y_n = \varrho_n \frac{\pi D^2}{4} ut, \tag{4.86}$$

By equation (4.86) determine the diameter of grain, while knowing the displacement time of liquid

$$t_{ж}=\varrho_{ж}\frac{V_{б}}{G_{ж}}, \tag{4.87}$$

where G_{x} - the flow rate of liquid from the tank.

During calculation by formulas (4.86) and (4.87) one should consider that the burning time of grain in the accumulator t is equal to the displacement time of liquid t_{x}.

By using formula (4.85) calculate the required nozzle throat area of the accumulator.

Let us note that during derivation of equations the system on the whole was considered, i.e., acculator and tanks. During investigation of laws of change of intratank parameters with time we write the equations of accumulators and tanks separately with consideration of the kinetic energy of gas flow. If it is necessary to study the processes of outflow of gases from the accumulator into tanks more comprehensively, then one should draw on additional equations of thermodynamics and heat transfer. In a number of cases the condensation of products and the vaporization of liquid are considered. However, one should comprehensively describe the entire complex of processes as applied to particular constructions.

CHAPTER V

INTRATANK PROCESSES

5.1. Heat and Mass Exchange

During calculation, design and research of feed systems it is necessary to know the conditions of heat exchange in the tank, mass transfer between gas and liquid, located in the tank, conditions of the influence of gas on liquid, the character of displacement of liquid from the tank.

The study of the indicated processes allows correctly organizing the storage of liquid in the tank and the supply of an engine with propellant during its operation.

Calculation is performed for a number of characteristic periods, which can include the following.

Storage of a full tank at constant ambient temperature. Here it is assumed that the tank is under certain pressure. In such conditions heat exchange is absent, there takes place molecular diffusion, which continues until the saturation of liquid by gas is finished and equilibrium is established between gas and vapors of liquid in the upper cavity of the tank.

Storage of a full tank at variable ambient temperature. In these conditions heat exchange sets in between the environment and the gas, located in the tank, and also between the environment and liquid. If the intensity of heat exchange with gas and liquid is different, then heat exchange between gas and liquid appears. As a result of heat exchange with the environment the temperature in boundary layers varies, which moves the gas and liquid. During heating, for example, the gas and liquid in areas adjacent to a wall will be lifted upward and move downward in the central part of the tank. The direction of motion of the gas and liquid on a mirror of liquid will be reciprocally opposite, which will lead to the intensification of diffusion.

Increase of pressure in the tank before engine starting. The displacement of liquid is not yet fulfilled. The change of the temperature of gas with time will depend on the type of pressurized system and heat exchange between gas and the wall, gas and liquid. This period is rather short and any noticeable diffusion processes here are not observed.

The displacement of liquid during operation engine on a stand or at launch, but before takeoff of the rocket. A change of temperature and volume of gas occurs. The ambient temperature is assumed constant. Heat exchange takes place inside the tank between gas, the wall and liquid. Part of the energy of gas, the quantity of which increases with time, is spent for displacement of liquid; in this case hydraulic resistances and mass forces are overcome and the kinetic energy of outflowing liquid is increased.

Displacement of liquid during operation engine in flight. In comparison with the previous case the calculation is complicated in view of heat exchange between the environment and the wall of the tank.

5.2. Heat Exchange

In the last, most complex case heat exchange occurs between gas and the tank wall, gas and liquid, environment and wall, wall and

liquid, and also the transfer of heat along the wall.

In certain cases heat exchange between gas and liquid is considered.

A rather complete picture of heat exchange can be obtained by considering two-dimensional nonstationary heat flow in the tank wall. By excluding heat exchange between gas and liquid, we arrive at the following equations.

The intensity of heating of the tank wall on the part of gases and from the external side is different. The temperature field in the wall will be unsymmetric. Inside the wall, along axis x (Fig. 51) let us separate the boundary, on which the derived wall temperatures along axis y become zero. Considering the diameter of the tank incommensurably large in comparison with the wall thickness, let us write calculation equations in Cartesian coordinate system.

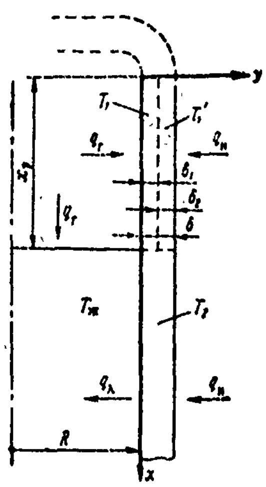

Fig. 5.1. Diagram of heat transfer in a tank.

The equation of thermal conductivity for part of the wall, located to the left of the considered boundary, will be written so:

$$\frac{\partial}{\partial t}[T_1(x,y,t)]=a_{\text{ст}}\left[\frac{\partial^2}{\partial x^2}T_1(x,y,t)+\frac{\partial^2}{\partial y^2}T_1(x,y,t)\right], \tag{5.1}$$

where the coefficient of temperature transfer of material of the wall

$$a_{\text{ст}}=\frac{\lambda_{\text{ст}}}{c_{\text{с1}}\varrho_{\text{ст}}}. \qquad (5.2)$$

The equation of thermal conductivity for the part of the wall, located to the right of the considered boundary,

$$\frac{\partial}{\partial t}[T_1'(x,y,t)]=a_{\text{ст}}\left[\frac{\partial^2}{\partial x^2}T_1'(x,y,t)+\frac{\partial^2}{\partial y^2}T_1'(x,y,t)\right]. \qquad (5.3)$$

Below the liquid mirror convective heat exchange occurs between the environment and wall. From the wall liquid heat is transferred due to thermal conductivity. For this part of the wall the equation of thermal conductivity will be written so:

$$\frac{\partial}{\partial t}[T_2(x,y,t)]=a_{\text{ст}}\left[\frac{\partial^2}{\partial x^2}T_2(x,y,t)+\frac{\partial^2}{\partial y^2}T_2(x,y,t)\right]. \qquad (5.4)$$

When performing calculations one should bear in mind that

$$a_{\text{ст}}=f(T). \qquad (5.5)$$

The following equation describes thermal conductivity in liquid:

$$\frac{\partial}{\partial t}[T_{\text{ж}}(x,y,t)]=a_{\text{ж}}\left[\frac{\partial^2}{\partial x^2}T_{\text{ж}}(x,y,t)+\frac{\partial^2}{\partial y^2}T_{\text{ж}}(x,y,t)\right]. \qquad (5.6)$$

For determination of the boundaries, within which the written equations act, let us take the origin of coordinates in the upper part of the tank on the inside of the wall. Current value of $x=x_{\text{р}}$, characterizing the position of the boundary of gas and liquid, or, which is the same, the position of the liquid mirror, will be determined by equation

$$x_{р}=x_{0}+\frac{1}{\rho_{ж}F_{б}}\int_{0}^{t}G_{ж}dt, \tag{5.7}$$

where x_0 — the position of liquid mirror at the initial moment of time; $G_{ж}$ — the current value of flow rate liquid from the tank; $F_{б}$ — the cross-sectional area of the tank.

At the initial moment of time the temperatures of the wall and liquid are equal to a certain prescribed temperature T_0. Therefore, initial conditions will be written so:

$$T_1(x,y,0)=T_0; \tag{5.8}$$

$$T_1'(x,y,0)=T_0; \tag{5.9}$$

$$T_2(x,y,0)=T_0; \tag{5.10}$$

$$T_{ж}(x,y,0)=T_0. \tag{5.11}$$

On the boundary between gas and liquid above the liquid mirror the heat flow, directed from gases to the wall, is completely transferred to the wall, therefore the boundary condition will be written so:

$$\alpha_{г}\left[T_{г}-T_1(x,0,t)\right]=\lambda_{ст}\frac{\partial}{\partial y}[T_1(x,0,t)]. \tag{5.12}$$

The external surface of the tank is heated convectively from the environment. For the part of the wall, located above the level of liquid, we have boundary condition:

$$\alpha_{н}\left[T_{н}-T_1'(x,0,t)\right]=\lambda_{ст}\frac{\partial}{\partial y}[T_1'(x,0,t)]. \tag{5.13}$$

For the wall, located below the level of liquid, the boundary condition will be written so:

$$\alpha_{н}\left[T_{н}-T_2(x,0,t)\right]=\lambda_{ст}\frac{\partial}{\partial y}[T_2(x,0,t)]. \tag{5.14}$$

For the boundary of temperatures T_1 and T_1' the condition can be written in the form

$$\frac{\partial}{\partial y}[T_1(x,\delta_1,t)]=0; \tag{5.15}$$

$$\frac{\partial}{\partial y}[T_1'(x,\delta_1,t)]=0 \tag{5.16}$$

or in this form:

$$T_1(x,\delta_1,t)=T_1'(x,\delta_1,t). \tag{5.17}$$

Below the liquid level for the interior wall of the tank, considering the steady change of temperature, we have boundary condition

$$T_2(x,\ 0,\ t)=T_{ж}(x,\ 0,\ t). \tag{5.18}$$

Preheating of liquid in the tank is symmetric relative to the axis of the tank, consequently, for the axis below the liquid level the boundary condition will be written so:

$$\frac{\partial}{\partial y}[T_{ж}(x,-R,t)]=0. \tag{5.19}$$

Now it remains to write the conditions for boundaries, directed along axis y.

In the upper part of the tank wall, where the origin of coordinates is located, it can be considered that

$$\frac{\partial}{\partial x}[T_1(0,y,t)]=0; \tag{5.20}$$

$$\frac{\partial}{\partial x}[T_1'(0,y,t)]=0. \tag{5.21}$$

Boundary condition (5.20) corresponds to section $0<y<\delta_1$, and condition (5.21) - to section $\delta_1<y<\delta$. These conditions are justified when heat flow of the top of the tank to section $(0,y)$ when $0<y<\delta$ is equal to the flow from the wall to the same section.

For boundary (x_p, y) when $0<y<\delta$ equality of temperatures should be fulfilled:

$$T_1(x_p^-,y,t)=T_2(x_p^-,y,t); \tag{5.22}$$

$$T_1'(x_p^-,y,t)=T_2'(x_p^-,y,t), \tag{5.23}$$

moreover boundary condition (5.22) corresponds to section $0<y<\delta_1$, and condition (5.23) - to section $\delta_1<y<\delta$.

The following boundary conditions are written for the lower part of the tank. If it is possible to consider that there are no changes in temperatures along axis x, then

$$\left.\begin{aligned} &\frac{\partial}{\partial x}[T_2(L,y,t)]=0;\\ &\frac{\partial}{\partial x}[T_{\text{ж}}(L,y,t)]=0, \end{aligned}\right\} \tag{5.24}$$

where L - distance to the lower part of the tank, i.e., $x_p=L$.

The last boundary remains - the demarcation line of liquid and gas. In the absence of convective heat exchange between gas and liquid the boundary condition will be written so:

$$\frac{\partial}{\partial x}[T_r(x_p^+,-y,t)]=\frac{\partial}{\partial x}[T_{\text{ж}}(x_p,-y,t)]. \tag{5.25}$$

If the liquid mirror is convectively heated from gas, which is in the tank, then, inasmuch as all the heat, supplied to the mirror, will be realized in the preheating process of liquid, we obtain

$$\alpha_{г}[T_{г}-T_{ж}(x_{p},-y,t)]=\lambda_{ж}\frac{\partial}{\partial x}[T_{ж}(x_{p},-y,t)]. \tag{5.26}$$

Both boundary conditions correspond to section $-R<y<0$.

Let us note that boundary conditions, with the solution of particular problems, should be refined with consideration of the tank construction and the specific character of heat exchange.

Sometines to get a preliminary approximate idea about heat exchange in the tank we consider a one-dimensional equation of thermal conductivity under the assumption of heating of the wall on one side or in the presence of equal intensity heating both on the inside and outside of the tank.

5.3. The Solution of One-Dimensional Equation of Thermal Conductivity

If we disregard heat flows, directed along axis x, then the equation of thermal conductivity takes the form

$$\frac{\partial}{\partial t}T_{ст}(y,t)=a_{ст}\frac{\partial^{2}}{\partial y^{2}}T_{ст}(y,t), \tag{5.27}$$

where $T_{ст}$ — the wall temperature.
Let us determine initial and boundary conditions. In the initial moment of time

$$T_{ст}(y,0)=T_{ст0}.$$

By analogy with equation (5.12) let us take boundary condition

$$\alpha[T_r - T_{ст}(0,t)] = \lambda_{ст}\frac{\partial}{\partial y}T_{ст}(0,t). \qquad (5.28)$$

Let us examine a symmetric problem, which corresponds to identical heating of the wall both inside and outside (Fig. 5.2). For $y=\delta$ we have boundary condition

$$\frac{\partial}{\partial y}T_{ст}(\delta,t)=0. \qquad (5.29)$$

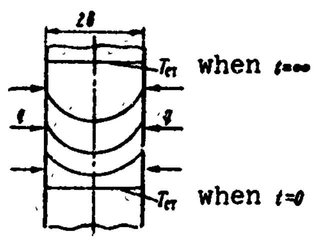

Fig. 5.2. Temperature distribution in the wall in the absence of heat exchange with the environment.

If we consider half the wall thickness (Fig. 5.3), then the conditions of temperature distribution along the thickness will correspond to the absence of heat exchange of the wall with the environment. The solution has the form [59]

$$T_{ст}(y,t)=T_r-(T_r-T_{ст0})\sum_{1}^{\infty}\frac{2\sin\mu_n\cdot\cos\mu_n}{\mu_n+\sin\mu_n\cdot\cos\mu_n}\times$$
$$\times 2\frac{y}{\delta}\exp\left(-\mu_n^2\frac{4a_{ст}}{\delta^2}t\right). \qquad (5.30)$$

The roots of characteristic equation μ_n are determined graphically (Fig. 5.4) with respect to points of intersection of contangent curve

$$y=\mathrm{ctg}\,\mu \qquad (5.31)$$

with a straight line

$$y=\mu\frac{\lambda_{ст}}{2\alpha_{ст}\delta}. \qquad (5.32)$$

Usually in expression (5.30) the series rapidly converges. Values of $\mu\frac{\lambda_{ст}}{2\alpha_{ст}\delta}$ are tabulated [59].

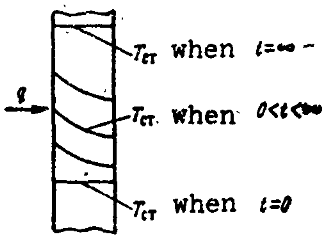

Fig. 5.3. Temperature distribution in the wall in the absence of heat exchange with the environment.

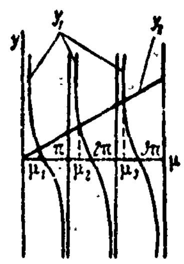

Fig. 5.4. The graphic method of determination of roots of characteristic equation.

5.4. The Heat Transfer Coefficient. Convective Heat Emission in Stationary Conditions

In the process of convective heat exchange the working medium transfers heat to the wall, so the quantity of heat, received by the wall, is equal to the quantity of heat which is returned to the working medium. In heat exchange there takes part the boundary layer, the thickness of which is designated δ. First let us examine the abrupt change of temperature from T to $T_{ст}$, which in Fig. 5.5 is represented by line *abcd*. Under the assumption of such an abrupt

change of temperature the process of heat exchange is isothermal. In actual conditions the temperature of the working medium varies along the thickness of the boundary layer approximately along line *bd*. Such a process of heat exchange is nonisothermal. The change of mass of working medium with the change of thickness of boundary layer on the element of length dx will comprise

$$dm = 2\pi r \rho d\delta dx. \qquad (5.33)$$

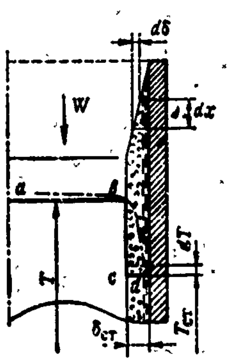

Fig. 5.5. For derivation of the equation for determining the heat emission coefficient.

The elementary change of heat in this case

$$dq = 2\pi r \rho c d\delta \frac{dx}{dt} dT. \qquad (5.34)$$

The element of the heated wall surface

$$dF = 2\pi r dx. \qquad (5.35)$$

The change of heat, received by the wall,

$$dq = \alpha dF dT, \qquad (5.36)$$

or

$$dq = \alpha 2\pi r dx dT. \tag{5.37}$$

By equating the right sides of equation (5.34) and (5.37), we obtain

$$\varrho c d\delta \frac{dx}{dt} = \alpha dx. \tag{5.38}$$

Consequently,

$$\alpha = \varrho c \frac{d\delta}{dx} \frac{dx}{dt}, \tag{5.39}$$

or

$$\alpha = \varrho c W \frac{d\delta}{dx}. \tag{5.40}$$

Now let us multiply the right and left sides of equation (5.40) by complex

$$\frac{d}{\lambda} \nu.$$

After transformations we obtain

$$\mathrm{Nu} = \mathrm{Re} \cdot \mathrm{Pr} \frac{d\delta}{dx}, \tag{5.41}$$

where the Nusselt number $\mathrm{Nu} = \frac{\alpha d}{\lambda}$, Reynolds number $\mathrm{Re} = \frac{Wd}{\nu}$ and Prandtl number $\mathrm{Pr} = \frac{\varrho \nu c}{\lambda}$. The thickness of the layer, which takes part in heat exchange, is proportional to the thickness of boundary layer. In turbulent flow

$$\delta = a_{\text{т}} 0{,}37 x^{0{,}8} \frac{d^{0{,}2}}{\mathrm{Re}^{0{,}2}}, \tag{5.42}$$

where a_τ — the proportionality factor.
Having taken the derivative with respect to x, we obtain

$$\frac{d\delta}{dx}=a_\tau 0{,}296\left(\frac{d}{x}\right)^{0,2}\mathrm{Re}^{-0,2}. \tag{5.43}$$

By substituting the value of the derivative from expression (5.43) in equation (5.41), we find

$$\mathrm{Nu}=a_\tau 0{,}296\left(\frac{d}{x}\right)^{0,2}\mathrm{Re}^{0,8}\cdot\mathrm{Pr}. \tag{5.44}$$

In the examination of actual processes into calculation there is introduced the coefficient of nonisothermicity β, considering the decrease of heat, yielded by the working medium in the nonisothermal process, in comparison with heat characteristic for isothermal heat exchange, moreover

$$\beta=\frac{T-\frac{1}{\delta}\int\limits_0^\delta T(\delta)\,d\delta}{T-T_{\text{ст}}}. \tag{5.45}$$

Now the calculation equation will be written so:

$$\mathrm{Nu}=a_\tau 0{,}296\left(\frac{d}{x}\right)^{0,2}\mathrm{Re}^{0,8}\cdot\mathrm{Pr}\cdot\beta. \tag{5.46}$$

In that area of heat exchange, where there is a laminar boundary layer,

$$\delta=a_л 5\sqrt{\frac{xd}{\mathrm{Re}}}. \tag{5.47}$$

Having taken the derivative with respect to x, we obtain

$$\frac{d\delta}{dx}=2{,}5\sqrt{\frac{d}{x\,\mathrm{Re}}}\,. \qquad (5.48)$$

The calculation formula will be written so:

$$\mathrm{Nu}=a_x\sqrt{\frac{d}{x}}\,\mathrm{Re}^{0,5}\cdot\mathrm{Pr}\cdot\beta. \qquad (5.49)$$

During derivation of equations (5.46) and (5.47) there were not considered many features, characterizing the heat transfer conditions; in each separate concrete case these features are manifested differently. Therefore, the calculation formula should be written so:

$$\mathrm{Nu}=A\left(\frac{d}{x}\right)^{k}\mathrm{Re}^{m}\cdot\mathrm{Pr}^{n}\cdot\beta, \qquad (5.50)$$

where factor A and exponents k, m and n are determined by results of processing of experimental data.

Convective heat emission of stabilized flow

With steady state the thermal and high-speed stabilization of flow can be forcibly created and started at any distance from the place of entry of the working medium into the heat-exchange device.

During motion along tubes the stabilization is begun at such a distance $x=l_{\text{кр}}$ from the place of entry, for which $\delta=\frac{d}{2}=R$. In this case

$$\left(\frac{d}{l_{\text{кр}}}\right)^{0,2}=(0{,}37\cdot 2)^{0,25}\,\mathrm{Re}^{-0,05}\approx 0{,}925\,\mathrm{Re}^{-0,05}.$$

The obtained value $(d/l)^{0.2}$ is retained for other values of $l > l_{нр}$ and, thus, characterizes the conditions of stabilized heat emission.

For determination of the coefficient of nonisothermicity let us use formula [94]:

$$W(\delta)=W(R)\left(\frac{\delta}{R}\right)^{1/\chi};\quad T(\delta)=T(R)[W(\delta)/W(R)]^{1/\chi}, \tag{5.48}$$

moreover χ scarcely depends on the Re number.

With change of temperature from T_0 to $T_{ст}$

$$T(\delta)=(T_0-T_{ст})\left(\frac{\delta}{R}\right)^{1/\chi}+T_{ст}. \tag{5.49}$$

It is logical to consider [63] χ as a power function of the temperature transfer coefficient at prescribed viscosity or, generally, of the Prandtl number, i.e.,

$$\chi=\chi_0 \Pr^{1-n}. \tag{5.50}$$

Now we find that

$$\beta=\frac{1}{\chi_0 \Pr^{1-n}+1},$$

where χ_0 corresponds to χ when Pr = 1.

If $\chi_0 \Pr^{1-n} \gg 1$, then

$$\beta\approx\frac{1}{\chi_0 \Pr^{1-n}}.$$

The effect of the direction of heat flow and a number of other factors on the intensity of heat emission is considered by power function

$$\left(\frac{Pr_{ж}}{Pr_{ст}}\right)^{s}.$$

Thus, for turbulent flow

$$Nu = A\,Re^{m} \cdot Pr^{n}. \tag{5.51}$$

If we accept the values of χ_0, m, n recommended in [63] and [94], then calculation by formula (5.51) will give good convergence with results of processing of experimental data.

With free convection

$$Nu = A(Pr \cdot Gr)^{c}, \tag{5.52}$$

where Grashof number

$$Gr = \frac{l^3}{\nu^2} B\Delta T; \tag{5.53}$$

B - the volumetric expansion coefficient of liquid.

If the difference of densities of liquid is determined by the difference of temperatures, then

$$B\Delta T = \frac{\varrho - \varrho_{н}}{\varrho}. \tag{5.54}$$

Tables 5.1 and 5.2 contain the necessary data for calculation of forces and free convection in fuel tanks.

Table 5.1. Forces convection in a fuel tank [according to formula (5.51)].

Heat exchange condition	A	m
$1<Re<4$	0,891	0,33
$4<Re<40$	0,821	0,385
$40<Re<4\cdot10^3$	0,615	0,466
$4\cdot10^3<Re<4\cdot10^4$	0,174	0,618
$4\cdot10^4<Re<2,5\cdot10^5$	0,0239	0,805

Note: $Re=\frac{Gd}{\mu}$, $k=0$, $n=0$.

Table 5.2. Free convection in a fuel tank [according to formula (5.52)].

Heat exchange conditions		A	C	Area of application
Vertical surface	Laminar motion	0,59	0,25	$10^4<(Pr\cdot Gr)<10^9$
	Turbulent motion	0,13	0,33	$10^9<(Pr\cdot Gr)<10^{12}$
Horizontal surface	Laminar motion	0,54	0,25	$10^5<(Pr\cdot Gr)<2\cdot10^7$ $T_ж>T_г$
		0,27	0,25	$3\cdot10^5<(Pr\cdot Gr)<3\cdot10^{10}$ $T_ж<T_г$
	Turbulent motion	0,14	0,33	$10^7<(Pr\cdot Gr)<3\cdot10^{10}$ $T_ж>T_2$

Note: $Nu=\frac{\alpha l}{\lambda}$.

Convective heat emission in nonstationary conditions

Separate works, for example [20], [35] have been dedicated to the study of heat emission conditions under nonstationary conditions. Examination of the effect of derivatives $\dot{G}$ and $\dot{q}$ on the heat emission coefficient presents the greatest interest. Under the effect of $\dot{G}$ there occurs a change of the velocity profile; if $\dot{G}>0$,

then increase of the coefficients of heat emission α and hydraulic resistance ξ. If heat flow q varies with time, then convective heat exchange is affected by nonstationary preheating of the working medium, which leads to an increase of α when $\dot{q}>0$. Therefore, for determination of the Nusselt number in nonstationary conditions it is possible to recommend formula

$$Nu(t) = Nu\,\varphi_1(\dot{G})\varphi_2(\dot{q}),$$

where Nu - the value of Nusselt number under steady conditions.

If $\dot{G}=0$, then $\varphi_1(G)=1$. If $\dot{G}$ affects the magnitude of the heat emission coefficient, then with increase of $\dot{G}$ function $\varphi_1(\dot{G})$ will approach a certain limit, equal to ψ_1. If $\dot{q}=0$, then $\varphi_2(\dot{q})=1$, since in this case

$$Nu(t) = Nu.$$

With increase of $\dot{q}$, even with instantaneous growth of heat flow, function $\varphi_2(\dot{q})$ takes some limiting value ψ_2.

In engineering calculations it is possible to take the exponential law of change of functions in the form

$$\varphi_1(\dot{G}) = 1 + (\psi_1 - 1)[1 - \exp(-\bar{\dot{G}})];$$
$$\varphi_2(\dot{q}) = 1 + (\psi_2 - 1)[1 - \exp(-\bar{\dot{q}})].$$

By using the theory of dimension, we find

$$\bar{\dot{G}} = \dot{G}\frac{d^2}{G\nu}, \quad \bar{\dot{q}} = \dot{q}\frac{d^2}{a(T - T_{cт})c},$$

where a — the temperature transfer coefficient of working medium; $T - T_{cт}$ — the temperature difference, characterizing heat exchange.

Values of $\psi_1, \psi_2, \bar{G}, \bar{q}$ are determined experimentally. Actually, B. M. Galitseyskiy, E. K. Kalinin and others [20], [35] in experiments with tubes for air when $1.8 \cdot 10^4 < Re < 2.9 \cdot 10^5$, feed pressure 0.46-1.6 Mn/m^2 and relationship $T/T_{ст} = 1-1.44$ obtained the law of change of the considered functions, close to expotential. By generalizing the results of a number of experiments, for approximate calculations it is possible to recommend $\psi_2 = 1.1-1.2$ and $\bar{q} = 0.05-0.10$.

5.5. The Method of Regular Conditions

In engineering practice we use approximate methods of calculation of heat exchange in the tank, one of which is the method of regular conditions.

The tank along axis x is divided conditionally into a number of sections, the length of each of which is equal to Δx. Sometimes it proves to be convenient to divide the tank into sections, the lengths of which are various.

Calculation is performed under the assumption of abrupt change of the height of the liquid level in the tank. The emptying time of a section with height Δx is equal to Δt. For each section of the wall parameters are averaged, and the gas parameters are averaged for each interval of time Δt. Let us assume section Δx is heated during time Δt. The heat flow, being directed from gases to a wall,

$$dq = \alpha (T_г - T_{ст1}) dt, \tag{5.55}$$

where $T_{ст1}$ — wall temperature on the part of gases. The increase of enthalpy of the section wall

$$di = 2\pi R \delta \Delta x \varrho_{ст} c_{ст} dT_{ст1}, \tag{5.56}$$

where δ — wall thickness; $\varrho_{\text{ст}}$ — density of wall material; $c_{\text{ст}}$ — specific heat of wall material; $\bar{T}_{\text{ст}}$ — mean wall temperature, moreover

$$\bar{T}_{\text{ст}}=\frac{T_{\text{ст}1}+T_{\text{ст}2}}{2}, \tag{5.57}$$

$T_{\text{ст}2}$ — temperature of external wall surface.

The method of regular conditions is based on the assumption that

$$T_{\text{ст}1}-T_{\text{ст}2}=\Delta T_{\text{ст}}=\text{const}. \tag{5.58}$$

Consequently

$$dT_{\text{ст}}=dT_{\text{ст}1}. \tag{5.59}$$

Considering

$$dq=di, \tag{5.60}$$

we find

$$\frac{\alpha}{2\pi R\delta\Delta x\varrho_{\text{ст}}c_{\text{ст}}}\Delta t=\int\limits_{T_{\text{ст}10}}^{T_{\text{ст}1}}\frac{dT_{\text{ст}1}}{T_{\text{г}}-T_{\text{ст}1}}. \tag{5.61}$$

Hence through time interval Δt

$$T_{\text{ст}}=T_{\text{г}}[1-\exp(-B\Delta t)]+T_{\text{ст}10}\exp(-B\Delta t), \tag{5.62}$$

where $T_{\text{ст}10}$ — initial value of wall temperature on the part of gas;

$$B=\frac{a}{2\pi R\delta\Delta x\varrho_{\text{ст}}c_{\text{ст}}}. \tag{5.63}$$

5.6. Calculation of the Heating of the Tank Wall Along Sections

During calculation of heating of the entire surface of the tank along sections, the number of sections is designated by Roman numerals, and the time intervals - Arabic.

The internal surface of the first annular section Δx will be heated for Δt s. Heat, transferred from gas to the wall in the first period,

$$\Delta Q_{11}=\alpha_1 2\pi R\Delta x(T_{\text{г}1}-T_{\text{ст}11})\Delta t, \tag{5.64}$$

where α_1 - the coefficient of heat emission from gases to the wall in the first period of calculation;
the increase of wall temperature in this case

$$\Delta T_{\text{ст}\,11}=\frac{\Delta Q_{11}}{2\pi R\delta\Delta x\varrho_{\text{ст}}c_{\text{ст}}}. \tag{5.65}$$

The wall temperature of the first section at the beginning of the second period

$$T_{\text{ст}12}=T_{\text{ст}11}+\Delta T_{\text{ст}11}; \tag{5.66}$$

the mass of gas of gas in the tank during the first period

$$m_1=\frac{p_1}{R_1T_{\text{г}1}}(V_0+2\pi R\Delta x), \tag{5.67}$$

where V_0 - initial free volume.

The decrease of gas tempetature will be equal to

$$\Delta T_{r1} = \frac{\Delta Q_{11}}{m_1 c_{V1}}, \qquad (5.68)$$

where c_{V1} — the specific heat of gas in the first period of calculation.

In the second period of time there occurs heating of two sections, but temperature $T_{\text{ст}I2}$ of the first section will be higher than temperature $T_{\text{ст}II2}$ of the second section, just as appeared from under the liquid mirror. The mass of gas, heating the wall in the second period,

$$m_2 = m_1 + \int_0^{\Delta t_1} G_r dt. \qquad (5.69)$$

Inasmuch as time interval Δt is rather small, during calculation it is possible to be guided by the average and constant value of specific heat.

The gas temperature in the tank at the beginning of the second period of heating

$$T_{r2} = \frac{m_1 (T_{r1} - \Delta T_{r1}) + T_{r0} \int_0^{\Delta t_1} G_r dt}{m_2}, \qquad (5.70)$$

where T_{r0} — temperature of a fresh batch of gas.
Heat, transferred to the first section in the second period of time,

$$\Delta Q_{12} = \alpha_2 2\pi R \Delta x (T_{r2} - T_{\text{ст}12}) \Delta t. \qquad (5.71)$$

Heat, transferred to the second section in the second period of time,

$$\Delta Q_{II2}=\alpha_2 2\pi R\Delta x(T_{r3}-T_{\text{cr}\,II2})\,\Delta t. \quad (5.72)$$

Total heat, transferred to the wall, is equal to

$$\Delta Q_2=\Delta Q_{I2}+\Delta Q_{II2}. \quad (5.73)$$

The decrease of gas temperature in the tank

$$\Delta T_{r2}=\frac{\Delta Q_2}{m_2 c_2}. \quad (5.74)$$

The new value of gas temperature is initial for determining the temperature in the third period:

$$T_{r3}=\frac{m_2(T_{r2}-\Delta T_{r2})+T_{r0}\int_{\Delta t_1}^{\Delta t_2} G_r\,dt}{m_3}, \quad (5.75)$$

moreover

$$m_3=m_2+\int_{\Delta t_1}^{\Delta t_2} G_r\,dt. \quad (5.76)$$

In the third period of time there is calculated the heating of even three sections, and calculation equations are written by analogy with the previous.

For performing of more precise calculations it is necessary to draw upon equations, characterizing the chemical composition and the

thermodynamic parameters of gas, which are continuously changed in the process of displacement of liquid.

Let us note that when using the formula of regular conditions (5.62) there is calculated the change of wall temperature with time. During calculation of heating of the tank wall along sections in the case of utilization of formulas (5.64)-(5.66) there is assumed an abrupt change of temperatures. The results can be refined, by using the method of regular conditions in calculation with respect to sections.

5.7. Mass Transfer, Diffusion of Gas into Liquid

Diffusion can be molecular or turbulent.

Molecular diffusion is observed in the absence of heat exchange of the tank with the environment and the mechanized mixing of liquid or circulation of gas. Turbulent diffusion is characterized by the presence of liquid or gas currents.

For the tank the equation of molecular diffusion in cylindrical coordinate system has the form

$$\frac{\partial c}{\partial t}=D_{м}\left[\frac{\partial^2 c}{\partial x^2}+\frac{1}{r}\,\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}(cr)\right)\right], \qquad (5.77)$$

where c - the concentration of gas in liquid; $D_{м}$ — coefficient of molecular diffusion

In equation (5.77) the first term $\partial^2 c/\partial x^2$ considers the change of concentration along the depth, and the second term - along the radius of the tank. However, in the case of molecular diffusion the flow of the mass of gas is practically uniform over the entire section of the tank; in this case instead of equation (5.77) we

arrive at nonstationary, one-dimensional equation

$$\frac{\partial c}{\partial t}=D_{\text{ж}}\frac{\partial^2 c}{\partial x^2}. \qquad (5.78)$$

At the initial moment of time (t = 0) along the entire depth of the tank (x) the concentration of gas c = 0, i.e.,

$$c(x, 0)=0. \qquad (5.79)$$

If the tank was serviced with liquid and filled with gas before beginning the research, then at moment of time t = 0, i.e., at the beginning of calculation, the concentration of gas in liquid will be nonzero and instead of condition (5.79) we will have

$$c(x, 0)=c_0.$$

At relatively low pressures the concentration is proportional to the partial pressure of substance, diffusing from the gas cushion of the tank into liquid, i.e.,

$$c=Kp_t, \qquad (5.80)$$

where K - Henry coefficient.
Therefore the boundary condition can be written so:

$$c(0, t)=Kp_t. \qquad (5.81)$$

At rather large depths on the bottom of the tank the concentration of gas in liquid is practically equal to zero; this leads to boundary condition

$$\frac{\partial}{\partial x}c(\infty,t)=0. \qquad (5.82)$$

The diffusion coefficient is inversely proportional to viscosity, which depends on the pressure. The theoretical relationship of the pressure coefficient to Eyring and Frenkel temperature has the form

$$D = D_0 \exp\left(-\frac{E_D}{RT}\right), \qquad (5.83)$$

where E_D — the activation energy of the diffusion process. After transformations we obtain equation

$$\ln D = A - \frac{B}{T}.$$

During the study of diffusion in hydrocarbon fuel, but under actual conditions, i.e., with vibration of tanks, V. P. Logvinyuk obtained the following approximate dependences:

$$\mathrm{Nu}_D \approx 0{,}05\,\mathrm{Re}\,S_c^{0{,}6};$$
$$D_т \approx D_м (1 + 0{,}03\,\mathrm{Re}\,S_c^{0{,}4})$$ [1],

where Nu_D — Nusselt diffusion criterion, moreover

$$\mathrm{Nu}_D = \frac{kh}{D};$$

k - mass transfer coefficient; h - the depth of penetration of diffusing gas, measured from the liquid mirror; S_c — Prandtl diffusion criterion, moreover

$$S_c = \frac{\nu}{D};$$

$D_т$ — turbulent diffusion coefficient; $D_м$ — molecular diffusion coefficient.

[1]The numerical values of coefficients and exponents are rounded-off by us.

The molecular diffusion coefficient in [15] is determined by formula

$$D_м = D_0 \frac{760}{p} \left(\frac{T}{273} \right)^n,$$

where D_0 – diffusion coefficient with $p=760$ mm Hg and $T=273°K$. The exponent

$$0,75 < n < 1,0.$$

In the presence of mechanized mixing or with noticeable change of ambient temperature turbulent diffusion takes place. During heating of tank walls natural convection appears, characterized by velocity components. The equation of turbulent diffusion in cylindrical coordinate system has the form

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(W_x c) + \frac{1}{r} \frac{\partial}{\partial r}(W_r rc) = D \left\{ \frac{\partial^2 c}{\partial x^2} + \frac{\partial}{\partial r} \left[\frac{\partial}{\partial r}(cr) \right] \right\}, \tag{5.84}$$

where

$$D = D_м + D_т.$$

If the change of concentration along the depth is caused mainly by the longitudinal motion of liquid, then

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(W_x c) = D \frac{\partial^2 c}{\partial x^2}. \tag{5.85}$$

With mean value of velocity

$$\frac{\partial c}{\partial t} + \overline{W}_x \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2}. \tag{5.86}$$

It is possible to solve equation (5.86) by the following method. Let us assume that concentration

$$c = TX, \tag{5.87}$$

where T varies only with time, and X is a function only of x. Instead of equation (5.86) we obtain

$$X \frac{\partial T}{\partial t} - \overline{W}_x T \frac{\partial X}{\partial x} = DT \frac{\partial^2 X}{\partial x^2} \tag{5.88}$$

or

$$\frac{1}{T} \frac{\partial T}{\partial t} = D \frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \overline{W}_x \frac{1}{X} \frac{\partial X}{\partial x}. \tag{5.89}$$

Now the left side depends only on t, and the right - only on x, consequently, both sides represent some constant quantity. By designating it through $-n^2$, we arrive at two equations, into which equation (5.89) is decomposed:

$$-\frac{1}{T} \frac{dT}{dt} = n^2; \tag{5.90}$$

$$D \frac{d^2 X}{\partial x^2} + \overline{W}_x \frac{\partial X}{\partial x} + n^2 X = 0. \tag{5.91}$$

The solution of equation (5.90) has the form

$$T = A' \exp(-n^2 t), \tag{5.92}$$

where A' - integration constant.

Let us rewrite equation (5.91) so:

$$\frac{d^2 X}{dx^2} + 2h_1 \frac{dX}{dx} + k^2 X = 0, \tag{5.93}$$

where

$$2h=\frac{W_x}{D};\tag{5.94}$$

$$k^2=\frac{n^2}{D}.\tag{5.95}$$

If $h^2>k^2$, then in this case the solution takes the form

$$X=A_1\exp[(q-h)t]+A_2\exp[-(q+h)t],\tag{5.96}$$

where A_1 and A_2 - integration constants;

$$q=\sqrt{h^2-k^2}.\tag{5.97}$$

In accordance with equalities (5.87), (5.91) and (5.96) concentration

$$c=\exp(-n^2t)[A\exp(q-h)t+B\exp(-q-h)t],\tag{5.98}$$

where new constants

$$A=A'A_1,\tag{5.99}$$

$$B=A'A_2\tag{5.100}$$

are determined by boundary conditions depending on a particular formulation of the problem.

5.8. Overflowing of Liquid from the Tank into the Line

The connection of the tank with the line is carried out with the aid of intake devices, which provide smooth, uniform overflowing of liquid. Intake devices should guarantee cavitation-free motion of

liquid, provide uniform lowering of the level of liquid in the tank and prevent funnel formation.

Cavitation appears when static pressure turns out to be lower than the saturated vapor pressure. The static pressure with overflow of liquid from the tank in the line is decreased as a result of increase of the flow velocity and under the effect of friction forces.

For two sections of a convergent channel of the intake device the balance of pressures is written so (Fig. 5.6):

$$p_1+\varrho_ж \frac{W_1^2}{2}=p_2+\varrho_ж (j_x+g_x)l+\varrho_ж \frac{W_2^2}{2}-\varrho_ж \int_0^l \dot{W}(l)dl, \qquad (5.101)$$

where p_1 and p_2 - static pressure in the considered sections; (j_x+g_x) - projection of acceleration to the axis of intake device; $\varrho_ж \int_0^l \dot{W}(l)dl$ - pressure, equivalent to mass forces.

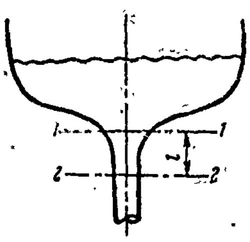

Fig. 5.6. Diagram of overflow of liquid from the tank into the line.

According to the equation of continuity

$$\dot{W}(l)=\frac{\dot{G}}{F(l)\varrho_ж}. \qquad (5.102)$$

Consequently,

$$\varrho_ж \int_0^l \dot{W}(l)dl=\dot{G}\int_0^l \frac{dl}{F(l)}. \qquad (5.103)$$

In order to provide constancy of pressure in all sections of the intake device, in equation (5.103) one should take $p_1 = p_2$. In this case we obtain

$$\frac{W_1^2}{2} = (j_x + g_x)l + \frac{W_2^2}{2} - \frac{\dot{G}}{\varrho_ж}\int_0^l \frac{dl}{F(l)}. \qquad (5.104)$$

By using the equation of continuity, we find

$$\frac{G^2}{2F(l)\varrho_ж^2} = (j_x + g_x)l + \frac{G}{2F_2^2\varrho_ж^2} - \frac{\dot{G}}{\varrho_ж}\int_0^l \frac{dl}{F(l)}. \qquad (5.105)$$

Equation (5.105) allows constructing the profile of the intake device, if the constant value of the derivative of flow rate in time is assigned. Inasmuch as when launching a rocket the acceleration of flow rate is changed, and the engine operates the longest at $G \approx \mathrm{const}$, then this case is of the greatest interest when selecting the profile of intake device.

Taking into account that the area of section

$$F_2 = \frac{\pi}{4} d_{тр}^2, \qquad (5.106)$$

by using equation (5.105) and assuming $G = 0$, for steady state we find [97]

$$d_{тр}(l) = \left[\frac{G^2}{\frac{G^2}{d_{тр}^4} - \frac{\pi^2}{8}(j_x + g_x)l\varrho_ж^2}\right]^{0,25}. \qquad (5.107)$$

Equation (5.107) allows determining the current value of the diameter of intake device, i.e., to profile it.

The next stage of the study of further motion of liquid is research of the conditions of filling of the line with liquid and interaction between elastic liquid and the deformable line.

CHAPTER VI

THE MOTION OF LIQUID THROUGH THE LINE

6.1. Equation of Continuity for Elastic Liquid, Moving in a Deformable Line

At arbitrary distance l from the inlet to the line let us separate an element with length ∂x (Fig. 6.1).

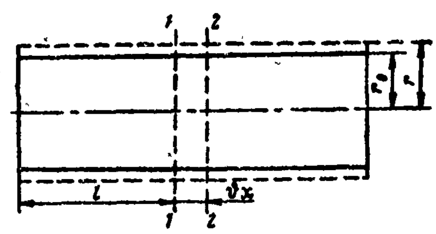

Fig. 6.1. For derivation of the equation of continuity for elastic liquid, moving in a deformable line.

The flow rate of liquid per second through section *1-1* according to the equation of continuity will be

$$\dot{Y}_1 = G_1 = F\rho_{ж}W. \tag{6.1}$$

On section of path ∂x there can occur change of density and velocity, therefore the flow rate through section *2-2* will be

$$\dot{Y}_2 = G_2 = F\rho_{ж}W - \frac{\partial}{\partial x}(\rho_{ж}W)F\partial x. \tag{6.2}$$

Change of the quantity of liquid per second, which is located between sections *1-1* and *2-2*, appearing as a result of change of the liquid density with time will be

$$\dot{Y}_{\varrho}=\frac{\partial \varrho_{\text{ж}}}{\partial t}F\vartheta x. \qquad (6.3)$$

Change of the quantity of liquid per second in the same volume as a result of change in the cross-sectional area with time will be

$$\dot{Y}_{Ft}=\frac{\partial F}{\partial t}\varrho_{\text{ж}}\vartheta x. \qquad (6.4)$$

If the cross-sectional area changes lengthwise, then the change of flow rate will be

$$\dot{Y}_{Fx}=\frac{\partial F}{\partial x}\varrho_{\text{ж}}W\vartheta x. \qquad (6.5)$$

Change of the quantity of liquid per second as a result of lengthening of the line will be

$$\dot{Y}_{x}=\frac{\partial}{\partial t}(\vartheta x)\varrho_{\text{ж}}F. \qquad (6.6)$$

The equation of the law of conservation of mass of liquid will be written so:

$$\dot{Y}_{1}-\dot{Y}_{2}+\dot{Y}_{\varrho}+\dot{Y}_{Ft}+\dot{Y}_{Fx}+\dot{Y}_{x}=0. \qquad (6.7)$$

By substituting the values of the found derivatives $\dot{Y}_i$, we obtain

$$\frac{\partial}{\partial t}\varrho_{\text{ж}}F+\frac{\partial}{\partial x}(\varrho_{\text{ж}}W)F+\frac{\partial F}{\partial t}\varrho_{\text{ж}}+\frac{\partial F}{\partial x}\varrho_{\text{ж}}W+\frac{\partial}{\partial t}(\vartheta x)\frac{\varrho_{\text{ж}}F}{\vartheta x}=0. \qquad (6.8)$$

Inasmuch as all elements of the line ϑx are drawn out equivalently,

$$\frac{\frac{\partial}{\partial t}(\vartheta x)}{\vartheta x}=\frac{\frac{\partial}{\partial t}x}{x}. \qquad (6.9)$$

Let us take into account that

$$\dot{\varrho}_{\text{ж}}=\frac{\partial\varrho_{\text{ж}}}{\partial t}+\frac{\partial\varrho_{\text{ж}}}{\partial x}W; \qquad (6.10)$$

$$\dot{F}=\frac{\partial F}{\partial t}+\frac{\partial F}{\partial x}W. \qquad (6.11)$$

The equation of continuity assumes the form

$$\frac{\dot{\varrho}_{\text{ж}}}{\varrho_{\text{ж}}}+\frac{\dot{F}}{F}+\frac{\partial W}{\partial x}+\frac{1}{x}\,\frac{\partial}{\partial t}\,x=0. \qquad (6.12)$$

6.2. Law of Elastic Deformations

The condition of proportionality between elementary changes of pressure and density has the form

$$\frac{dp}{d\varrho_{\text{ж}}}=\frac{E_{\text{ж}}}{\varrho_{\text{ж}}}=C^2, \qquad (6.13)$$

where C – the speed of sound; $E_{\text{ж}}$ – modulus of elasticity of liquid.

The law of elastic deformations for the pipeline in the case of radial deformation

$$P=\varepsilon ES, \qquad (6.14)$$

where ε – relative elongation, moreover

$$\varepsilon=\frac{r-r_0}{r_0}; \qquad (6.15)$$

r_0 – initial value of radius of the line;

E – modulus of elasticity of material;

S – the area being elongated, moreover

$$S=2\vartheta x\delta, \tag{6.16}$$

δ – wall thickness of line.

Thus,

$$P=2E\delta\frac{r-r_0}{r_0}\vartheta x. \tag{6.17}$$

The force, elongating the wall,

$$P=P_1-P_{\text{ст}}-P_{\text{ж}}. \tag{6.18}$$

Static force

$$P_1=2r_0p\vartheta x, \tag{6.19}$$

where $p = f(x)$ – static pressure in the region of the considered sections.

The dynamic force, caused by accelerated motion of the wall of the line,

$$P_{\text{ст}}=2\pi r_0\vartheta x\delta\varrho_{\text{м}}\dot{W}_{\text{ст}}, \tag{6.20}$$

where $2\pi r_0\vartheta x\delta\varrho_{\text{м}}$ – mass of wall; $\varrho_{\text{м}}$ – density of wall material; W_{CT} – the rate of radial displacement of the wall. The rate of radial displacement of liquid $W_r(r)$ is a function of the radius. On the axis of a tube $W_r(0) = 0$, and near the wall $W_r(r_0) = W_{CT}$.

Dynamic force, caused by the acceleration of liquid,

$$P_{\text{ж}}=2\pi\vartheta x\int_0^{r_0}\varrho_{\text{ж}}\dot{W}_r(r)\,r\,dr. \tag{6.21}$$

For determination of force, elongating the wall, let us write

$$P=2\pi r_0 p \delta x. \qquad (6.22)$$

For determination of coefficient ξ, having equated the right sides of equations (6.18) and (6.22), we obtain

$$\xi=1-\frac{\pi \varrho_{\text{ж}} \dot{W}_{\text{ст}} \delta}{p}-\frac{\pi \int_0^{r_0} \varrho_{\text{ж}} \dot{W}_r(r) r dr}{r_0 p}. \qquad (6.23)$$

By equating the right sides of equations (6.17) and (6.22), we find

$$\xi p=E\delta \frac{r-r_0}{r_0^2}. \qquad (6.24)$$

By taking the average value for ξ and having taken its derivative, we obtain

$$\xi \frac{\partial p}{\partial t}=E \frac{\delta}{r_0^2} \frac{\partial r}{\partial t}. \qquad (6.25)$$

Taking into account that

$$F=\pi r^2 \approx \pi r_0^2, \qquad (6.26)$$

instead of expression (6.25) we find

$$\frac{\partial F}{\partial t}=\frac{2\pi r_0^3}{E\delta} \xi \frac{\partial p}{\partial t}. \qquad (6.27)$$

According to the law of elastic deformations with elongation of line

$$P_l=\varepsilon_l E S_l, \qquad (6.28)$$

moreover

$$S_l=2\pi r_0 \delta; \qquad (6.29)$$

$$\frac{{}^0x}{{}^0x-x}=l_s \qquad (6.30)$$

The tensile stress depends on the conditions of elongation deformation. If the reduced pressure is designated through p_l, then

$$P_l = \pi r_0^2 p_l. \tag{6.31}$$

In this case

$$p_l = 2\frac{\delta}{r_0}\,\frac{x - x_0}{x_0}\,E. \tag{6.32}$$

Having taken the derivative, we find

$$\frac{1}{x_0}\,\frac{\partial}{\partial t}x = \frac{r_0}{2\delta E}\,\frac{\partial p_l}{\partial t}. \tag{6.33}$$

According to formula (6.13) we find

$$\frac{\dot{\varrho}_ж}{\varrho_ж} = \frac{\dot{p}}{E_ж}. \tag{6.34}$$

According to equations (6.11), (6.26) and (6.27) we have

$$\frac{\dot{F}}{F} = \frac{2r_0}{E\delta}\,\xi\,\frac{\partial p}{\partial t} + \frac{\partial F}{\partial x}\,\frac{W}{\pi r_0^2}. \tag{6.35}$$

By considering relationships (6.33), (6.34) and (6.35), instead of equation (6.12) we will have

$$\frac{\dot{p}}{E_ж} + \frac{\partial W}{\partial x} + \frac{2r_0}{E\delta}\,\xi\,\frac{\partial p}{\partial t} + \frac{\partial F}{\partial x}\,\frac{W}{\pi r_0^2} + \frac{r_0}{2E\delta}\,\frac{\partial p_l}{\partial t} = 0. \tag{6.36}$$

The third term in equation (6.36), i.e.,

$$\frac{2r_0}{E\delta}\,\xi\,\frac{\partial p}{\partial t},$$

reflects radial deformation of the line with time, caused by the presence of local derivative $\partial p/\partial t$. The following term, i.e.,

$$\frac{\partial F}{\partial x}\frac{W}{\pi r_0^2},$$

is caused by the presence of change of the cross-sectional area of the duct lengthwise.

Elongation deformation is characterized by partial derivative $\partial p_l/\partial t$. At velocities $W \ll C$ instead of total derivative $\dot{p}$ it is possible to be guided by the partial derivative, i.e., to use replacement of $\dot{p}$ by $\partial p/\partial t$. In this case equation (6.36) will be written so:

$$\frac{\partial p}{\partial t}+\lambda^2\left[\frac{\partial W}{\partial x}+\frac{W}{\pi r_0^2}\frac{\partial F}{\partial x}+\frac{r_0}{2E\delta}\frac{\partial p_l}{\partial t}\right]=0, \qquad (6.37)$$

where

$$\lambda^2=\frac{1}{\frac{1}{E_ж}+\frac{2r_0}{E\delta}\xi}, \qquad (6.38)$$

and

$$\frac{\lambda}{\sqrt{\varrho_ж}}=\frac{1}{\sqrt{\frac{\varrho_ж}{E_ж}+\frac{\varrho_ж}{\varrho}\frac{2r_0}{\delta}\xi\frac{\varrho}{E}}}. \qquad (6.39)$$

Quantity $\frac{E_ж}{\varrho_ж}=C_ж^2$ is the square of the speed of sound in liquid; ratio E/ϱ is equal to the square of the speed of sound in the pipe material. Thus, $\lambda/\sqrt{\varrho_ж}$ characterizes the value of the reduced speed of sound $C_{пр}$. The coefficient of equivalence, which allows taking into account the effect of walls of the line, will be

$$K_э=\frac{\varrho_ж}{\varrho}\frac{2r_0}{\delta}\xi, \qquad (6.40)$$

Now we have

$$C_{\text{пр}}^2 = \frac{1}{\frac{1}{C_{\text{ж}}^2} + \frac{K_l}{C^2}} \qquad (6.41)$$

or

$$\frac{\varrho^3}{C_{\text{пр}}^2} = \frac{\varrho^3}{C_{\text{ж}}^2} + \frac{\varrho_{\text{ж}} d_0}{C^2}\xi.$$

By using the equation of continuity, we find

$$\frac{\partial p}{\partial t} + \lambda^2\left[\frac{1}{\pi r_0^2 \varrho_{\text{ж}}}\frac{\partial G}{\partial t} + \frac{G}{\pi^2 r_0^4 \varrho_{\text{ж}}}\frac{\partial F}{\partial x} + \frac{r_0}{2E\delta}\frac{\partial p_l}{\partial t}\right] = 0. \qquad (6.42)$$

In engineering calculations it is possible to accept $p_l = p$; in this case instead of equation (6.42) we receive:

$$\frac{\partial p}{\partial t} + \lambda^2\left[\frac{1}{\pi r_0^2 \varrho_{\text{ж}}}\frac{\partial G}{\partial t} + \frac{G}{\pi^2 r_0^4 \varrho_{\text{ж}}}\frac{\partial F}{\partial x}\right] = 0,$$

but now

$$\lambda^2 = \frac{1}{\frac{1}{E_{\text{ж}}} + \frac{5}{2}\frac{r_0}{E\delta}\xi}.$$

6.3. Wave Equation

For derivation of the equation let us use the equation of motion and equation (6.37), which are written so:

$$\frac{\partial W}{\partial t} - (j_x + g_x) + \frac{1}{\varrho_{\text{ж}}}\frac{\partial p}{\partial x} - \nu\frac{\partial^2 W}{\partial x^2} = 0; \qquad (6.43)$$

$$\frac{\partial p}{\partial t} + \lambda^2\left[\frac{\partial W}{\partial x} + \varphi\right] = 0, \qquad (6.44)$$

where there is designated

$$\varphi=\frac{W}{\pi r_0^2}\,\frac{\partial F}{\partial x}+\frac{r_0}{2E\delta}\,\frac{\partial p_l}{\partial t}. \qquad (6.45)$$

Having taken partial derivatives with respect to x from equation (6.43) and with respect to t from equation (6.44), we obtain

$$\frac{\partial^2 W}{\partial t\partial x}-\frac{\partial}{\partial x}(j_x+g_x)+\frac{1}{\rho_{ж}}\,\frac{\partial^2 p}{\partial x^2}-\nu\,\frac{\partial^3 W}{\partial x^3}=0; \qquad (6.46)$$

$$\frac{\partial^2 p}{\partial t^2}+\lambda^2\left[\frac{\partial^2 W}{\partial t\partial x}+\frac{\partial\varphi}{\partial t}\right]=0. \qquad (6.47)$$

By excluding $\partial^2 W/\partial t\partial x$ from expression (6.47) with the aid of equation (6.46), after transformations we arrive at heterogeneous wave equation for sound pressure:

$$\frac{1}{\lambda^2}\,\frac{\partial^2 p}{\partial t^2}-\frac{1}{\rho_{ж}}\,\frac{\partial^2 p}{\partial x^2}=-U, \qquad (6.48)$$

where

$$U=\frac{\partial}{\partial x}(j_x+g_x)+\nu\frac{\partial^3 W}{\partial x^3}+\frac{\partial}{\partial t}\left(\frac{W}{\pi r_0^2}\,\frac{\partial F}{\partial x}\right)+\frac{r_0}{2E\delta}\,\frac{\partial^2 p_l}{\partial t^2}. \qquad (6.49)$$

The change of external volumetric force along the length of the duct is so small, that it is possible to accept

$$\frac{\partial}{\partial x}(j_x+g_x)=0. \qquad (6.50)$$

The augend of the right side of equation (6.49) during solution of engineering problems will be written so:

$$\nu\frac{\partial^3 W}{\partial x^3}=-\frac{\partial}{\partial x}\,\frac{\Delta p}{\rho_{ж} l} \qquad (6.51)$$

or

$$\nu\frac{\partial^3 W}{\partial x^3}=-\frac{\partial}{\partial x}\left(\frac{\lambda_{\text{тр}}}{d}\,\frac{W^2}{2}\right).$$

Here Δp – hydraulic losses; $\lambda_{\text{тр}}$ – friction coefficient. The third term of equation (6.49) characterizes the change of flow velocity with time

$$\frac{\partial}{\partial t}\left(\frac{W}{\pi r_0^2}\,\frac{\partial F}{\partial x}\right)=\frac{1}{\pi r_0^2}\left(\frac{\partial W}{\partial t}\,\frac{\partial F}{\partial x}+W\,\frac{\partial^2 F}{\partial t\,\partial x}\right). \qquad (6.52)$$

The addend of the right side of equation (6.52) corresponds to the change of velocity with consideration of change of the cross-sectional area along the length of the channel. The augend considers the change of cross-sectional area lengthwise and with time. For estimation of the fourth component of equation (6.49) let us examine such a case (Fig. 6.2).

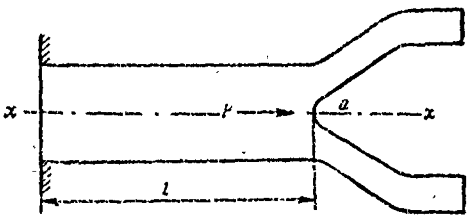

Fig. 6.2. Diagram of the action of liquid flow on a branching of the line.

Let us assume the elongation of the pipe occurs under the action of ballistic force P, caused by the action of the liquid flow and applied to conical branching a, moreover

$$P=c_x m(j_x+g_x)+c_x' \rho_{\text{ж}} F_a \frac{W^2}{2}, \qquad (6.53)$$

where c_x and c_x' – coefficients, which characterize the interaction of liquid flow with branching; F_a – area of projection normal to axis x of branching sections, being under the action of force P; m – the

mass of liquid in the line with cross-sectional area F at length l, moreover

$$m = Fl\varrho_{ж}; \tag{6.54}$$

(j_x+g_x) – projection of external force to axis x, equal to

$$(j_x+g_x) = j\cos\beta - g\cos\alpha. \tag{6.55}$$

Here α – the angle, depending on the attitude of a rocket; β – the angle, depending on the position of the line in the rocket. Inasmuch as the flight speed of the rocket V varies with time, instead of equation (6.55) it is better to write

$$(j_x+g_x) = \dot{V}\cos\beta - g\cos\alpha. \tag{6.56}$$

Pressure

$$p_l = \frac{P}{F_a} = c_x\frac{F}{F_a}l\varrho_{ж}(\dot{V}\cos\beta - g\cos\alpha) + c_x'\varrho_{ж}\frac{W^2}{2}. \tag{6.57}$$

Thus, for a case when g = const, we find

$$\frac{\partial^2 p_l}{\partial t^2} = c_x\frac{F}{F_a}l\varrho_{ж}\frac{\partial^3 V}{\partial t^3}\cos\beta + c_x'\varrho_{ж}\left[\left(\frac{\partial W}{\partial t}\right)^2 + W\frac{\partial^2 W}{\partial t^2}\right]. \tag{6.58}$$

If from equations (6.43) and (6.44) we take partial derivatives for t and x respectively, then after transformations we arrive at the wave equation for speed:

$$\frac{1}{\lambda^2}\frac{\partial^2 W}{\partial t^2} - \frac{1}{\varrho_{ж}}\frac{\partial^2 W}{\partial x^2} - U_W = 0, \tag{6.59}$$

Moreover

$$U_W = \frac{1}{\lambda^2}\left[\frac{\partial}{\partial t}(j_x+g_x) + \nu\frac{\partial}{\partial t}\frac{\partial^2 W}{\partial x^2}\right] + \\ + \frac{1}{\varrho_{ж}}\left[\frac{\partial}{\partial x}\left(\frac{W}{\pi r_0^2}\frac{\partial F}{\partial x}\right) + \frac{r_0}{2E\delta}\frac{\partial^2 p_l}{\partial t\partial x}\right]; \tag{6.60}$$

$$\frac{\partial^2 p_t}{\partial t \partial x} = c_x \frac{F}{F_a} l \varrho_{ж} \frac{\partial}{\partial x} \frac{\partial^2 V}{\partial t^2} \cos\beta + c_x' \varrho_{ж} \left(\frac{\partial W}{\partial t} \frac{\partial W}{\partial x} + W \frac{\partial^2 W}{\partial t \partial x} \right). \tag{6.61}$$

Thus, the nonhomogeneity of equations is determined by term U or U_W. Let us note that in the wave equation for velocity there is a term, considering the change of external volumetric force with time, i.e.,

$$\frac{\partial}{\partial t}(j_x + g_x).$$

There appeared a component, considering the change of hydraulic losses with time:

$$-\nu \frac{\partial}{\partial t} \frac{\partial^2 W}{\partial x^2} \approx \frac{\partial}{\partial t} \frac{\Delta p}{\varrho_{ж} l}. \tag{6.62}$$

During the study of wave processes it proves to be convenient to be guided by the velocity potential ψ, moreover

$$\bar{W} = -\operatorname{grad} \bar{\psi}. \tag{6.63}$$

Consequently, in one-dimensional formulation

$$W = -\frac{\partial \psi}{\partial x}. \tag{6.64}$$

The equation of motion has the form

$$-\frac{\partial}{\partial t} \frac{\partial \psi}{\partial x} = (j_x + g_x) - \frac{1}{\varrho_{ж}} \frac{\partial p}{\partial x} + \frac{\partial^2}{\partial x^2} \frac{\partial \psi}{\partial x}. \tag{6.65}$$

If we do not consider external volumetric forces and forces of viscous friction, then after integration of equation (6.65) we find

$$\frac{p}{\varrho_{ж}} = \frac{\partial \psi}{\partial t}. \tag{6.66}$$

Wave equation for velocity potential has the form

$$\frac{1}{\lambda^2}\frac{\partial^2\psi}{\partial t^2}-\frac{1}{\nu_{\text{ж}}}\frac{\partial^2\psi}{\partial x^2}=-U_{\psi}. \tag{6.67}$$

6.4. Nonstationary Motion of Liquid in The Line

The solution of equations, which describe the motion of liquid in the line, is connected with considerable difficulties. They appear when determining boundary conditions, during compilation of an algorithm in the course of utilization of the net-point method and when performing calculations on digital computers.

Therefore, during formulation of the problem we try, by using assumptions, to simplify both equations and boundary conditions. Let us examine the problem at various assumptions.

In the simplest case there is assumed one-dimensional nonstationary flow of incompressible liquid in a cylindrical tube. In this case

$$\varrho_{\text{ж}}=\text{const} \tag{6.68}$$

and velocity components

$$W_r=0; \tag{6.69}$$

$$W_\varphi=0. \tag{6.70}$$

The flow will be one-dimensional if external volumetric force

$$(j_r+g_r)=0. \tag{6.71}$$

The equation of continuity under such conditions gives

$$\frac{\partial W_x}{\partial x}=0. \tag{6.72}$$

Consequently, the velocity, directed along the axes of the tube, does not depend on x; thus,

$$W=W(r,t) \tag{6.73}$$

and, as a consequence,

$$\frac{\partial}{\partial x} W(r,t)=0. \tag{6.74}$$

The equation of motion, written relative to axis x for an axisymmetrical problem, takes the form

$$\frac{\partial}{\partial t} W(r,t)=(j_x+g_x)-\frac{1}{\varrho_{\text{ж}}}\frac{\partial}{\partial x}p+ \\ +\nu\left[\frac{\partial^2}{\partial r^2}W(r,t)+\frac{1}{r}\frac{\partial}{\partial r}W(r,t)\right]. \tag{6.75}$$

For simplification of the solution we sometimes consider a plane problem, in this case from equation (6.75) the last term drops out, i.e.,

$$\nu\frac{1}{r}\frac{\partial}{\partial r}W(r,t).$$

From equation (6.75) it follows that

$$\frac{\partial}{\partial x}p=\text{const}; \tag{6.76}$$

since in it all the terms do not depend on x. If when $t = 0$ the flow velocity is equal to zero, then the initial condition has the form

$$W(r,0)=0. \tag{6.77}$$

With two variables and four boundaries eight boundary conditions should be assigned. Equation (6.75) is solved analytically.

More complex is the condition at which W_r is nonzero. For an axisymmetrical problem we take $(j_r+g_r)=0$. The equation of continuity will be written so:

$$\frac{\partial}{\partial x}W_x(x,r,t)+\frac{\partial}{\partial r}W_r(x,r,t)+\frac{1}{r}W_r(x,r,t)=0. \tag{6.78}$$

For a plane problem from the left side of equation (6.78) one should remove the last term.

Inasmuch as derivatives $\frac{\partial}{\partial x}W_x$ and $\frac{\partial}{\partial r}W_r$ are nonzero, equations of motion will be written so [47]:

$$\begin{aligned}&\frac{\partial}{\partial t}W_x(x,r,t)+\frac{\partial}{\partial x}W_x(x,r,t)W_x(x,r,t)+\frac{\partial}{\partial r}W_x(x,r,t)W_r(x,r,t)=\\&=(j_x+g_x)-\frac{1}{\rho_{\text{ж}}}\frac{\partial}{\partial x}p(x,r,t)+\nu\left[\frac{\partial^2}{\partial x^2}W_x(x,r,t)+\right.\\&\left.+\frac{\partial^2}{\partial r^2}W_x(x,r,t)+\frac{1}{r}\frac{\partial}{\partial r}W_x(x,r,t)\right];\end{aligned} \tag{6.79}$$

$$\begin{aligned}&\frac{\partial}{\partial t}W_r(x,r,t)+\frac{\partial}{\partial x}W_r(x,r,t)W_x(x,r,t)+\\&+\frac{\partial}{\partial r}W_r(x,r,t)W_r(x,r,t)=(j_r+g_r)-\frac{1}{\rho_{\text{ж}}}\frac{\partial}{\partial r}p(x,r,t)+\\&+\nu\left[\frac{\partial^2}{\partial x^2}W_r(x,r,t)+\frac{\partial^2}{\partial r^2}W_r(x,r,t)+\right.\\&\left.+\frac{1}{r}\frac{\partial}{\partial r}W_r(x,r,t)-\frac{1}{r^2}W_r(x,r,t)\right].\end{aligned} \tag{6.80}$$

With consideration of the axisymmetrical problem in equation (6.80) one should accept $(j_r+g_r)=0$. During examination of plane problem from the right side of equation (6.79) the last term is excluded, and from equation (6.80) the last two terms are excluded. Now, i.e., for plane problem, the system of equations will be written so:

$$\begin{aligned}&\dot{W}_x(x,r,t)=(j_x+g_x)-\frac{1}{\rho_{\text{ж}}}\frac{\partial}{\partial x}p(x,r,t)+\\&+\nu\left[\frac{\partial^2}{\partial x^2}W_x(x,r,t)+\frac{\partial^2}{\partial r^2}W_x(x,r,t)\right];\end{aligned} \tag{6.81}$$

$$\dot{W}_r(x,r,t)=(j_r+g_r)-\frac{1}{\rho_{ж}}\frac{\partial}{\partial r}p(x,r,t)+$$

$$+\nu\left[\frac{\partial^2}{\partial x^2}W_r(x,r,t)+\frac{\partial^2}{\partial r^2}W_r(x,r,t)\right]; \qquad (6.82)$$

$$\frac{\partial}{\partial x}W_x(x,r,t)+\frac{\partial}{\partial r}W_r(x,r,t)=0. \qquad (6.83)$$

Let us note that here $W_\varphi=0$.

The given systems of equations are used for solution of various problems of applied character. After formulation of the problem for calculation or research one should estimate the value of separate terms, and those which have a nonessential effect on the result of the solution should be dropped. First one should evaluate the role of nonlinear convective terms of type $\frac{\partial}{\partial x}W_x(x,r,t)W_x(x,r,t)$ and those like them. Sometimes it is possible to drop the second derivities of velocity with respect to x. All this simplifies working with equations.

Plane problems are solved easier than axisymmetrical, but for round lines the solution of plane problem differs from the solution of axisymmetrical.

Equations contain the first time derivatives from two variables, consequently, it is necessary to prescribe two initial conditions. If at the initial moment of time the liquid is motionless, then the initial conditions will be written so:

$$W_x(x,r,0)=0; \qquad (6.84)$$

$$W_r(x,r,0)=0. \qquad (6.85)$$

With the onset of the process of acceleration of liquid, in the case of rapid valve opening at the line outlet or penetration of the diaphragm, installed there, a pressure wave appears, running at

first in the direction opposite the direction of motion of liquid. In order to exclude this process from examination, calculation begins from the moment of time, somewhat different from zero. In most cases at this moment of time velocities W_x and W_r are so low that it is possible to leave them in view of condition (6.84) and (6.85).

With three variables and four boundaries it is necessary to assign twelve boundary conditions.

At the boundary of "inlet" $(0, r, t)$ there is assigned constant or time variable pressure

$$p(0, r, t)=p_0(t). \qquad (6.86)$$

The diagram of inlet velocities, if we consider any section, parallel to the axis and passing through the axis, is represented in the form of a rectangle. Consequently,

$$\frac{\partial}{\partial r} W_x(0,r,t)=0; \qquad (6.87)$$

$$W_r(0, r, t)=0. \qquad (6.88)$$

However, having examined the flow of liquid in the intake device, it is possible to modify the laws of change of velocity components at the line inlet.

If when using the net-point method condition (6.87) is inconvenient and incompatible with the boundary condition at boundary "wall," at the point of connection of boundaries "inlet" and "wall," then it can be replaced by the following boundary condition:

$$W_x(0,r,t)=W_x(0,0,t)(1-e^{-\alpha(R-r)}), \qquad (6.89)$$

having taken a rather high value of α.

The dynamic boundary condition at the "wall" will be written in the following manner. Tangential stress $\tau_{r,x}$, effective in liquid at the boundary of liquid with the wall (x, R, t) is equal, but opposite in sign, to tangential stress $\tau_{\text{ст}}$, effective on the wall, i.e., [73]

$$|\tau_{R,x}| = |\tau_{\text{ст}}|. \qquad (6.90)$$

Tangential stress, affecting the liquid,

$$\tau_{R,x} = \mu\left[\frac{\partial}{\partial x} W_r(x,R,t) + \frac{\partial}{\partial r} W_x(x,R,t)\right]; \qquad (6.91)$$

tangential stress on the wall

$$\tau_{\text{ст}} = \lambda_{\text{тр}} \frac{x}{2R} \varrho_{\text{ж}} \frac{\overline{W}_x}{2}, \qquad (6.92)$$

where $\lambda_{\text{тр}}$ – friction coefficient; mean velocity

$$\overline{W}_x = \frac{1}{2R} \int_{-R}^{+R} W_x(x,r,t)\,dr. \qquad (6.93)$$

Thus,

$$\tau_{\text{ст}} = \lambda \frac{x}{8R^2} \varrho_{\text{ж}} \int_{-R}^{R} W_x(x,r,t)\,dr. \qquad (6.94)$$

By virtue of the action of friction forces there occurs "sticking" of liquid to the wall, which allows writing these boundary conditions:

$$W_x(x,R,t) = 0; \qquad (6.95)$$

$$W_r(x,R,t) = 0. \qquad (6.96)$$

In order to take into account the effect of wall roughness, we assign viscosity, changing along axes x and r [94].

At the boundary "axis" $(x, 0, t)$ with the presence of symmetry

$$\frac{\partial}{\partial r} p(x,0,t)=0; \qquad (6.97)$$

$$\frac{\partial}{\partial r} W_x(x,0,t)=0; \qquad (6.98)$$

$$\frac{\partial}{\partial r} W_r(x,0,t)=0. \qquad (6.99)$$

If the projection of accelerations (j_r+g_r) is nonzero, then the symmetricity of flow relative to the axis is disturbed. Therefore, instead of two equations of motion (6.79) and (6.80) it is necessary to have three equations or to solve a plane problem, using equations (6.81), (6.82) and (6.83).

The solution of the system, consisting of three equations of motion and equation of continuity, is connected with considerable difficulties, and the solution to the problem in the plane variant leads to solutions, the errors of which are not always possible to evaluate.

If the projection of accelerations (j_r+g_r) is nonzero, then, inasmuch as the symmetricity of flow relative to the axis is disturbed, instead of the "axis" boundary one should select the "wall" boundary $(x, -R, t)$. In this instance the boundary conditions will be written so:

$$|\tau_{-R,x}|=|\tau_{\text{ст}}|; \qquad (6.100)$$

$$W_x(x,-R,t)=0; \qquad (6.101)$$

$$W_r(x,-R,t)=0. \qquad (6.102)$$

Further complication of the problem is associated with the consideration of compressibility of liquid. In this case the equation

of continuity will be written in this form:

$$\frac{\partial}{\partial t}\varrho_x(x,r,t)+\frac{\partial}{\partial x}W_x(x,rt)+\frac{\partial}{\partial r}W_r(x,r,t)+\frac{1}{r}W_r(x,r,t)=0. \qquad (6.103)$$

The initial condition can be prescribed in the form of

$$\varrho(x,r,0)=\varrho_0, \qquad (6.104)$$

where ϱ_0 — initial value of liquid density.

The greatest difficulties appear when determining conditions at the "outlet" boundary (L,r,t). They depend on the specific character of liquid motion during passage from the line to the pump inlet, to the injector assembly of the combustion chamber, or to the collector of the cooling jacket.

The dynamic boundary condition with the presence of a free surface is expressed by the equality of normal stress in liquid on the boundary with free surface to ambient pressure p_0 on the free surface itself, i.e., [73]

$$p_{nn}=-p_0, \qquad (6.105)$$

where p_{nn} — normal stress in liquid on the boundary with free surface, moreover

$$p_{nn}=-p_0-\frac{2}{3}\mu\left[\frac{\partial}{\partial x}W_x(x,r,\varphi,t)+\frac{1}{r}\frac{\partial}{\partial r}W_r(x,r,\varphi,t)r+\frac{\partial}{r\partial\varphi}W_\varphi(x,r,\varphi,t)\right]+2\mu\frac{\partial}{\partial n}W_n;\ (n=x,r,\varphi). \qquad (6.106)$$

6.5. Simulation of Hydraulic Flows

Physical phenomena are similar, if they differ only in scales of quantities and functions, which determine their qualitative characteristics. In order to formulate the conditions of similarity of liquid

flows, it is necessary to have an equation (or system of equations), describing motion, and to know the boundary conditions. The accuracy of solution of the problem about similarity will depend on how correctly the equation reflects the physical process, and on the degree of correspondence of boundary conditions to the actual situation. One should be guided by differential equations in partial derivatives, inasmuch as they characterize the change of parameters both with time and with respect to coordinates. The differential equation of motion includes velocity components and their derivatives, and also terms which characterize various forces, affecting the flow. Boundary conditions reflect the time and geometric characteristics of the duct. Thus, conditions of similarity are determined by time, geometric, kinematic and dynamic factors.

Geometrically similar flows are flows, all linear dimensions of which are proportional to each other, i.e., all the linear dimensions of one flow are changed an identical number of times in comparison with the corresponding linear dimensions of another flow [73].

Kinematically similar flows are flows, in which the velocities of particles at all similar points are proportional to each other.

In dynamically similar flows the particles of liquid at similar points are affected by forces of the same type, i.e., forces of the same nature (volumetric, surface), moreover the relationships between all similar forces, affecting similar points of these flows, pertaining to an unit of volume of liquid, are equal.

If the noted conditions are satisfied, then the solutions of equations will be similar, i.e., the character of time variation of all parameters of flow at similar points will be identical, and the values of the appropriate parameters and time will differ only in scale.

For realization of strict simulation it is necessary to provide similarity of equations, which describe the process being simulated,

and the similarity of boundary conditions, i.e., initial and boundary conditions.

However, with solution of engineering problems it frequently proves to be sufficient to provide partial similarity, in this case there is allowed the nonfulfillment of a number of conditions, determined by theory. Sometimes we are limited to the similarity of processes, being characterized by the change of averaged values of parameters with time.

On an example let us consider the method of obtaining criteria and conditions of similarity.

Let us assume that for the investigated flow in real piping with a sufficient degree of accuracy it is possible to accept that the motion of liquid conforms to the following equation:

$$\varrho_{н}\frac{\partial W_{н}}{\partial t_{н}}+\varrho_{н}\frac{\partial W_{н}}{\partial l_{н}}W_{н}=\varrho_{н}(j+g)_{н}-\frac{\partial p_{н}}{\partial l_{н}}+\varrho_{н}\nu_{н}\frac{\partial^2 W_{н}}{\partial l_{н}^2}. \tag{6.107}$$

Between the parameters of real and model flow there should be constant relationships, determined by coefficients c_i, moreover

$$\left.\begin{array}{l} t_{н}=c_t t_{м};\ \varrho_{н}=c_{\varrho}\varrho_{м};\ W_{н}=c_W W_{м}; \\ (j+g)_{н}=c_j(j+g)_{м};\ p_{н}=c_p p_{м}\text{, etc.} \end{array}\right\} \tag{6.108}$$

By sutstituting in natural equation (6.107) the values of parameters from equation (6.108), we obtain an equation for model flow:

$$\left[c_{\varrho}\frac{c_W}{c_t}\right]\varrho_{м}\frac{\partial W_{м}}{\partial t_{м}}+\left[c_{\varrho}\frac{c_W^2}{c_l}\right]\varrho_{м}\frac{\partial W_{м}}{\partial l_{м}}W_{м}=[c_{\varrho}c_j]\varrho_{м}(j+g)_{м}-$$

$$-\left[\frac{c_p}{c_l}\right]\frac{\partial p}{\partial l}+\left[c_{\varrho}c_{\nu}\frac{c_W}{c_l^2}\right]\varrho_{м}\nu_{м}\frac{\partial^2 W_{м}}{\partial l_{м}^2}. \tag{6.109}$$

The conditions of similarity of nature and models can be expressed by different means. Frequently the conditions of similarity are

expressed in the form of criteria of similarity, and sometimes in the form of relationships of separate parameters or their combinations, if this proves to be convenient during the solution of a particular problem.

The equations of nature and model will be similar, if the factors, enclosed in brackets [ ], are equal. It is possible to equate all coefficient to each other or divide by some constant coefficient so as to obtain new coefficients in dimensionless form after division, and then to equate them together. If as the divider we select one of the coefficients of equation (6.109), then we receive a number of dimensionless coefficients, equal to one. Let us subdivide all the coefficients of equation (6.109) by a second coefficient, i.e., by $c_\varrho \frac{c_W^2}{c_l}$, we obtain

$$\frac{c_l}{c_t c_W} = 1 = \frac{c_j c_l}{c_W^2} = \frac{c_p}{c_\varrho c_W^2} = \frac{c_\nu}{c_l c_W}. \tag{6.110}$$

Hence follows:

$$\frac{c_W c_t}{c_l} = 1; \quad \frac{c_W^2}{c_j c_l} = 1; \quad \frac{c_\varrho c_W^2}{c_p} = 1; \quad \frac{c_l c_W}{c_\nu} = 1. \tag{6.111}$$

By substituting in these equations the values of parameters from system (6.108), we find the conditions of similarity:

$$\left.\begin{aligned} &\frac{W_{\text{н}} t_{\text{н}}}{l_{\text{н}}} = \frac{W_{\text{м}} t_{\text{м}}}{l_{\text{м}}}; \quad \frac{W_{\text{н}}^2}{(j+g)_{\text{н}} l_{\text{н}}} = \frac{W_{\text{м}}^2}{(j+g)_{\text{м}} l_{\text{м}}}; \\ &\frac{p_{\text{н}}}{\varrho_{\text{н}} W_{\text{н}}^2} = \frac{p_{\text{м}}}{\varrho_{\text{м}} W_{\text{м}}^2}; \quad \frac{W_{\text{н}} l_{\text{н}}}{\nu_{\text{н}}} = \frac{W_{\text{м}} l_{\text{м}}}{\nu_{\text{м}}}. \end{aligned}\right\} \tag{6.112}$$

The conditions of similarity (6.112) are criteria, which should be identical (idem) for nature and the model. The homochromicity number

$$H = \frac{Wt}{l} = \text{idem}; \tag{6.113}$$

Froude number

$$F=\frac{W^2}{(j+g)l}=\text{idem}, \tag{6.114}$$

and usually it is written so [73]: $F=\frac{W^2}{gl}$;

Euler number

$$E=\frac{p}{\varrho W^2}=\text{idem}; \tag{6.115}$$

Reynolds number

$$Re=\frac{Wl}{\nu}=\text{idem}. \tag{6.116}$$

With simulation of processes in liquid-propelleant rocket engine units [ZhRD] (ЖРД) the criteria of similarity are conveninetly expressed through flow rate

$$G=FW\varrho. \tag{6.117}$$

In this case we obtain

$$\left.\begin{aligned} H&=\frac{Gt}{lF\varrho}=\text{idem};\\ F&=\frac{G^2}{(j+g)F^2l\varrho^2}=\text{idem};\\ E&=\frac{pF^2\varrho}{G^2}=\text{idem};\\ Re&=\frac{Gl}{\nu F}=\text{idem}. \end{aligned}\right\} \tag{6.118}$$

During solution of problems of practical character it is necessary to select combinations of criteria (6.118), which correspond to specific requirements of operation. Let us assume that it is necessary to impose requirements on the pipeline, which connects the tank with the engine on the firing stand. Tests should be conducted in natural conditions, i.e., time of operation, flow rate of propellant and

the propellant components on the stand should be the same as on the rocket. Consequently

$$t_c=t_p;\ G_c=G_p;\ \varrho_c=\varrho_p;\ v_c=v_p. \tag{6.119}$$

Having multiplied the homochronicity number (criterion) by the Euler number (criterion), we obtain

$$\mathrm{H}\cdot\mathrm{E}=\frac{ptF}{Gl}=\text{idem}. \tag{6.120}$$

With consideration of system (6.119) we can obtain two more such conditions:

$$p_c=p_p; \tag{6.121}$$

$$\frac{l_c}{F_c}=\frac{l_p}{F_p}. \tag{6.122}$$

By multiplying the Euler number by the Roude number, we find

$$\mathrm{E}\cdot\mathrm{F}=\frac{p}{(j+g)\,l\varrho}=\text{idem}. \tag{6.123}$$

Thus, it is necessary to satisfy condition

$$(j+g)_c l_c=(j+g)_p l_p. \tag{6.124}$$

If $(j+g)_c=(j+g)_p$, then $l_c=l_p$ and according to equation (6.122) we obtain $F_c=F_p$.

If F_c is not equal to F_p, then it is possible to satisfy condition (6.124) only when $j+g=0$.

6.6. The Peculiarities of Intrachamber Processes During Bench Tests and in Flight Conditions

During operation of an engine on a rocket, in flight, intrachamber processes sometimes do not occur, as during tests on a firing stand. This is explained by a number of differences of conditions of bench testing and in flight.

In most cases the layout of the feed system of the stand and its working conditions differ from those accepted in the rocket. The stand is designed for repeated conducting of tests, therefore thick-walled lines are used, the length, diameter and configuration of which are different than the rocket.

Under the action of geometrical factors in a transient process the component ratio can rather deeply deviate from nominal. On the stand are used reusable valves, additional filters, measuring instruments, etc., are installed. All this reflects the feed conditions of propellant components into the combustion chamber.

The rigidity of mounting the engine on the stand and in the rocket is different, therefore, vibrations appearing during operation of the engine differently reflect the character of occurrence of intrachamber processes.

In flight under the effect of external mass forces the rate of increase of pressure in the chamber and the acceleration of motion of liquid burning particles of propellant are different than on the stand.

Let us examine the effect of replacement of volumetric force

$$P_V = (j+g)l \tag{6.125}$$

by equivalent surface force

$$P_F = \frac{r_6}{\varrho_ж} \tag{6.126}$$

on wave processes in a straight tube. The wave equation has the form

$$\frac{\partial^2 \varrho_ж}{\partial t^2} = \lambda^2 \frac{\partial^2 \varrho_ж}{\partial x^2}. \tag{6.127}$$

With the action of surface force the density of liquid does not change along the length of the tube and is equal to $\varrho_ж$. The force, compressing the liquid, with the presence of external volumetric force

$$P_{Vx} = F\varrho_ж(j+g)x. \tag{6.128}$$

The increase of pressure

$$\Delta p_V = \varrho_ж(j+g)x. \tag{6.129}$$

Consequently, the liquid density

$$\varrho_ж(x) = \varrho_ж\left[1 + \frac{\varrho_ж}{E_ж}(j+g)x\right]; \tag{6.130}$$

where $E_ж$ – modulus of elasticity of liquid. In case of the action of pressure forces with sinusoidal disturbance of density in the form

$$\Delta\varrho_ж \sin\left(\frac{\omega}{\lambda}x\right)$$

the initial conditions will be written so:

$$\left.\begin{aligned} &\varrho_ж(x,0) = \varrho_ж + \Delta\varrho_ж \sin\left(\frac{\omega}{\lambda}x\right); \\ &\frac{\partial}{\partial t}\varrho_ж(x,0) = \Delta\varrho_ж\omega\cos\left(\frac{\omega}{\lambda}x\right). \end{aligned}\right\} \tag{6.131}$$

Here ω – frequency; λ – coefficient of equation (6.127). With external mass forces

$$
\left.\begin{aligned}
\varrho_ж(x,0) &= \varrho_ж\left[1+\frac{\varrho_ж}{E_ж}(j+g)x\right]+\Delta\varrho_ж \sin\left(\frac{\omega}{\lambda}x\right);\\
\frac{\partial}{\partial t}\varrho_ж(x,0) &= \frac{\varrho_ж^2}{E_ж}(j+g)+\Delta\varrho_ж\omega\cos\left(\frac{\omega}{\lambda}x\right).
\end{aligned}\right\} \tag{6.132}
$$

Inasmuch as initial conditions (6.131) differ from conditions (6.132), then the solution of wave equation, written, for example, in the form of equation

$$\varrho(x,t)=A\cos\left(\frac{\omega}{\lambda}t+\varphi_x\right)\cos\left(\frac{\omega}{\lambda}t+\varphi_t\right), \tag{6.133}$$

with the presence of pressure forces will differ from the solution, in which external mass forces are considered.

6.7. Balance of Pressures for the Tank-Combustion Chamber Hydraulic Circuit

The motion of liquid is examined in some section of the line, inside which for parameters their mean values are taken. As the basic let us take the one-dimensional equation of motion in the form

$$\dot{W}-(j+g)+\frac{1}{\varrho_ж}\frac{\partial p}{\partial x}-\nu\nabla^2 W=0. \tag{6.134}$$

In engineering formulation in accordance with accepted assumptions we will have [67]

$$\dot{W}\delta x-(j+g)\delta x+\frac{1}{\varrho_ж}\delta p-\nu\nabla^2 W\delta x=0. \tag{6.135}$$

For the entire hydraulic circuit with length

$$L=\sum\delta x=\int_0^l dx \tag{6.136}$$

instead of expression (6.135) we receive

$$\varrho_ж\int_0^l \dot{W}dx-\varrho_ж\int_0^l (j+g)dx+\int_{p_1}^{p_2} dp-\varrho_ж\nu\int_0^l \nabla^2 W dx=0. \tag{6.137}$$

Equation (6.137) represents the balance of pressures on a section from the tank to the chamber. The addend expresses the pressure, equivalent to mass forces.

$$p_W = \varrho_{\text{ж}} \int_0^l \dot{W} dx. \tag{6.138}$$

By expanding the expression of substantial derivative, we obtain

$$p_W = \varrho_{\text{ж}} \int_0^l \frac{\partial W}{\partial t} dx + \varrho_{\text{ж}} \int_0^l \frac{\partial W}{\partial x} W dx. \tag{6.139}$$

Considering the increases of velocity in the cross section and along the length of the duct independent from one another, we obtain

$$p_W = \varrho_{\text{ж}} \int_0^l \frac{dW}{dt} dx + \varrho_{\text{ж}} \int_{W_1}^{W_2} W dW. \tag{6.140}$$

According to the equation of continuity it is possible to write that

$$\frac{dW}{dt} = \frac{\dot{G}}{F(x) \varrho_{\text{ж}}}, \tag{6.141}$$

inasmuch as the flow rate of incompressible liquid in a rigid line does not depend on the length and, consequently,

$$\frac{\partial G}{\partial t} = \frac{dG}{dt} = \dot{G}. \tag{6.142}$$

Thus,

$$p_W = \dot{G} \int_0^l \frac{dx}{F(x)} + \varrho_{\text{ж}} \frac{W_2^2 - W_1^2}{2}. \tag{6.143}$$

The addend of the right side of equation (6.143) characterizes pressure, equivalent to mass forces, appearing with change of the flow rate with time, i.e.,

$$p_G = \dot{G} \int_0^l \frac{dx}{F(x)}; \tag{6.144}$$

the augend represents the change of pressure, which appears due to change of the kinetic energy of flow under the effect of geometric factors:

$$p_L = \varrho_{ж} \frac{W_2^2 - W_1^2}{2}. \tag{6.145}$$

With the aid of the equation of continuity equation (6.145) can be written so:

$$p_L = \frac{G^2}{2\varrho_{ж}} \left(\frac{1}{F_2^2} - \frac{1}{F_1^2} \right). \tag{6.146}$$

Pressure, equivalent to external mass forces,

$$p_j = \varrho_{ж} \int_0^l (j+g)\,dx. \tag{6.147}$$

Inasmuch as $(j+g)$ does not depend on x, then

$$p_j = \varrho_{ж} (j+g) l, \tag{6.148}$$

where

$$(j+g) = \dot{V}_x + g_x; \tag{6.149}$$

$\dot{V}_x$ – projection of acceleration $\dot{V}$ to axis x; g_x – projection of acceleration of gravity to the same axis. Thus,

$$p_j = \varrho_{ж} (\dot{V}_x + g_x) l. \tag{6.150}$$

The following integral (6.137) is the difference of external pressures. With the presence of tank pressurization $p_б$ and with excess pressure $p_н$, created by a pump,

$$\int_{p_1}^{p_2} dp = -\int_{p_б + p_н}^{p_к} dp = p_б + p_н - p_к, \tag{6.151}$$

where $p_к$ – pressure in the chamber. The last integral (6.137) characterizes hydraulic losses:

$$\Delta p=-\varrho_ж \nu \int_0^l \nabla^2 W dx. \qquad (6.152)$$

Thus, the balance of pressures (6.137) can be written so:

$$p_б+p_н-p_к-p_G-p_L-\Delta p=0. \qquad (6.153)$$

The equation of balance of pressures can be derived different, having taken as the basis the law of conservation of energy in the form

$$\sum dL_{ai}-\sum dL_{ci}=0, \qquad (6.154)$$

where dL_{ai} – elementary work, being performed over the liquid; dL_{ci} – elementary work of resistance forces.

Elementary work

$$dL_i=P_i dx; \qquad (6.155)$$

force

$$P_i=p_i F. \qquad (6.156)$$

Inasmuch as

$$W=\frac{dx}{dt}, \qquad (6.157)$$

then

$$dL_i=p_i F W dt, \qquad (6.158)$$

where p_i — the appropriate pressure. For incompressible liquid

$$FW = \frac{G}{\varrho_{ж}} = \mathrm{idem}, \qquad (6.159)$$

i.e., it is constant along the length of the duct. By substituting expression (6.158) in equation (6.154) and bearing in mind equality (6.159), after division of both sides of the equality by

$$\frac{G}{\varrho_{ж}} dt$$

we receive

$$\sum p_{ai} - \sum p_{ci} = 0. \qquad (6.160)$$

Having determined the active specific forces (pressures p_{ai}) and the specific forces of resistance (p_{ci}), we arrive at balance (6.153).

6.8. Determination of Separate Components of Pressure Balance

Let us examine pressure p_G, equivalent to mass forces, appearing with change of the flow rate with time [67].

For a complex duct, in which elements with characteristic cross-sectional areas are series-connected,

$$p_G = \dot{G} \sum_{1}^{n} \int_{0}^{l_i} \frac{dx}{F_i(x)}, \qquad (6.161)$$

or

$$p_G = b\dot{G}. \qquad (6.162)$$

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For each element of the duct it is possible to calculate

$$b_i=\int_0^{l_i}\frac{dx}{F_i(x)}. \qquad (6.163)$$

Consequently, for the whole duct

$$b=\sum_1^n b_i. \qquad (6.164)$$

Let us determine the coefficient of internal mass forces for a number of particular cases.

1. Channel of constant section –

$$b=\frac{l}{F}. \qquad (6.165)$$

2. Duct, consisting of n series-connected segments of constant section, –

$$b=\sum_1^n\frac{l_i}{F_i}. \qquad (6.166)$$

3. The segment of the line with radius r and length l –

$$b=\frac{l}{\pi r^2}. \qquad (6.167)$$

4. Duct, consisting of n series-connected lines, characterized by dimensions l_i and r_i, –

$$b=\frac{1}{\pi}\sum_1^n\frac{l_i}{r_i^2}. \qquad (6.168)$$

5. Conical duct with inlet radius r_1 and exit radius r_2 –

$$dx=\frac{dr}{\operatorname{tg}\alpha};\quad F=\pi r^2,$$

consequently,

$$b=\int_{r_1}^{r_2}\frac{dr}{\pi r^2\operatorname{tg}\alpha}=\frac{1}{\pi\operatorname{tg}\alpha}\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \tag{6.169}$$

6. Duct of a cylindrical chamber –

$$b=\frac{l}{2\pi r\Delta}, \tag{6.170}$$

where Δ – width of flow duct.

7. The cooling duct of a conical chamber or conical nozzle –

$$b=\int_{r_1}^{r_2}\frac{dr}{2\pi r\Delta\operatorname{tg}\alpha}=\frac{\ln\left|\frac{r_2}{r_1}\right|}{2\pi\Delta\operatorname{tg}\alpha}. \tag{6.171}$$

8. Duct, consisting of n identical parallel-connected channels; inasmuch as the resistances of all channels are equal, flow rate through each channel

$$G_i=\frac{G}{n}. \tag{6.172}$$

Consequently,

$$\dot{G}_i=\frac{\dot{G}}{n}. \tag{6.173}$$

Inasmuch as the pressure losses for a channel are equal to pressure losses for the entire group of channels, then

$$(1+\varepsilon)\frac{G^2}{2\varrho_ж F^2}+b\dot{G}=(1+\varepsilon_i)\frac{G_i^2}{2\varrho_ж F_i^2}+b_i\dot{G}_i, \tag{6.174}$$

where ε — resistance coefficient. Consequently,

$$b=\frac{(1+\varepsilon_l)\frac{G_l^2}{2\varrho_ж F_l^2}-(1+\varepsilon)\frac{G^2}{2\varrho_ж F^2}+b_l\dot{G}_l}{\dot{G}}\approx\frac{b_l}{n}. \qquad (6.175)$$

6.9. Filling of Hydraulic Lines

During preparation of an engine for starting a number of preliminary operations are performed, the content and sequence of which depend on the engine layout, the tactical and technical problem of rocket launch, the peculiarities of propellant, the working conditions of the engine and other factors. Some operations, however, are typical for many liquid-propellant rocket engines. They, in particular, include filling of lines with propellant components during or after the opening of the main valve [67]. The features of filling of the line depend on the mode of valve opening, the increase of pressure before the valve, characteristics of propellant component, lines and elements of the line. Depending on the change of flow rates of liquids with time the conditions of their arrival into the combustion chamber are determined. They are evaluated by values of flow rates and component ratios at the moment of their entry into the combustion chamber, at the moment of ignition and during the progress of the engine to steady state. It is understandable, that after the ignition of propellant the motion of components will depend not only on the parameters of the feed system, but also on the parameters of the combustion chamber.

Let us examine the motion of one of the components from the main valve or blowout diaphragm to the injector edge.

For any moment of time the equation of balance of pressures of the hydraulic line can be written so:

$$p-\Delta p-p_G-p_п=0, \qquad (6.176)$$

where p — pressure before the valve; Δp — hydraulic losses in the line, being filled with liquid; p_G — pressure, equivalent to mass

forces; $p_п$ – pressure in the hydraulic circuit before the liquid flow front.

Filling of the line with high-boiling component

For determination of hydraulic losses Δp, which appear during motion of liquid through the flow duct of an arbitrary section, let us use known relationship

$$\Delta p = aG^2. \tag{6.177}$$

With filling of the duct with liquid to each moment of time t there corresponds the current value of l of the section of the duct, already filled with liquid. Therefore, let us represent the coefficient of hydraulic losses by product

$$a = a^* l, \tag{6.178}$$

where a^* – the value of the hydraulic loss coefficient for a unit of length of the flow duct.

For determination of this coefficient we have

$$a^* = \frac{\lambda_{тр}}{2\varrho_ж d_э F^2}, \tag{6.179}$$

where $\lambda_{тр}$ – friction coefficient; F – cross-sectional area of the duct; $d_э$ – equivalent diameter:

$$d_э = 4\frac{F}{\Pi};$$

Π – duct perimeter.

Let us examine the motion of liquid through a round tube. Here $F = \frac{\pi}{4}d^2$; $\Pi = \pi d$, consequently, $d_э = d$.

By substituting values of F and $d_э$ in equation (6.179), we find

$$a^* = \frac{8}{\pi^2}\frac{\lambda}{\varrho_ж d^5}. \tag{6.180}$$

At moment of time t the liquid will fill a line at length

$$l=\frac{4}{\pi\varrho_{ж}d^{2}}\int_{0}^{t}Gdt. \qquad (6.181)$$

For determination of hydraulic losses now we have

$$\Delta p=\frac{32}{\pi^{3}}\,\frac{\lambda}{\varrho_{ж}^{2}d^{7}}\,G^{2}\int_{0}^{t}Gdt. \qquad (6.182)$$

In a more complex duct the cross-sectional area is changed lengthwise and the volume, filled with liquid at moment of time t, will be

$$V=\frac{1}{\varrho_{ж}}\int_{0}^{t}Gdt.$$

On the other hand,

$$V=\int_{0}^{l}F(l)\,dl,$$

consequently,

$$\int_{0}^{l}F(l)\,dl=\frac{1}{\varrho_{ж}}\int_{0}^{t}Gdt. \qquad (6.183)$$

By knowing the geometry of the flow duct, one should determine the integral of volume

$$\int_{0}^{l}F(l)\,dl$$

and find the evident relationship of F to l.

Thus, for instance, for the conical part of the flow duct of a nozzle the current value of the nozzle radius

$$r=l\sin\frac{\alpha}{2},$$

where l – length of the flow duct; α – the total nozzle cone angle.

The current value of cross-sectional area of the duct

$$F(l)=2\pi r\Delta,$$

where Δ – width of annular flow duct (cooling clearance), or

$$F(l)=2\pi\Delta l\sin\frac{\alpha}{2}. \tag{6.184}$$

On a certain section of the duct from l_0 to l:

$$\int_{l_0}^{l}F(l)\,dl=2\pi\sin\frac{\alpha}{2}\int_{l_0}^{l}\Delta l\,dl. \tag{6.185}$$

If the width of the flow duct is constant, then

$$\int_{l_0}^{l}F(l)\,dl=\pi\Delta\sin\frac{\alpha}{2}(l^2-l_0^2). \tag{6.186}$$

By substituting the value of integral of volume in equation (6.183), after transformation we find

$$l=\sqrt{\frac{1}{\pi\varrho_{ж}\Delta\sin\frac{\alpha}{2}}\int_0^t G\,dt+l_0^2}. \tag{6.187}$$

The equivalent diameter for annular duct

$$d_э=4\Delta,$$

consequently,

$$a^*=\frac{\lambda}{32\pi^2\varrho_{ж}\Delta^3 l^2\left(\sin\frac{\alpha}{2}\right)^2}. \tag{6.188}$$

Inasmuch as here $a^* = f(l)$, then

$$\Delta p = G^2 \int_{l_0}^{l} a^* dl. \tag{6.189}$$

By substituting the value of a^*, we find

$$\Delta p = \frac{\lambda}{32\pi^2 \varrho_{\text{ж}} \Delta^3 \left(\sin\frac{\alpha}{2}\right)^2} G^2 \int_{l_0}^{l} \frac{dl}{l^2} \tag{6.190}$$

or

$$\Delta p = \frac{\lambda}{32\pi^2 \varrho_{\text{ж}} \Delta^3 \left(\sin\frac{\alpha}{2}\right)^3} G^2 \left(\frac{1}{l_0} - \frac{1}{l}\right), \tag{6.191}$$

where l – is determined by formula (6.181).

Pressure, being used for overcoming mass forces,

$$p_G = \dot{G} \sum \int_0^l \frac{dl}{F(l)}. \tag{6.192}$$

For a line of constant diameter

$$p_G = \frac{16\dot{G}}{\pi^2 \varrho_{\text{ж}} d^4} \int_0^t G dt. \tag{6.193}$$

By substituting values of Δp and p_G from equations (6.182) and (6.193) in expression (6.176), we obtain

$$p - p_n = \frac{32}{\pi^3} \frac{\lambda}{\varrho_{\text{ж}}^2 d^7} \dot{Z}^2 Z + \frac{16}{\pi^2 \varrho_{\text{ж}} d^4} \ddot{Z} Z, \tag{6.194}$$

where $Z = \int_0^t G dt.$ (6.195)

Let us examine the graphical-analytical method of solution of equation (6.194). If we accept

$$a_1 = \frac{(p - p_n)\pi^2 \varrho_ж^2 d^7}{32\lambda}; \qquad (6.196)$$

$$a^2 = \frac{\pi}{2} \frac{\varrho_ж}{\lambda} d^5, \qquad (6.197)$$

then equation (6.194) will be written so:

$$a_1 = a_2 Z\ddot{Z} + Z\dot{Z}^2 \qquad (6.198)$$

or

$$\frac{a_1}{Z} = a_2\ddot{Z} + \dot{Z}^2. \qquad (6.199)$$

To each moment of time corresponds a fully determined value of Z and its derivatives. Consequently, equation (6.199) is expanded into two equations:

$$\frac{a_1}{Z} = A; \qquad (6.200)$$

$$a_2\ddot{Z} + \dot{Z}^2 = A. \qquad (6.201)$$

The region of change of A can be determined according to equation (6.200), which is written so:

$$A = \frac{a_1}{\int G dt}. \qquad (6.202)$$

To the least value of t corresponds $A = A_{max}$. The least value of $A = A_{min}$ is determined according to the condition of filling of the whole hydraulic duct with liquid, the volume of which

$$V = \frac{1}{\varrho_ж} \int G dt. \qquad (6.203)$$

Consequently,

$$A_{min}=\frac{a_1}{V}Q_x. \tag{6.204}$$

Taking into account that $\dot{Z} = G$, $Z = \int G dt$, equation (6.201) is written so:

$$a_2\dot{G}+G^2=A. \tag{6.205}$$

After separation of variables and integration we arrive at tabular integral

$$t=a_2\int\frac{dG}{A-G^2}. \tag{6.206}$$

By solving equation (6.206), let us construct graph G with respect to t for several selected values of A, which lie from A_{min} to A_{max}. The area, limited by curve $G(t)$ and calculated for various randomly selective values of t, allows constructing a graph of Z with respect to t also for various (selected earlier) values of A (Fig. 6.3). But to every value of A, as follows from equation (6.200), corresponds one value of Z. Now on the graph of function $Z(t)$ let us plot the values of Z, obtained from equation (6.200). By connecting points a, b, c, plotted on the graph, to a smooth curve, we find the final solution of equation (6.198) in the form of graph $Z = f(t)_A$. Thus, by points a, b, c for the accepted quantities of A we can find a number of values t_a, t_b, t_c... Now, by transposing values t_a, t_b, t_c to graph $G(t)$ and using curves G with respect to t, corresponding to various values of A, we find flow rates G_a, G_b, G_c... and construct the relationship of flow rate to time in the form of $G = \Phi(t)$.

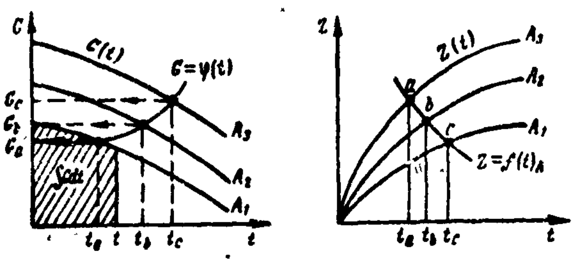

Fig. 6.3. For the graphical analytical solution of equation (6.194).

Filling of lines with low-boiling component

During motion of cryogenic liquids, as a result of heat exchange with the wall a certain quantity of liquid will be vaporized. The vapors occupy as if three regions. Part of the forming vapors is before the front of the moving component and creates some counterpressure in the line in the region between liquid and the injector edge. On the initial section of moving liquid there can be formed a region of film boiling, and following it – a region of nucleate boiling. However, the character of vaporization will depend on the thermal properties of the line, the properties of cryogen, the velocity of its motion, the amount of counterpressure, etc. The conditions of liquid motion are determined now not only by the pressure balance (6.176), but also by the equation of law of conservation of vapor-gaseous phase in the form

$$\dot{Y}_m-\dot{Y}_0-\dot{Y}_н=0, \tag{6.207}$$

where Y_m – mass of vapors, formed as a result of gasification of component; Y_0 – the quantity of gas and vapor, located in the line; $Y_н$ – the quantity of vapor-gaseous mixture, escaping through the injectors during filling of the line with liquid.

The wall temperature of the line varies from initial value $T_{ст0}$ to a temperature, practically equal to the liquid temperature, on a rather short section of path. If we consider that the liquid temperature $T_ж$ is equal to the boiling point T_s and that the transition from $T_{ст0}$ to T_s has an intermittent character, then heat, given off by the wall,

$$q=\pi dl\Delta\varrho_{ст}c_{ст}(T_{ст0}-T_s), \tag{6.208}$$

where l – length of the line, filled with liquid at moment of time t. Inasmuch as the liquid is at the boiling point, then all the heat q is spent for converting the liquid into vapor, i.e.,

$$q=Y_m r^*. \tag{6.209}$$

where r^* – latent heat of vaporization. By equating the right sides of equations (6.208) and (6.209), we find

$$Y_m = \frac{\pi d \Delta \rho_{\text{ст}} c_{\text{ст}} (T_{\text{ст}0} - T_s)}{r^*} l. \qquad (6.210)$$

The path, being passed by the liquid front,

$$l = \frac{4}{\pi d^2 \rho_{\text{ж}}} \int_0^t G\,dt. \qquad (6.211)$$

Consequently,

$$Y_m = \frac{4 \Delta \rho_{\text{ст}} c_{\text{ст}} (T_{\text{ст}0} - T_s)}{r^* \rho_{\text{ж}} d} \int_0^t G\,dt. \qquad (6.212)$$

Having taken the derivative with respect to t, we find

$$\dot{Y}_m = \frac{4 \Delta \rho_{\text{ст}} c_{\text{ст}} (T_{\text{ст}0} - T_s)}{r^* \rho_{\text{ж}} d} G. \qquad (6.213)$$

The quantity of vapors in the line is determined by the equation of state:

$$Y_0 = \frac{p_{\text{п}} V_0}{R T_s}. \qquad (6.214)$$

The free volume in the line

$$V_0 = V_{\text{н}} - \frac{1}{\rho_{\text{ж}}} \int_0^t G\,dt, \qquad (6.215)$$

where $V_{\text{н}}$ – initial volume of line. Therefore

$$Y_0 = \frac{p_{\text{п}}}{R T_s} \left(V_{\text{н}} - \frac{1}{\rho_{\text{ж}}} \int_0^t G\,dt \right). \qquad (6.216)$$

Consequently,

$$\dot{Y}_0=\frac{1}{RT_s}\left[\dot{p}_{\text{н}}\left(V_{\text{н}}-\frac{1}{\varrho_{\text{ж}}}\int_0^t G\,dt\right)-\frac{p_{\text{н}}}{\varrho_{\text{ж}}}G\right]. \tag{6.217}$$

Being guided by a small pressure drop on the injectors, we accept

$$\dot{Y}_{\text{н}}=G_{\text{н}}=\mu F_{\phi}\sqrt{2\varrho_{\text{г}}(p_{\text{п}}-p_{\text{к}})}, \tag{6.218}$$

where $p_{\text{к}}$ — pressure in the chamber; $\varrho_{\text{г}}$ — gas density:

$$\varrho_{\text{г}}=\frac{p_{\text{п}}}{RT_s}. \tag{6.219}$$

Consequently,

$$\dot{Y}_{\text{н}}=\mu F_{\phi}\sqrt{2\frac{p_{\text{п}}}{RT_s}(p_{\text{п}}-p_{\text{к}})}. \tag{6.220}$$

Now the initial equation of the law of conservation of mass assumes the form

$$4\frac{\varrho_c}{\varrho_{\text{ж}}}\frac{c_{\text{ст}}}{r^*}\frac{\Delta}{d}(T_{\text{ст}0}-T_s)\dot{Z}-\frac{1}{RT_s}\left[\dot{p}_{\text{п}}\left(\dot{V}_{\text{н}}-\frac{1}{\varrho_{\text{ж}}}Z\right)-\frac{p_{\text{п}}}{\varrho_{\text{ж}}}\dot{Z}\right]-\mu F_{\phi}\sqrt{2\frac{p_{\text{п}}}{RT_s}(p_{\text{п}}-p_s)}. \tag{6.221}$$

Equation (6.221) contains two variables: $p_{\text{п}}$ and Z and their derivatives. Therefore, the considered equation should be solved together with the equation of pressure balance (6.176).

As already mentioned, in actuality the process of change of the wall temperature from its initial value $T_{\text{ст}0}$ to final, equal to T_s, proceeds with time. Consequently, there is a section of the tube,

along the length of which the wall temperature is variable. The beginning of this section approximately coincides and moves with the front of liquid, and the region of lowest temperatures is arranged from the liquid front to the side opposite the direction of motion of liquid. To every current value of temperature T_{CT} there corresponds its distance x, measured from the liquid front; besides this, every value of T_{CT} will characterize a definite intensity of gas formation. Let us derive the equation for constructing the graph of function $T_{\text{ст}}(x)$.

Heat, transferred from the wall to liquid,

$$Q=\alpha F_{\text{ст}}(T_{\text{ст}}-T_{s}). \tag{6.222}$$

The heat transfer coefficient depends on particular conditions of heat exchange. On the path of gas formation the intensity of heat exchange varies, and when conducting detailed research it is necessary to separately examine the regions of film, nucleate and liquid heat exchange. Let us be limited to the case when it is possible to accept

$$\alpha=AG^{m}, \tag{6.223}$$

moreover for separate time intervals A, m, they are assigned in the form of constant quantities. Mass of the wall being cooled

$$Y_{\text{ст}}=\rho_{\text{ст}}F_{\text{ст}}\Delta, \tag{6.224}$$

heat, given off by the wall,

$$Q=-Y_{\text{ст}}c_{\text{ст}}\frac{dT_{\text{ст}}}{dt}. \tag{6.225}$$

By equating the right sides of equations (6.211) and (6.214), we find

$$dt=-\frac{\rho_{\text{ст}}c_{\text{ст}}\Delta}{AG^{m}}\,\frac{dT_{\text{ст}}}{T_{\text{ст}}-T_{s}}. \tag{6.226}$$

After integration we obtain

$$t=\frac{\varrho_{\text{ст}}c_{\text{ст}}\Delta}{AG^{m}}\ln\frac{T_{\text{ст}0}-T_s}{T_{\text{ст}}-T_s}. \tag{6.227}$$

We find this time by using the equation of continuity

$$t=\frac{\pi d^2\varrho_{\text{ж}}}{4G}x. \tag{6.228}$$

By equating the right sides of equations (6.227) and (6.228), we find the length of the section, on which the wall temperature is changed from $T_{\text{CT}0}$ to T_s:

$$x=B\ln\frac{T_{\text{ст}0}-T_s}{T_{\text{ст}}-T_s}, \tag{6.229}$$

where

$$B=\frac{4\varrho_{\text{ст}}c_{\text{ст}}\Delta}{\pi Ad^2\varrho_{\text{ж}}}G^{1-m}. \tag{6.230}$$

By raising formula (6.229) to power, after transformations we obtain the sought expression for constructing the graph of function $T_{\text{ст}}(x)$ in the form

$$T_{\text{ст}}=T_{\text{ст}0}\exp\left(-\frac{x}{B}\right)+T_s\left[1-\exp\left(-\frac{x}{B}\right)\right]. \tag{6.231}$$

With a blister being formed on the surface of the wall of boiling the coefficient of heat emission can be determined by an equation, obtained on the basis of the theory of heat transfer:

$$\alpha_{\text{пуз}}=\alpha_{\text{ж}}\left[\frac{1}{1+\chi}\left(1+\varrho_{\text{ж}}\frac{RT_s}{p}\chi\right)\right]^m G^m+ \\ +\frac{\chi}{1+\chi}\left(1+\frac{r^*}{T_{\text{ст}}-T_{\text{ж}}}\right)G. \tag{6.232}$$

Here coefficient $a_{ж}$ is calculated under the assumption of only liquid cooling:

$$a_{ж}=\frac{A}{\lambda d^{1-m}}\left(\frac{4}{\pi\nu\varrho_{ж}}\right)^{m}\mathrm{Pr}^{n}.$$

The relative quantity of vapors $\chi=\frac{G_{п}}{G_{ж}}$, where $G_{п}, G_{ж}$ – the flow rate of vapor and liquid in the considered section of the line. With nucleate boiling $0<\chi<\chi_{кр}$. If $\chi>\chi_{кр}$, then film boiling appears, at which the coefficient of heat emission sharply drops. The value of $\chi_{кр}$ is determined experimentally.

With nucleate boiling the liquid velocity is increased due to constraint of the flow duct by bubbles of vapor; this is considered by the addend of equation (6.232). The augend considers the increase of the coefficient of heat emission, which appears because of preheating and vaporization of part of the cooling liquid, the relative quantity of which is equal to χ. Turbulization of flow by bubbles of vapor is considered by exponent m, determined experimentally.

CHAPTER VII

TURBOPUMP UNIT

A. CENTRIFUGAL PUMP

One of the most important units of the feed system, which provides motion of propellant components along the hydraulic passage, is the centrifugal pump. The pump is actuated from a turbine and at nominal revolutions provides the design head (pressure) and the required flow rate. With deviation from nominal revolutions both the head and flow rate are changed. While maintaining nominal revolutions, but with change of head the flow rate changes. In all these cases, as a rule, the power consumed by the pump, and its efficiency are changed.

The change of operating conditions of the pump is caused by various factors, which can be divided into:

a) depending on the characteristic of the turbine and the peculiarities of its operation;

b) depending on the parameters of hydraulic lines and pressurized systems;

c) depending on parameters of the pump itself.

The pump parameters are selected while designing and are refined experimentally. Under nonsteady conditions they vary with time. Small systematic deviations from nominal values are

associated with the action of external factors, and they can also be caused by industrial, technological and operational factors.

The study of nonsteady operating conditions of an engine and the effect of external and internal influences is impossible without knowledge of the interconnection between power, consumed by the pump, its efficiency created by head and flow rate of liquid. This interconnection is a characteristic of a centrifugal pump.

The complexity of mathematical expression of the characteristic depends upon the accepted theory and the assumptions made, which in turn are determined by the purpose of research and by the required accuracy of calculation. In this work is discussed one of the possible engineering methods, making it possible to obtain the solution in the form of convenient calculation formulas.

7.1. Head, Created by the Pump

During operation of the turbopump unit [TNA] (THA) to the pump shaft which revolves with angular velocity ω, there is applied torsional moment M. Elementary energy, transferred by the turbine to the pump, will be

$$dE=M\omega dt. \tag{7.1}$$

If all the mechanical energy, supplied to the pump, is transformed into energy of liquid flowing through the pump, then

$$dE=H_t G dt, \tag{7.2}$$

where H_t - theoretical head, created by the pump; G - mass flow rate of liquid per second.

Inasmuch as

$$Gdt=dm, \tag{7.3}$$

then

$$\frac{dE}{dm}=H_t. \qquad (7.4)$$

Thus, the theoretical head is equal to energy, supplied to the pump and pertaining to a unit of mass of liquid.

By equating the right sides of equations (7.1) and (7.2), we find

$$H_t=\frac{M\omega}{G}. \qquad (7.5)$$

The actual head, created by the pump,

$$H=H_t\eta_н, \qquad (7.6)$$

where the pump efficiency

$$\eta_н=\eta_V\eta_h\eta_м. \qquad (7.7)$$

Here η_V - efficiency, considering volumetric losses, i.e., the leakage of liquid from the flow area or the presence of dead (parasitic) circulation; η_h - efficiency, considering hydraulic losses in the flow passage; $\eta_м$ - efficiency, considering losses, which appear due to mechanical friction.

From the theory of centrifugal pumps it is known that

$$\eta_н=f(H, G, \omega). \qquad (7.8)$$

Now equation (7.5) can be rewritten so:

$$M=\frac{G}{\omega}\frac{H}{\eta_н}. \qquad (7.9)$$

It is known that power

$$N=M\omega. \qquad (7.10)$$

Consequently, power, supplied to the pump from the turbine,

$$N=\frac{HG}{\eta_{н}}. \tag{7.11}$$

The actual head, created by the pump, under steady state conditions is made up of static head

$$H_{ст}=\frac{p}{\rho_{ж}} \tag{7.12}$$

and dynamic head

$$H_{д}=\frac{C^2}{2}. \tag{7.13}$$

Usually we measure pressures and velocities at the pump inlet (p_1, C_1) and at the pump exit (p_2, C_2). The increase of head in the pump

$$H_{н}=\frac{p_2-p_1}{\rho_{ж}}+\frac{C_2^2-C_1^2}{2}. \tag{7.14}$$

With nonsteady conditions, i.e., with change of angular velocity of the impeller with time, moment $M_{Ж}$, imparted to liquid by the impeller, will be smaller than moment M, supplied to the pump shaft:

$$M_{ж}=M-J_{н}\dot{\omega}, \tag{7.15}$$

where J_{H} - moment of inertia of revolving parts of the pump.

The increase of the pump head under conditions of nonsteady state

$$H_{н}=\frac{p_2-p_1}{\rho_{ж}}-\frac{C_2^2-C_1^2}{2}-\int_0^l \dot{C}(t)\,dl, \tag{7.16}$$

where $C(t)$ - acceleration of liquid flow.

The last component in equation (7.16) represents energy, referred to a unit of mass of liquid and being spent for overcoming of mass (volumetric) forces. In order to visually represent the value of the last component, let us accept that the acceleration of flow is identical for all points of the passage. In this case

$$\int_0^l \dot{C}(l)\,dl=\dot{C}l. \qquad (7.17)$$

Let us multiply and divide the right side of equality (7.17) by $F\rho_{ж}$, then

$$\int_0^l \dot{C}(l)\,dl=m\,\frac{\dot{C}}{F\rho_{ж}}. \qquad (7.18)$$

The mass of liquid, which fills the flow area of the pump,

$$m=F\rho_{ж}l.$$

Force, being spent on overcoming mass forces,

$$P=m\dot{C}.$$

The pressure equivalent to it

$$p=\frac{m\dot{C}}{F}.$$

Consequently, the drop of head, caused by the action of mass forces, will be

$$H_{\dot{C}}=\frac{m\dot{C}}{F\rho_{ж}},$$

and this is the last component in equation (7.16), i.e.,

$$\int_0^l \dot{C}(l)\,dl=H_{\dot{C}}. \qquad (7.19)$$

Equation (7.16) is widely used during research of nonsteady states, especially, connected with the starting of liquid-propellant rocket engine. With rapid starting of the engine acceleration $\dot{G}$ can be so great, that it will cause a severe decrease of static pressure, which sometimes leads to rather deep cavitation.

Let us first determine the separate components of the equation of law of conservation of energy of a centrifugal pump, and then let us write the general equation of energy in collected form.

7.2. Energy, Transferred to Liquid by a Vane Wheel at Steady State

The resultant moment of interaction of a wheel with flow is expressed by the equation of moments of momentum:

$$M_{\text{к}}=G'[(C_U r)_2-(C_U r)_1], \tag{7.20}$$

where G' - flow rate through the wheel; C_U - projection of absolute velocity of liquid motion to the direction of the velocity of following (the peripheral velocity of the wheel);

$$(C_U r)_i=\frac{\int C_U r\, dG'}{G'} \tag{7.21}$$

is the mean value of the velocity moment along the section. Energy, transferred to liquid by the vane wheel,

$$E_{\text{к}}=\frac{M_{\text{к}}}{G'}\omega. \tag{7.22}$$

By using expressions (7.20) and (7.22), we obtain

$$E_{\text{к}}=C_{U2}U_2-C_{U1}U_1, \tag{7.23}$$

where peripheral velocities

$$U_2=\omega r_2, \tag{7.24}$$

$$U_1=\omega r_1. \tag{7.25}$$

7.3. Energy Conversion in the Flow Area of a Wheel at Steady State

From velocity triangles, constructed for the impeller inlet and outlet (Fig. 7.1), follows:

$$C_{U2}U_2=\frac{C_2^2}{2}+\frac{U_2^2-W_2^2}{2}; \qquad (7.26)$$

$$C_{U1}U_1=\frac{C_1^2}{2}+\frac{U_1^2-W_1^2}{2}, \qquad (7.27)$$

where U_i - velocity of following (peripheral velocity); W_i - relative velocity of liquid; C_i - absolute velocity of liquid; C_{Ui} - projection of C_i to the direction of velocity U_i.

By subtracting the obtained expression (7.27) from equality (7.26), we find

$$C_{U2}U_2-C_{U1}U_1=\frac{C_2^2-C_1^2}{2}+\left(\frac{U_2^2-W_2^2}{2}-\frac{U_1^2-W_1^2}{2}\right). \qquad (7.28)$$

The left side of equation (7.28) according to relationship (7.23) represents energy, transferred to liquid by a vane wheel in steady motion. The right side of equation (7.28) characterizes the types of energy, which the liquid possesses. The addend represents the increase of kinetic energy in absolute motion, equal to kinetic head

$$E_{\text{кин}}=H_{\text{кин}}=\frac{C_2^2-C_1^2}{2}. \qquad (7.29)$$

The augend of the right side represents the energy of interaction of flow with external forces, equal to static head

$$E_{\text{ст}}=H_{\text{ст}}=\frac{U_2^2-W_2^2}{2}-\frac{U_1^2-W_1^2}{2}=\frac{p_2-p_1}{\varrho_{\text{ж}}}. \qquad (7.30)$$

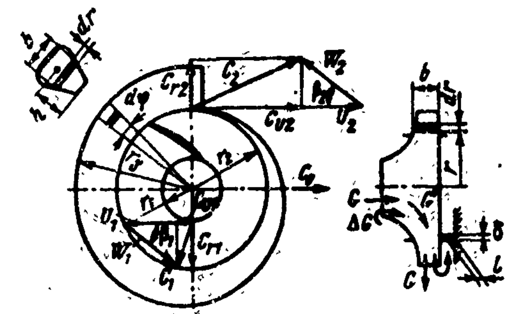

Fig. 7.1. Velocity triangle for the flow area of a centrifugal pump.

Proof of condition (7.30)

The equation of motion in the direction of the movement of element of liquid is written so:

$$\frac{\partial W}{\partial t}+\frac{\partial W}{\partial S}W=(j_S+g_S)'-\frac{1}{\varrho_ж}\frac{\partial p}{\partial S}. \qquad (7.31)$$

In relative motion $(j_S+g_S)'$ is the sum of external volumetric, centrifugal and and Coriolis forces.

External volumetric forces are determined by acceleration of rocket flight and acceleration of gravity. It is expedient to consider their effect in the examination of the entire hydraulic circuit.

Coriolis force of inertia is perpendicular to the direction of relative velocity, if an "ideal" centrifugal wheel is considered, consequently, the work of Coriolis force in the direction of movement of liquid will be equal to zero.

Centrifugal force

$$\omega^2 r\cos\Theta=\omega^2 r\frac{dr}{dS}, \qquad (7.32)$$

where Θ - angle between segments Δr and ΔS.

Under steady state conditions equation (7.31) with consideration of formula (7.32) will be written so:

$$\frac{dW}{dS}W=\omega^2 r\frac{dr}{dS}-\frac{1}{\varrho_ж}\frac{dp}{dS}. \tag{7.33}$$

Multiplying by dS, taking into account that

$$U=\omega r, \tag{7.34}$$

and integrating, we find

$$\int_{W_1}^{W_2} W\,dW=\int_{U_1}^{U_2} U\,dU-\int_{p_1}^{p_2}\frac{dp}{\varrho_ж}. \tag{7.35}$$

After conversions when $\varrho_ж$ = const the expression for static head takes the form

$$E_{ст}=\frac{U_2^2-W_2^2}{2}-\frac{U_1^2-W_1^2}{2}=\frac{p_2-p_1}{\varrho_ж}, \tag{7.36}$$

which is identical to formula (7.30).

7.4. The Working Formula of Energy, Transferred to Liquid by a Vane Wheel at Steady State

The work, accomplished by a wheel when ω = const,

$$E_к=\omega\int_{r_1}^{r_2} C_U\,dr. \tag{7.37}$$

From the velocity triangle it follows that

$$C_U=U-\frac{C_r}{\operatorname{tg}\beta}. \tag{7.38}$$

According to the equation of continuity

$$C_r=\frac{G'}{2\pi r b\varrho_ж}k, \tag{7.39}$$

where b - width of flow part of the impeller; G' - flow rate through the wheel (Fig. 7.2):

$$G'=G+\Delta G_1+\Delta G_2. \qquad (7.40)$$

Here G - flow rate through the pump; ΔG_1 - leakage in the region of vane wheel, caused by circulation through the inlet throat; ΔG_2 - leakage in the region of vane wheel, caused by circulation through openings of the vane web; k - constriction coefficient:

$$k=\frac{h}{h-\frac{\Delta}{\sin\beta_л}}, \qquad (7.41)$$

where Δ - blade thickness; $\beta_л$ - current value of blade angle; h - blade pitch:

$$h=\frac{2\pi r}{z}; \qquad (7.42)$$

r - current value of radius; z - number of blades.

After the necessary conversions we receive

$$E_к=\frac{\omega^2}{2}(r_2^2-r_1^2)-\frac{\omega}{2\pi}\int\frac{G'k}{\rho_ж rb\,\mathrm{tg}\,\beta}dr. \qquad (7.43)$$

By assuming

$$\frac{G'k}{\rho_ж b\,\mathrm{tg}\,\beta}$$

the mean value on the section of path S from r_1 to r_2, we obtain

$$E_к=\frac{\omega^2}{2}(r_2^2-r_1^2)-\frac{\omega}{2\pi\rho_ж}\frac{G'k}{b\,\mathrm{tg}\,\beta}\ln\left(\frac{r_2}{r_1}\right)=D_1\omega^2-D_1'\omega G', \qquad (7.44)$$

where D_1 and D_1' are easily determined by formula (7.44).

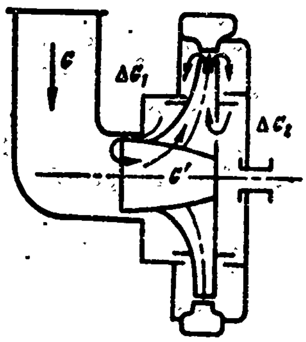

Fig. 7.2. Leakage of liquid in the region of the impeller of a centrifugal pump.

7.5. Determination of Kinetic Head

Part of the energy, transferred to liquids, is spent, as was noted, on increase of its kinetic energy. From velocity triangles (see Fig. 7.1) follows:

$$\frac{C_2^2-C_1^2}{2}=\frac{C_{U2}^2-C_{U1}^2}{2}+\frac{C_{r2}^2-C_{r1}^2}{2}. \tag{7.45}$$

Thus, the increase of kinetic energy of liquid in absolute motion

$$E_{\text{кин}}=\frac{C_2^2-C_1^2}{2} \tag{7.46}$$

is equal to the increase of kinetic energy in rotatory motion

$$E_{u}=\frac{C_{U2}^2-C_{U1}^2}{2} \tag{7.47}$$

plus the increase of kinetic energy of radially directed flows

$$E_{r}=\frac{C_{r2}^2-C_{r1}^2}{2}. \tag{7.48}$$

By substituting in equation (7.45) the values of projections of velocities, after conversions we receive

$$E_{\text{кин}}=\omega^2(r_2^2-r_1^2)-\frac{G'}{\pi\rho_{ж}}\omega\left(\frac{k_2}{b_2\operatorname{tg}\beta_2}-\frac{k_1}{b_1\operatorname{tg}\beta_1}\right)+$$
$$+\left(\frac{G'}{2\pi\rho_{ж}}\right)^2\left[\left(1+\frac{1}{\operatorname{tg}^2\beta_2}\right)\left(\frac{k_2}{r_2b_2}\right)^2-\right.$$
$$\left.-\left(1+\frac{1}{\operatorname{tg}^2\beta_1}\right)\left(\frac{k_1}{r_1b_1}\right)^2\right]. \tag{7.49}$$

7.6. Determination of Static Head

According to the velocity triangle (see Fig. 7.1)

$$W = \frac{U - C_U}{\cos\beta}. \qquad (7.50)$$

After conversions of equation (7.36) we receive

$$E_{\text{ст}} = \omega^2(r_2^2 - r_1^2) - \frac{G'}{\pi\varrho_{ж}}\,\omega\left(\frac{k_2}{b_2\sin^2\beta_2} - \frac{k_1}{b_1\sin^2\beta_1}\right) + \frac{1}{2}\left(\frac{G'}{2\pi\varrho_{ж}}\right)^2\left[\left(\frac{k_2}{r_2 b_2\sin\beta_2}\right)^2 - \left(\frac{k_1}{r_1 b_1\sin\beta_1}\right)^2\right]. \qquad (7.51)$$

Degree of reaction of vane wheel

The ratio of static head to total is called the degree of reaction of vane wheel:

$$\varepsilon = \frac{E_{\text{ст}}}{E_{\text{к}}} = \frac{p_2 - p_1}{(p_2 - p_1) + \varrho_{ж}\left(\frac{C_2^2 - C_1^2}{2}\right)}. \qquad (7.52)$$

For determination of static head it is possible to write

$$E_{\text{ст}} = E_{\text{к}} - E_{\text{кин}} = C_{U2}U_2 - C_{U1}U_1 - \frac{C_{U2}^2 - C_{U1}^2}{2} - \frac{C_{r2}^2 - C_{r1}^2}{2}. \qquad (7.53)$$

Thus,

$$\varepsilon = 1 - \frac{(C_{U2}^2 - C_{U1}^2) + (C_{r2}^2 - C_{r1}^2)}{2(C_{U2}U_2 - C_{U1}U_1)}. \qquad (7.54)$$

In that particular case, when it is possible to assume $C_{r2} = C_{r1}$ and $C_{U1} = 0$,

$$\varepsilon \approx 1 - \frac{C_{U2}}{2U_2}. \qquad (7.55)$$

After simple conversions we obtain

$$\varepsilon=\frac{1}{2}\left(1-\frac{G' k_2}{2\pi\omega r_2^2 b_2 \rho_{\text{ж}} \operatorname{tg}\beta_2}\right). \tag{7.56}$$

7.7. The Energy of Liquid, Caused by Angular Acceleration of Vane Wheel

The moment of momentum (Fig. 7.3) -

$$M_{\dot\omega}=\frac{\partial}{\partial t}\int(C_U r)\,dm; \tag{7.57}$$

elementary mass of liquid -

$$dm=\frac{2\pi r b \rho_{\text{ж}}}{k}\,dr; \tag{7.58}$$

transmitted desired energy -

$$E_{\dot\omega}=\frac{M_{\dot\omega}\omega}{G'}=2\pi\rho_{\text{ж}}\frac{\partial}{\partial t}\int_{r_1}^{r_2} G'(\omega C_U r)\frac{b}{k}\,r\,dr. \tag{7.59}$$

By taking the mean value for $G'\frac{b}{k}$ on section of path S from r_1 to r_2, we obtain

$$E_{\dot\omega}=2\pi\rho_{\text{ж}}G'\frac{b}{k}\frac{\partial}{\partial t}\int_{r_1}^{r_2}(\omega C_U r)\,r\,dr \tag{7.60}$$

or

$$E_{\dot\omega}=2\pi\rho_{\text{ж}}G'\frac{b}{k}\frac{\partial}{\partial t}\int_{r_1}^{r_2}(\omega C_U)\,r^2\,dr. \tag{7.61}$$

With filling of the entire interior cavity of the wheel with liquid the solution of equation (7.61) has the form

$$E_{\dot\omega}=\pi\rho_{\text{ж}}G'\frac{b}{k}\Big[\dot\omega\omega(r_2^4-r_1^4)-\frac{(\dot\omega G'+\dot G'\omega)k}{2\pi b\rho_{\text{ж}}\operatorname{tg}\beta}(r_2^2-r_1^2)\Big]. \tag{7.62}$$

In order to characterize the energy distribution E_{ω} by types, let us examine equation

$$\frac{d}{dt}(\omega C_U r) = C_U \dot{C}_U + C_r \dot{C}_r + \frac{d}{dt}\left(\frac{U^2 - W^2}{2}\right), \tag{7.63}$$

since

$$C^2 = C_U^2 - C_r^2. \tag{7.64}$$

The addend of the right side of equation (7.63) characterizes energy $E_{\omega ж}$ being consumed on change of angular velocity of rotation of liquid in the flow part of the impeller with time, inasmuch as $C_U = \omega_ж r$. The augend characterizes energy E_r, being consumed on change of radial velocity component of liquid with time, and the third component - energy E_p, caused by change of external surface forces, affecting the flow, with time. Let us show that

$$\frac{d}{dt}\left(\frac{U^2 - W^2}{2}\right) = \frac{1}{\varrho_ж} \frac{\partial p}{\partial t}. \tag{7.65}$$

Equation (7.31) taking into consideration equation (7.33) and with the absence of external volumetric forces is written so:

$$\dot{W} = \omega^2 r \frac{dr}{dS} - \frac{1}{\varrho_ж} \frac{\partial p}{\partial S}. \tag{7.66}$$

After multiplication by dS, taking into account that

$$\frac{dS}{dt} = W, \tag{7.67}$$

we obtain

$$W\,dW = U\,dU - \frac{1}{\varrho_ж} \frac{\partial p}{\partial S} dS. \tag{7.68}$$

Having divided equality (7.68) by dt, after conversions we find

$$U\dot{U} - W\dot{W} = \frac{d}{dt}\left(\frac{U^2 - W^2}{2}\right) = \frac{1}{\varrho_ж} \frac{\partial p}{\partial t}. \tag{7.69}$$

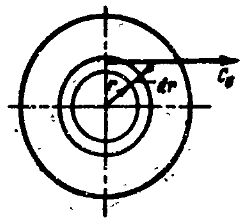

Fig. 7.3. For derivation of formula (7.59).

7.8. The Working Formula of Energy, Consumed on Change of the Angular Velocity of Rotation of Liquid in the Flow Area of Vane Wheel

As was shown,

$$E_{\dot{\omega}} = E_{\dot{\omega}\text{ж}} + E_{\dot{r}} + E_{\rho}. \qquad (7.70)$$

Thus, instead of computation of $E_{\dot{\omega}}$ it is possible to calculate the components of the right side of equality (7.70). Under nonsteady state conditions energy

$$E_{\dot{\omega}\text{ж}} = \int \frac{\omega_\text{ж}}{G'} \dot{\omega}_\text{ж} \, dJ_\text{ж}, \qquad (7.71)$$

where $\dot{\omega}_\text{ж}$ - angular velocity of rotation of liquid; $J_\text{ж}$ - moment of inertia of liquid:

$$dJ_\text{ж} = r^2 dm = 2\pi r^3 b \frac{\varrho_\text{ж}}{k} dr. \qquad (7.72)$$

The angular velocity of liquid

$$\omega_\text{ж} = \frac{c_U}{r}. \qquad (7.73)$$

According to the velocity triangle and equation of continuity

$$\omega_\text{ж} = \omega - \frac{G'}{2\pi r^2 b \varrho_\text{ж} \operatorname{tg} \beta} k. \qquad (7.74)$$

Derivative

$$\dot{\omega}_\text{ж} = \dot{\omega} - \frac{\dot{G}'}{2\pi r^2 b \varrho_\text{ж} \operatorname{tg} \beta} k. \qquad (7.75)$$

Now, instead of equation (7.71) we will have

$$E_{\dot{\omega}\text{ж}}=2\pi\rho_{\text{ж}}\int\limits_{r_1}^{r_2}\left(\omega-\frac{G'}{2\pi r^2 b\rho_{\text{ж}}\,\mathrm{tg}\,\beta}k\right)\times$$

$$\times\left(\dot{\omega}-\frac{\dot{G}'}{2\pi r^2 b\rho_{\text{ж}}\,\mathrm{tg}\,\beta}k\right)\frac{b}{G'k}r^3dr. \tag{7.76}$$

By taking for all parameters, except r, their mean values on the section of path S from r_1 to r_2, we obtain the solution in this form:

$$E_{\dot{\omega}\text{ж}}=\frac{\pi\rho_{\text{ж}}b}{2k}\left[\dot{\omega}\frac{\omega}{G'}(r_2^4-r_1^4)-\frac{k}{\pi b\rho_{\text{ж}}\,\mathrm{tg}\,\beta}\times\right.$$

$$\left.\times\left(\dot{\omega}+\dot{G}'\frac{\omega}{G'}\right)(r_2^2-r_1^2)+\left(\frac{k}{\pi b\rho_{\text{ж}}\,\mathrm{tg}\,\beta}\right)^2\times\dot{G}'\ln\left(\frac{r_2}{r_1}\right)\right]. \tag{7.77}$$

7.9. The Working Formula of Energy, Consumed on the Change of Radial Velocity Component of Liquid

Let us write the initial expression so:

$$E_{\dot{r}}=\int\frac{C_r\dot{C}_r}{G}dm=\int\frac{1}{\rho_{\text{ж}}}\frac{dP}{F}. \tag{7.78}$$

Elementary force -

$$dP=\dot{C}_r dm=\frac{\dot{G}'k}{2\pi rb\rho_{\text{ж}}}2\pi rb\frac{\rho_{\text{ж}}}{k}dr=\dot{G}'dr; \tag{7.79}$$

surface of application of force -

$$F=2\pi r\frac{b}{k}. \tag{7.80}$$

Consequently,

$$E_{\dot{r}}=\frac{1}{2\pi}\int\limits_{r_1}^{r_2}\dot{G}'k\frac{dr}{rb}. \tag{7.81}$$

By taking the mean value for $G'k/b$ on the section of path S from r_1 to r_2, we obtain solution

$$E_i = \frac{\dot{G}'k}{2\pi b} \ln\left(\frac{r_2}{r_1}\right). \qquad (7.82)$$

7.10. The Working Formula of Energy, Caused by Change of External Surface Forces, Affecting Flow

The external surface force, affecting flow, is equal to $\frac{1}{\varrho_{\text{ж}}} \frac{\partial p}{\partial x}$. By using equality (7.69), we obtain the initial expression in the form

$$E_p = 2\pi \int_{r_1}^{r_2} \varrho_{\text{ж}} G' \frac{b}{k} \frac{d}{dt}\left(\frac{U^2 - W^2}{2}\right) r\,dr. \qquad (7.83)$$

By taking the mean value for $\varrho_{\text{ж}} G' \frac{b}{k}$ on the section of path S from r_1 to r_2, we obtain

$$E_p = 2\pi \varrho_{\text{ж}} G' \frac{b}{k} \int_{r_1}^{r_2} \frac{d}{dt}\left(\frac{U^2 - W^2}{2}\right) r\,dr. \qquad (7.84)$$

Inasmuch as

$$W = \frac{G'k}{2\pi r b \varrho_{\text{ж}} \sin\beta}, \qquad (7.85)$$

the solution of equation (7.84) takes the form

$$E_p = \frac{\pi \varrho_{\text{ж}} G' b}{k} \dot{\omega}\omega \left(r_2^2 - r_1^2\right) - \frac{\dot{G}' (G')^2 k}{2\pi \varrho_{\text{ж}} b \sin^2\beta} \ln\left(\frac{r_2}{r_1}\right). \qquad (7.86)$$

7.11. The Working Formula of Energy, Caused by Change of Liquid Velocity in the Pump Inlet Throat

Additional head (or decrease of head), characterizing steady state in the flow area of vane wheel, was determined by integral (7.62). The mass forces affect flow in the inlet throat and in the spiral chamber. For inlet throat

$$E\dot{c}_0=\int\frac{F_0C_0}{\varrho_{\text{ж}}}\,dG=\int\frac{C_0\dot{C}_0}{G}\,dm, \tag{7.87}$$

where C_0 - liquid velocity in inlet throat; G - flow rate through the pump. For a passage of arbitrary shape

$$dm=\varrho_{\text{ж}}F_0(l)\,dl. \tag{7.88}$$

According to the equation of continuity

$$C_0=\frac{G}{\varrho_{\text{ж}}F_0(l)}; \tag{7.89}$$

in this case

$$\dot{C}_0=\frac{\dot{G}}{\varrho_{\text{ж}}F_0(l)}. \tag{7.90}$$

Thus

$$E\dot{c}_0=\dot{G}\int\frac{dl}{\varrho_{\text{ж}}F_0(l)}. \tag{7.91}$$

If $\varrho_{\text{ж}}F_0$ = const, then

$$E\dot{c}_0=\frac{l}{\varrho_{\text{ж}}F_0}\dot{G}. \tag{7.92}$$

7.12. The Working Formula of Energy, Caused by Change of Liquid Velocity in a Spiral Chamber

Similar to formula (7.87)

$$E\dot{c}=\int\frac{C\dot{C}}{G_l}\,dm. \tag{7.93}$$

Flow rate $G_l=G(\varphi)$.

Elementary mass of liquid

$$dm=Frd\varphi, \tag{7.94}$$

moreover (Fig. 7.4)

$$F=\int_{r_2}^{r_3} b\,dr. \tag{7.95}$$

In a spiral chamber r_2 = const, $r_3(\varphi)$ and $b(r, \varphi)$. Thus,

$$dm=\int_0^{2\pi}\int_{r_2}^{r_3(\varphi)} rb\,dr\,d\varphi. \tag{7.96}$$

Obviously we should take

$$G_i=\frac{\varphi}{2\pi}G, \tag{7.97}$$

where G - flow rate through the pump. Let us assume the spiral chamber is made so that

$$C=\frac{r_2}{r}C'_{U_2}, \tag{7.98}$$

where C'_{U_2} - tangential velocity component of liquid after exiting the impeller. It is evident that

$$C'_{U2}=\omega r-\frac{G}{2\pi r_2 b_2 \rho_{ж} \operatorname{tg}\beta_2}. \tag{7.99}$$

Now equation (7.93) assumes the form

$$E_{\dot{c}}=\frac{2\pi r_2^2}{G}\left(\omega r_2-\frac{G}{2\pi r_2 b_2 \rho_{ж} \operatorname{tg}\beta_2}\right)\times$$
$$\times\left(\omega r_2-\frac{G}{2\pi r_2 b_2 \rho_{ж} \operatorname{tg}\beta_2}\right)\int_0^{2\pi}\int_{r_2}^{r_3(\varphi)}\frac{1}{\varphi}\frac{b^2}{r}\,dr\,r\,d\varphi. \tag{7.100}$$

First we should take the internal integral, having substituted $b(r)$. In the solution one should substitute $r_3(\varphi)$ and find the solution of external integral

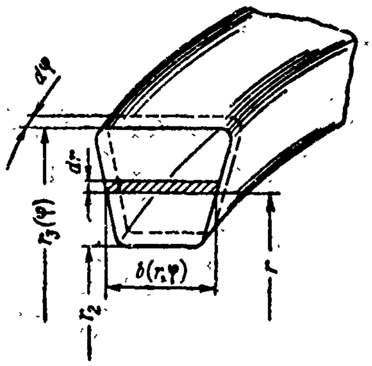

Fig. 7.4. Geometrical dimensions of spiral chamber.

7.13. Energy, Consumed on Overcoming Forces of Viscous Friction in the Pump Inlet Throat

Sought energy

$$E_{hC_0}=\frac{\Delta p_{C_0}}{\varrho_ж}. \tag{7.101}$$

For determination of hydraulic losses let us take

$$\Delta p_{C_0}=\frac{1}{2}G^2\int_0^{l_0}\frac{\lambda}{\varrho_ж}\frac{dl}{F^2 d}. \tag{7.102}$$

Let us also take

$$F=\xi\frac{\pi}{4}d^2, \tag{7.103}$$

where ξ - coefficient of the cross-sectional shape of the channel. Instead of formula (7.101) we have

$$E_{hC_0}=\frac{8}{\pi^2}G^2\int_0^{l_0}\frac{\lambda}{\xi^2\varrho_ж^2}\frac{dl}{d^5}. \tag{7.104}$$

By taking the mean value for $\lambda/\xi^2 d^5$, we obtain

$$E_{hC_0}=\frac{8}{\pi^2}\frac{G^2}{\varrho_ж^2}\frac{\lambda}{\xi^2 d^5}l. \tag{7.105}$$

If hydraulic losses are determined experimentally, then it is expedient to express the friction coefficient λ through resistance coefficient ε, having accepted $\lambda=\frac{\varepsilon d}{l}$. In this case solution (7.104) will differ from solution (7.105).

7.14. Energy, Consumed on Overcoming Forces of Viscous Friction in the Flow Area of Vane Wheel

Initial relationship

$$E_{hk}=\frac{(G')^2}{2}\int_{r_1}^{r_2}\frac{\lambda}{\varrho_ж^2 F^2 d_э}\,dr, \tag{7.106}$$

where $d_э$ - equivalent diameter:

$$d_э=4\frac{F}{\Pi}. \tag{7.107}$$

The area of the channel in relative motion

$$F=\frac{2\pi}{k}rb\sin\beta; \tag{7.108}$$

perimeter of channel

$$\Pi=4\pi\tau r, \tag{7.109}$$

where τ - coefficient, considering the presence of blades:

$$\tau=1-\frac{n\Delta}{4\pi r\cos\beta}+\frac{nb}{2\pi r}. \tag{7.110}$$

Now relationship (7.106) will be written so:

$$E_{hk}=\frac{(G')^2}{16\pi^2}\int_{r_1}^{r_2}\frac{\lambda\tau}{\varrho_ж^2}\,\frac{k^3}{r^2 b^3\sin^3\beta}\,dr. \tag{7.111}$$

By taking $\varrho_ж$ = const and

$$\lambda\tau\frac{k^3}{b^3\sin^3\beta}$$

for its mean value, we obtain

$$E_{hk}=\frac{\lambda\tau k^3(G')^2}{16\pi^2\varrho_ж^2 b^3\sin^3\beta}\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \tag{7.112}$$

7.15. Energy, Consumed on Overcoming of Forces of Viscous Friction in a Spiral Chamber

It is evident that

$$E_{hC}=\frac{1}{2}\int\limits_{0}^{2\pi}\lambda\frac{C^2}{d_э}r_{cp}\,d\varphi, \tag{7.113}$$

where r_{cp} - radius of center of gravity of cross section. Let

$$r_{cp}=r_2+a\varphi, \tag{7.114}$$

where

$$a=\frac{r_{cp\,max}-r_2}{2\pi}.$$

Velocity

$$C=\frac{G_i}{F_i\rho_ж}, \tag{7.115}$$

where

$$G_i=\frac{\varphi}{2\pi}G; \tag{7.116}$$

$$F_i=\frac{\varphi}{2\pi}F_3, \tag{7.117}$$

where F_3 - cross-sectional area of spiral chamber at the place of its connection to outlet throat.

Thus,

$$C=\frac{G}{F_3\rho_ж}, \tag{7.118}$$

For circular section

$$\Pi=2\pi r(\varphi)=\sqrt{2\varphi F_3}; \tag{7.119}$$

for noncircular section

$$\Pi=\zeta\sqrt{2\varphi F_3},\tag{7.120}$$

where ζ - coefficient of shape of section. After conversions expression (7.113) assumes the form

$$E_{hC}=\frac{\sqrt{2}\pi G^2}{4F_3^{2,5}}\left(r_2\int_0^{2\pi}\frac{\lambda\zeta}{\varrho_{\text{ж}\varphi}^2}\,d\varphi+a\int_0^{2\pi}\lambda\zeta\sqrt{\varphi}\,d\varphi\right)\tag{7.121}$$

By taking the mean value for $\lambda\zeta$ we find the solution for $\rho_{\text{ж}}$ = const in the form

$$E_{hC}=\frac{\pi^{1,5}G^2}{\varrho_{\text{ж}}^2F_3^{2,5}}\left(r_2+\frac{a\pi}{1,5}\right)(\lambda\zeta)_{\text{cp}}.\tag{7.122}$$

7.16. The Law of Conservation of Energy for a Centrifugal Pump

Energy E, supplied to the pump shaft under steady state conditions, is transferred to liquids by the vane wheel as $E_{\text{к}}$, determined by formula (7.44). This energy corresponds to the head of steady state, i.e.,

$$E_{\text{к}}=H_{\text{к}},\tag{7.123}$$

which is made up, in turn, of kinetic head $H_{\text{кин}}$, determined by formula (7.49), and static head $H_{\text{ст}}$ computed by equation (7.51).

Thus,

$$H_{\text{к}}=H_{\text{кин}}+H_{\text{ст}};\tag{7.124}$$

$$E_{\text{к}}=E_{\text{кин}}+E_{\text{ст}}.\tag{7.125}$$

Under nonsteady state conditions part of the energy E_0 supplied to the impeller is consumed on overcoming mass (volumetric) forces,

affecting the blading of the pump. The energy of liquid $E_{\dot\omega}$, caused by angular acceleration of vane wheel, is determined from formula (7.62); energy $E_{к.н}$, caused by change of velocity in the pump housing, is the same of energies $E_{\dot c_0}$ and $E_{\dot c}$, calculated according to formulas (7.87) and (7.100) respectively.

Thus,

$$E_{\dot Q} = E_{\dot\omega} + E_{к.н}; \qquad (7.126)$$

$$E_{к.н} = E_{\dot c_0} + E_{\dot c}, \qquad (7.127)$$

where $E_{\dot c_0}$ - energy, caused by change of liquid velocity in the inlet throat of the pump; $E_{\dot c}$ - energy, caused by change of liquid velocity in a spiral chamber.

During analysis of equation (7.63) it was shown that the energy of liquid, caused by the presence of angular acceleration of vane wheel, is equal to the sum of energy $E_{\dot\omega ж}$, going for change of angular velocity of rotation of liquid in the flow area of vane wheel, energy $E_{\dot r}$, consumed on change of radial velocity component of liquid, and energy E_p, which appears as a result of change of external surface forces, affecting the flow. The last three types of energies are determined by formulas (7.77), (7.82) and (7.86) respectively.

Thus,

$$E_{\dot\omega} = E_{\dot\omega ж} + E_{\dot r} + E_p. \qquad (7.128)$$

Under steady state conditions part of energy E_h is spent on overcoming forces of viscous friction, moreover

$$E_h = E_{hc_0} + E_{hк} + E_{hc}, \qquad (7.129)$$

where E_{hc_0} pertains to the inlet throat and is determined by formula (7.105), $E_{hк}$ - characterizes hydraulic losses in the flow area of impeller [formula (7.112)], E_{hc} corresponds to a spiral chamber and is calculated by formula (7.122).

Now it is possible to write the equation of the law of conservation of energy for a centrifugal pump, operating under conditions $\dot{\omega}>0$, in the following assembled form:

$$E=E_{к}-E_{\dot{\omega}}-E_{h}. \qquad (7.130)$$

In expanded form this equation will be written so:

$$E=E_{к}-E_{\dot{\omega}}-E_{\dot{C}_{0}}-E_{C}-E_{hC_{0}}-E_{hк}-E_{hC}. \qquad (7.131)$$

B. SCREW FOREPUMP

7.17. The Basic Equation of Dynamics of a Screw Forepump

The basic equation of dynamics of a screw forepump is the relationship of head of a screw conveyer to time-variable flow rate, rpm and their derivatives.

During derivation of the basic equation of dynamics of a screw forepump let us take the following:

- ideal, incompressible liquid;

- one-dimensional, continuous, axisymmetric motion of liquid;

- single-phase flow, without vapor-gas inclusions.

Let us separate (Fig. 7.5) the element of liquid with mass

$$dm=\rho_{ж}F_{п}dS \qquad (7.132)$$

and examine its motion in the channel of screw conveyer. Let us expand into a plane an infinitely thin layer of cylindrical cross section of vane cascade, being at arbitrary radius r_{i}, from the axis of screw conveyer.

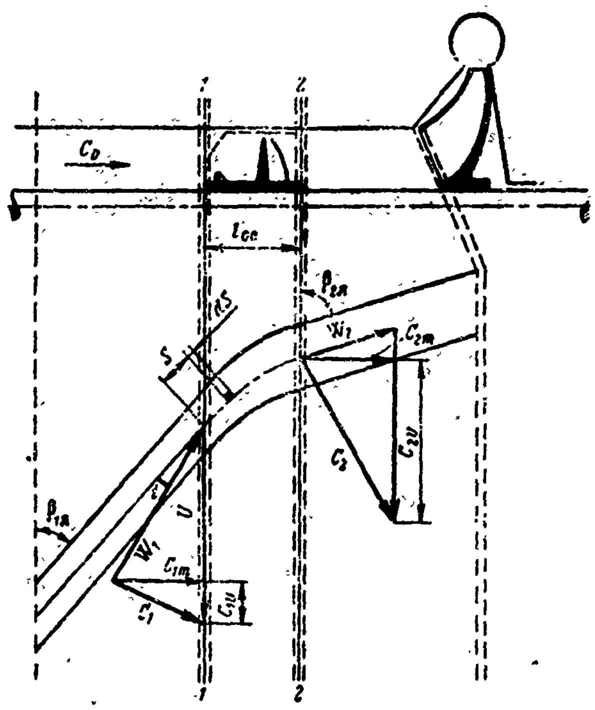

Fig. 7.5. For calculation of screw forepump.

To the separated element of liquid let us apply the theorem about the change of moment of momentum. According to the theorem the time derivative from the moment of momentum of liquid, flowing through the boundary (reference) sections of channels of the impeller, relative to the axis of screw conveyer is equal to the sum of moments of all external forces affecting the liquid relative to the same axis:

$$\frac{dL}{d\tau}=\sum_{j} M_j. \qquad (7.133)$$

The moment of momentum of liquid in impeller channels is equal to

$$L=\int_{S_1}^{S_2} rC_U\,dm. \qquad (7.134)$$

For fixed r with steady motion of liquid the quantity of time derivative from the moment of momentum is a function of two independent variables - time τ and coordinate S.

In this case

$$\left.\begin{aligned} &C_U=C_U(\tau, S);\quad C_m=C_m(\tau, S);\quad W=W(\tau, S);\\ &F_W=F_W(S);\\ &U=U(\tau)-\text{for axial-flow impellers;}\\ &U=U(\tau, S)-\text{for centrifugal impellers.}\end{aligned}\right\} \tag{7.135}$$

By substituting expression (7.132) in formula (7.134), we obtain

$$L(\tau)=\int_{S_1}^{S_2} r\varrho F_W(S)C_U(\tau, S)\,dS. \tag{7.136}$$

Let us differentiate expression (7.136) with respect to time:

$$\frac{dL}{d\tau}=\varrho\left|rF_W(S)C_U(\tau, S)W(\tau, S)\right|_1^2+\int_{S_1}^{S_2} rF_W(S)\frac{\partial C_U(\tau, S)}{\partial\tau}\,dS, \tag{7.137}$$

where

$$W(\tau, S)=\frac{dS}{d\tau}.$$

By substituting integration limits and omitting the entry of parameters (τ and S), we obtain

$$\frac{dL}{d\tau}=\varrho(r_2F_{2W}W_2C_{2U}-r_1F_{1W}W_1C_{1U})+\varrho\int_{S_1}^{S_2} rF_W\frac{\partial C_U}{\partial\tau}\,dS. \tag{7.138}$$

The moment of external forces, applied to every kilogram of liquid moving for a unit of time, creates increase of specific energy of liquid, i.e., it characterizes the magnitude of pump head. For steady state these quantities coincide:

$$\frac{\omega}{G'}\sum_j M_j=\frac{M_{\text{кр}}\omega}{G'}=H_\tau. \tag{7.139}$$

By substituting equalities (7.138) and (7.139) in relationship (7.133), we obtain the expression for theoretical head of screw forepump:

$$H_{т}=\frac{\omega}{G'}\frac{dL}{d\tau}=\frac{\varrho}{G'}(F_{2W}W_2U_2C_{2U}-F_{1W}W_1U_1C_{1U})+$$
$$+\frac{\varrho}{G'}\int_{S_1}^{S_2}\frac{dC_U}{d\tau}UF_W\,dS. \qquad (7.140)$$

The integral in expression (7.140) characterizes the amount of increase in the moment of momentum, cuased by time variation of the moment of momentum of the mass of pump rotor and mass of liquid flowing along channels of the impeller.

Generally the work on section *1-2*:

$$A=\int_{S_1}^{S_2}P\,dS,$$

where P - force; S - path. As applied to the impeller of a vane pump and the liquid flowing through it under transient conditions it is possible to write for the impeller:

$$P_{\text{к}}=m_{\text{к}}\frac{dU}{d\tau};\quad dS_{\text{к}}=U\,d\tau;$$

for liquid:

$$P_{\text{ж}}=m_{\text{ж}}\frac{dC_U}{d\tau};\quad dS_{\text{ж}}=C_U\,d\tau.$$

Then the work of impeller

$$A_{\text{к}}=\int_1^2P_{\text{к}}\,dS_{\text{к}}=\int_1^2m_{\text{к}}U\,dU;$$

the work of liquid

$$A_{\text{ж}}=\int_1^2P_{\text{ж}}\,dS_{\text{ж}}=\int_1^2m_{\text{ж}}C_U\,dC_U.$$

It is obvious that in the expression for pump head under transient conditions only the work of liquid should be considered. Taking into consideration the aforesaid and equality

$$F_{2W}W_2=F_{1W}W_1=\frac{G'}{Q},$$

equation (7.140) takes the form

$$H_{\tau}=(U_2C_{2U}-U_1C_{1U})+\frac{Q}{G'}\int\limits_{S_1}^{S_2}F_W C_U\frac{\partial C_U}{\partial\tau}\,dS. \qquad (7.141)$$

Expression (7.141) is the basic equation of dynamics of axial-flow forepump.

The theoretical head, developed by the screw forepump in transient conditions, is equal to the pump head at steady state according to the Euler equation (binomial in brackets) plus the head, obtained as a result of change of quantities C_U, F_W with time and along the length of the channel.

Let us convert the expression, standing under the integral sign of relationship (7.41). From velocity triangles of a screw forepump, operating in pumping mode, let us express the quantity of peripheral component of absolute velocity:

$$C_U=U-\frac{C_m}{\operatorname{tg}\beta_n}. \qquad (7.142)$$

By assuming C_U a function of two independent time variables $U(\tau)$ and $C_m(\tau)$, let us differentiate equality (7.142) in time. In this case we assume that the third quantity, which enters expression (7.142), - flow angle at pump impeller inlet β_n - does not have independent change with time, but is changed depending on the change of quantities $U(\tau)$ and $C_m(\tau, S)$ and is determined uniquely through them for any C_U = const.

Thus, as a result of differentiation of expression (7.142) we have

$$\frac{\partial C_U}{\partial \tau}=\frac{\partial U}{\partial \tau}-\frac{1}{\operatorname{tg}\beta_n}\frac{\partial C_m}{\partial \tau}. \qquad (7.143)$$

The flow angle varies along the length of the impeller channel, i.e.,

$$\beta_n=\beta_n(S).$$

In front of the screw conveyer vane inlet $\beta_n=\beta_{\text{л}}-i$ and impeller channel vane inlet (at a sufficient distance from the leading edges of vanes) it is possible to consider $\beta_n=\beta_{\text{л}}$, and at impeller channel exit $\beta_n=\beta_{2\text{л}}-\vartheta$, where ϑ - angle of lag of flow.

By substituting equalities (7.142), (7.143) in equation (7.141) and considering equality

$$F_W=F_m\sin\beta_n,$$

we receive the following integral equation for theoretical head of screw forepump, operating in pumping mode with unsteady motion:

$$H_\tau=U|C_U|_1^2+\frac{Q}{G'}\int_{S_1}^{S_2}\left(C_U\frac{\partial U}{\partial \tau}-C_U\frac{1}{\operatorname{tg}\beta_n}\frac{\partial C_m}{\partial \tau}\right)F_m\sin\beta_n\,dS. \qquad (7.144)$$

Equation (7.144) is the basic equation of dynamics of screw forepump. It permits determining the magnitude of screw conveyer head both during work at transient conditions, and at steady state, when

$$\frac{\partial U}{\partial \tau}=0;\quad \frac{\partial C_m}{\partial \tau}=0.$$

To get the calculation relationship of theoretical head of screw conveyer it is necessary to integrate equation (7.144) under fully defined boundary conditions, which correspond to the design of screw forepump and to its operating conditions.

7.18. Boundary Conditions for Integration of Basic Equation of Forepump

The theoretical head of the screw conveyer to a considerable extent depends, besides factors considered by equation (7.144), on the amount and direction of entry spin of flow at the screw conveyer inlet.

There are examined several basic cases of operation of screw forepump. As the positive direction of spin of flow before the screw conveyer there is taken the value of angle α, read clockwise from the direction of the vector of axial velocity component to the direction of the absolute velocity vector of flow. Depending on the amount of flow and the direction of this angle we determine the amount of the head of screw conveyer for four cases, characterized by the design of the pump and its operating conditions. In this case the head of screw conveyer as a mean integral quantity of energy along the cross section of flow is determined by the calculated diameter D_p of series-connected (not isolated) screw conveyer. In this case

$$D_p=\sqrt{\frac{D_ш^2+d_{вт}^2}{2}}, \tag{7.145}$$

where $D_ш$ - external diameter of screw conveyer; $d_{вт}$ - diameter of screw conveyer bushing.

For all the cases of operation of the screw conveyer stated below there takes place the following equality, ensuring from the design of axial-flow impeller:

$$U_{1p}=U_{2p}=U_p,$$

where U_{1p} and U_{2p} - peripheral velocities of screw conveyer at calculated diameter at the screw conveyer inlet and outlet. Let us suppose that C_m = const along the height of the screw conveyer vane.

The first case

$$\alpha_1=0^\circ;\quad \alpha_2>\alpha_1=0^\circ.$$

Boundary conditions

$$C_{1U}=0;\quad C_{2U}=U-\frac{C_{2m}}{\mathrm{tg}\,\beta_{2n}}. \qquad (7.146)$$

Equation (7.144) at boundary conditions (7.146) takes the form

$$H_{\tau}=\left(U_{p}^{2}-\frac{U_{p}C_{2m}}{\mathrm{tg}\,\beta_{2n}}\right)+\frac{\varrho}{G'}\int_{S_1}^{S_2}\left(C_U\frac{\partial U}{\partial\tau}-\frac{C_U}{\mathrm{tg}\,\beta_n}\frac{\partial C_m}{\partial\tau}\right)F_m\sin\beta_n\,dS. \qquad (7.147)$$

The first case of operation of screw conveyer corresponds to axial entry of flow on the vanes of screw conveyer, when flow spin at the inlet is absent (C_{1U} = 0).

The second case

$$0<\alpha_1<90^\circ;\quad \alpha_2<\alpha_1.$$

Boundary conditions

$$C_{1U}=U-\frac{C_{1m}}{\mathrm{tg}\,\beta_{1n}};\quad C_{2U}=U-\frac{C_{2m}}{\mathrm{tg}\,\beta_{2n}} \qquad (7.148)$$

Equation (7.144) at boundary conditions (7.148) takes the form

$$H_{\tau}=U_{p}\left(\frac{C_{1m}}{\mathrm{tg}\,\beta_{1n}}-\frac{C_{2m}}{\mathrm{tg}\,\beta_{2n}}\right)+$$
$$+\frac{\varrho}{G'}\int_{S_1}^{S_2}\left(C_U\frac{\partial U}{\partial\tau}-\frac{C_U}{\mathrm{tg}\,\beta_n}\frac{\partial C_m}{\partial\tau}\right)F_m\sin\beta_n\,dS. \qquad (7.149)$$

The second case of operation of screw conveyer corresponds to positive entry spin of flow to the screw conveyer, for example, with installation of an appropriate return-circuit rig before the screw conveyer.

The spin of flow is considered positive, if the vector of peripheral component of absolute velocity of liquid $\vec{C}_U$ coincides with the direction of the vector of peripheral (velocity of following velocity of impeller $\vec{U}$. And vice versa, - negative, if vector $\vec{C}_U$ is directed opposite to direction of rotation of the screw conveyer.

It is known that with the presence of positive entry spin of flow to the vane pump the inlet flow conditions to the vanes of impeller are improved. Losses of inlet head in this case are decreased, due to which the anticavitation properties of the pump are improved. The cavitation specific speed is substantially raised. Furthermore, the installation of a return-circuit rig before the screw conveyer reduces the eddy counterflows, which appear in the channels of the screw conveyer during operation at uncalculated conditions.

However, the installation of return-circuit rig before the screw conveyer leads to additional losses of pressure to hydraulic friction and others, due to which static head at the screw conveyer inlet is reduced.

The placement of the return-circuit rig is expedient in the case when, first, this is allowed by axial dimensions and the weight of the construction; secondly, when the total effect is positive, i.e., when the efficiency of the pumping unit is raised from the placement of the return-circuit rig.

The third case

$$\alpha_1<0;\quad 0>\alpha_2>\alpha_1.$$

Boundary conditions

$$C_{1U}=\frac{C_{1m}}{\operatorname{tg}\beta_{1n}}-U;\quad C_{2U}=\frac{C_{2m}}{\operatorname{tg}\beta_{2n}}-U. \tag{7.150}$$

Equation (7.144) at boundary conditions (7.150) takes the form

$$H_{\tau}=U_{p}\left(\frac{C_{2m}}{\operatorname{tg}\beta_{2n}}-\frac{C_{1m}}{\operatorname{tg}\beta_{1n}}\right)+$$

$$+\frac{\varrho}{G'}\int_{S_1}^{S_2}\left(C_U\frac{\partial U}{\partial r}-\frac{C_U}{\operatorname{tg}\beta_n}\frac{\partial C_m}{\partial r}\right)F_m\sin\beta_n\,dS. \qquad (7.151)$$

The third case of operation of screw conveyer corresponds to negative entry spin of flow to the screw conveyer, at which the entry conditions of flow to the vanes of the impeller are made worse. As a result of large losses at the screw conveyer inlet the antifriction properties of the pumping unit are also made worse.

With negative exit spin ($\alpha_2 < 0°$) the screw conveyer operates under conditions of hydraulic stagnation and positive head does not develop. The value of H_T, obtained by formula (7.151), in this instance shows the amount of head lost with stagnation.

The fourth case

$$\alpha_1<0°;\quad 0°<\alpha_2>\alpha_1.$$

Boundary conditions

$$C_{1U}=\frac{C_{1m}}{\operatorname{tg}\beta_n}-U;\quad C_{2U}=U-\frac{C_{2m}}{\operatorname{tg}\beta_n}. \qquad (7.152)$$

Equation (7.144) at boundary conditions (7.152) takes the form

$$H_{\tau}=2U_p^2-U_p\left(\frac{C_{2m}}{\operatorname{tg}\beta_{2n}}+\frac{C_{1m}}{\operatorname{tg}\beta_{1n}}\right)+$$

$$+\frac{\varrho}{G'}\int_{S_1}^{S_2}\left(C_U\frac{\partial U}{\partial r}-\frac{C_U}{\operatorname{tg}\beta_n}\frac{\partial C_m}{\partial r}\right)F_m\sin\beta_n\,dS. \qquad (7.153)$$

The fourth case of operation of screw conveyer, just as the third, corresponds to negative entry spin of flow to the screw conveyer. At the screw conveyer exit the spin of flow is positive ($\alpha_2 > 0°$). Consequently, the screw conveyer operates in pumping mode and

develops positive head. The operation of screw conveyer under conditions of the fourth case is characterized by low efficiency and low cavitation specific speed.

From the examined four cases of operation of screw conveyer it follows that head characteristics of screw forepump depending on impeller inlet and exit flow angles are substantially different. In this case there was not considered the effect of the magnitude of angle of attack on the head of screw conveyer.

Basis of selection of boundary conditions for screw conveyer

In existing pumps of general and special machine building the screw forepumps are applied as a rule without return-circuit rigs at the inlet. Therefore, with derivation of the equation for head of the screw conveyer it is possible to consider $C_{1U} = 0$, which corresponds to the first case - boundary conditions (7.146).

For small α, i.e., such that it is possible to permit equality at screw conveyer inlet

$$\operatorname{tg}\beta_n = \frac{C_m}{U},$$

head characteristics for 2nd and 3rd cases approximately coincide with the characteristic of screw conveyer, which operates under conditions of the first case.

The fourth case is characteristic for axial impellers of variable pitch with large angle of rotation of flow at impeller vanes. The operation of such impellers is practically very little studied and is associated with instability of flow in channels, increased tendency toward the appearance of cavitation and large hydraulic losses.

Thus, by elaborating on the first case of operation of screw forepump as the basic, typical for centrifugal screw pumps, let us express equation (7.147) through screw conveyer parameters. For this let us use equalities

$$U_p=\frac{\pi D_p}{60}n;\quad C_m=\frac{1}{\varrho F_m}G',$$

where

$$D_p=\sqrt{\frac{D_ш^2+d_{вт}^2}{2}};\quad F_{mш}=\frac{\pi}{4k_ш}(D_ш^2-d_{вт}^2);$$

$k_ш$ - coefficient, considering the constraint of flow by screw conveyer vanes ($k_ш > 1$):

$$k_ш=\frac{1}{1-z\dfrac{\delta_{cp}}{\pi D_{cp}\sin\beta_{л.cp}}},\tag{7.154}$$

z - number of vanes of screw conveyer.

After substitution of equalities (7.154) in equation (7.147) and some conversions we receive the equation for theoretical head of screw forepump in transient and steady conditions:

$$H_т=\frac{\pi^2 D_p^2}{3600}n^2-\frac{\pi D_p}{60\varrho F_{2m}\operatorname{tg}\beta_{2\,л.p}}nG'+$$
$$+\frac{\varrho}{G'}\int\limits_{S_1}^{S_2}\left[\left(U_p-\frac{C_m}{\operatorname{tg}\beta_{л.p}}\right)\frac{\partial U_p}{\partial\tau}-\left(U_p-\frac{C_m}{\operatorname{tg}\beta_{л.p}}\right)\times\right.$$
$$\left.\times\frac{1}{\operatorname{tg}\beta_{л.p}}\frac{\partial C_m}{\partial\tau}\right]F_m\sin\beta_л\,dS.\tag{7.155}$$

7.19. Integration of Expressions for Dynamic Components of the Head of a Screw Forepump

Let us write the integrand of relationship (7.155) in the following form:

$$I_1-I_2=\frac{\varrho}{G'}\frac{\partial U}{\partial\tau}\int\limits_{S_1}^{S_2}\left(U_p-\frac{C_m}{\operatorname{tg}\beta_л}\right)F_m\sin\beta_л\,dS-$$
$$-\frac{\varrho}{G'}\frac{\partial C_m}{\partial\tau}\int\limits_{S_1}^{S_2}\left(U_p-\frac{C_m}{\operatorname{tg}\beta_л}\right)F_m\cos\beta_л\,dS.$$

By stubstituting here the values of time derivatives:

$$\frac{\partial U}{\partial \tau}=\frac{\pi D_p}{60}\frac{\partial n}{\partial \tau};\quad \frac{\partial C_m}{\partial \tau}=\frac{1}{\varrho F_m}\frac{\partial G'}{\partial \tau} \tag{7.156}$$

and considering equalities (7.154), we obtain the expressions for the first and second integrals:

$$I_1=\frac{\varrho\pi D_p}{60}\frac{\partial n}{\partial \tau}\int_{S_1}^{S_2}\left(\frac{\pi D_p}{60}\frac{n}{G'}-\frac{1}{\varrho F_m \operatorname{tg}\beta_n}\right)F_m \sin\beta_n\, dS; \tag{7.157}$$

$$I_2=\frac{\partial G'}{\partial \tau}\int_{S_1}^{S_2}\left(\frac{\pi D_p}{60}\frac{n}{G'}-\frac{1}{\varrho F_m \operatorname{tg}\beta_n}\right)\cos\beta_n\, dS. \tag{7.158}$$

Into expressions for the first and second integrals enters the flow angle β_n, which under transient conditions varies with time along the length of the channel of screw conveyer (for noninterrupted flow the last circumstance concerns only a variable-pitch screw converter).

As was noted in Section 7.17, angle β_n is uniquely defined through parameters $U(\tau)$ and $C_m(\tau)$ or, which is the same, - through $n(\tau)$ and $G'(\tau)$ for any fixed C_U. Let us show that at zero angle of attack the dynamic components of head of constant-pitch screw conveyer are equal to zero. Let us express the value of tg β_n through geometric and energy parameters of the pump.

From the velocity triangle at the screw conveyer inlet when C_{1U} = 0 with consideration of relationship (7.154) we have

$$\operatorname{tg}\beta_{1n}=\frac{C_{1m}}{U}=\frac{60}{\varrho F_{1m}\pi D_p}\frac{G'}{n}. \tag{7.159}$$

at the screw conveyer exit approximately for small α and θ it is possible to write

$$\operatorname{tg}\beta_{2n}=\frac{C_{2m}}{U-C_{2m}\operatorname{tg}\alpha}\approx\frac{60}{\varrho F_{2m}\pi D_p}\frac{G'}{n}. \tag{7.160}$$

As follows from equalities (7.159) and (7.160), the difference of screw conveyer inlet and exit flow angles, which correspond to the same (calculated) diameter of screw conveyer,

$$\beta_{2л}-\beta_{1л}$$

is determined by the difference of inlet and exit cross-sectional areas

$$\frac{1}{F_{2mm}}-\frac{1}{F_{1mm}}.$$

Consequently, with accepted assumptions the flow angle during noninterrupted flow in impeller channels and assigned n and G is determined completely by the geometry of screw conveyer.

For any i-th section, perpendicular to the axis of screw conveyer, at zero angle of attack there is valid equality

$$\operatorname{tg}\beta_{iл}=\frac{G_0}{\varrho F_{mi}\pi D}\,\frac{G'}{n}. \qquad (7.161)$$

By substitution of expressions (7.161) into equations (7.157) and (7.158) we obtain

$$I_1=0;\quad I_2=0. \qquad (7.162)$$

Thus, at zero angle of attack a constant-pitch screw conveyer develops head neither at steady nor transient conditions.

Variable-pitch screw conveyer

For variable-pitch screw conveyer, when

$$\beta_{2л}\neq\beta_{1л};\quad F_{2m}\neq F_{1m}, \qquad (7.163)$$

it is not possible to take integrals (7.157) and (7.158) in general form. As experience shows, the values of these integrals with sufficient accuracy for practical purposes can be determined by numerical integration by the method of mean ΔS for sections:

$$I_1=\left(\frac{\rho\pi^2 D_p^2}{3600}\,\frac{n}{G'}\sum_{j=1}^{n}F_{mj}\sin\beta_{n.p\,j}\Delta S_j-\right.$$

$$\left.-\frac{\pi D_p}{60}\sum_{j=1}^{n}\Delta S\cos\beta_{n.p\,j}\right)\frac{dn}{d\tau}. \qquad (7.164)$$

Similarly for the second integral

$$I_2=\left(\frac{\pi D_p}{60}\,\frac{n}{G'}\sum_{j=1}^{n}\Delta S_j\cos\beta_{n.p\,j}-\right.$$

$$\left.-\frac{1}{\rho}\sum_{j=1}^{n}\frac{\Delta S_j\cos\beta_{n.p\,j}}{F_{mj}\,\mathrm{tg}\,\beta_{n.p\,j}}\right)\frac{dG'}{d\tau}. \qquad (7.165)$$

The curvature of channels of screw conveyers in existing pumps is small, not exceeding magnitude

$$\Delta\beta=\beta_{2л}-\beta_{1л}\leqslant 8^\circ\div 15^\circ.$$

The vanes of variable-pitch screw conveyers are profiled usually by the method of arcs of circumferences, at which the change of the magnitude of center line angle of the vane along its length occurs according to linear law

$$\frac{d\beta_л}{dS}=\mathrm{const}. \qquad (7.166)$$

The law of change of the active flow area in the meridion section along the length of the screw conveyer channels in this case can differ from linear because of the different thickness of vanes. Usually this difference is not substantial and it can be disregarded.

The remarks made allow substituting the current values of F_{mj} and $\beta_{n.p\,j}$ in expressions (7.164) and (7.165) without large error by their mean values, assuming that the laws of change of F_{mj} and $\beta_{n.p\,j}$ along the length of the channel l_{oc} of screw conveyer are linear. Then expressions (7.164) and (7.165) will take the form

$$I_1=\left(\frac{\varrho\pi^2 D_p^2 F_{m\,ш.ср}\, l_{oc}}{3600}\,\frac{n}{G'}-\frac{\pi D_p l_{oc}}{60\,\mathrm{tg}\,\beta_{л.эк}}\right)\frac{dn}{d\tau};\qquad (7.167)$$

$$I_2=\left(\frac{\pi D_p l_{oc}}{60\,\mathrm{tg}\,\beta_{л.эк}}\,\frac{n}{G'}-\frac{l_{oc}}{\varrho F_{m\,ш.ср}\,\mathrm{tg}^2\beta_{л.эк}}\right)\frac{dG'}{d\tau},\qquad (7.168)$$

where $\beta_{л.эк}$ - equivalent vane angle of screw conveyer:

$$\beta_{л.эк}=\beta+\mathrm{arctg}\,\frac{2f}{L};\qquad (7.169)$$

β - the screw conveyer vane chord angle; f - deflection of vane on 0.5 the length of chord L.

7.20. Theoretical Head of Screw Forepump for Transient and Steady State

The expression for theoretical head of a variable-pitch screw conveyer in transient mode [equation (7.155)] takes the form

$$H^{i}_{т.ш}=A_0 n^2-\frac{B_0}{\varrho}\,nG'+\left(E_0\,\varrho\,\frac{n}{G'}-F_0\right)\frac{dn}{d\tau}-\left(M_0\,\frac{n}{G'}-N_0\,\frac{1}{\varrho_0}\right)\frac{dG'}{d\tau},\qquad (7.170)$$

where

$$\left.\begin{aligned}
A_0&=\frac{\pi^2 D_p^2}{3600};\\
B_0&=\frac{\pi D_p}{60 F_{2m\,ш}\,\mathrm{tg}\,\beta_{л.эк}};\\
E_0&=\frac{\pi^2 D_p^2 F_{m\,ш.ср} L_{ср}\sin\beta_{л.эк}}{3600};\\
F_0&=\frac{\pi D_p L_{ср}\sin\beta_{л.эк}}{60\,\mathrm{tg}\,\beta_{л.эк}};\\
M_0&=\frac{\pi D_p L_{ср}\sin\beta_{л.эк}}{60\,\mathrm{tg}\,\beta_{л.эк}};\\
N_0&=\frac{L_{ср}\cos\beta_{л.эк}}{F_{m\,ш.ср}\,\mathrm{tg}\,\beta_{л.эк}}.
\end{aligned}\right\}\qquad (7.171)$$

7.21. Hydraulic Losses of Head in Vane Pumps

We distinguish the following types of hydraulic losses in vane pumps:

1) hydraulic friction losses;

2) losses connected with conversion of dynamic head into static (channel exit conicity losses);

3) hydraulic shock losses in the presence of leakage of flow to impeller vanes and with exit from impeller.

The first two types of hydraulic losses are observed during operation of a pump both at design and at partial load conditions. The numerical value of these losses is proportional to the square of the relationship of current flow rate to design, corresponding to shock-free, i.e.,

$$\Delta h_{1,2} \backsim \left(\frac{Q_i}{Q_p}\right)^2. \qquad (7.177)$$

The third type of losses is characteristic for partial load conditions. The amount of hydraulic shock losses is proportional to the square of the difference of design and current values of flow rate, i.e.,

$$\Delta h_3 \sim \left(1 - \frac{Q_i}{Q_p}\right)^2. \qquad (7.178)$$

According to such a presentation the hydraulic shock losses in design condition ($Qi = Q_p$) are equal to zero. In partial load conditions the hydraulic shock losses depending on the magnitude of angle of attack can reach large values.

It is known that a constant-pitch screw conveyer develops head because of the presence of the angle of attack. The following optimum values of angles of attack of vane pumps are established experimentally:

Constant-pitch screw conveyer

For constant-pitch screw conveyer

$$\beta_{1л}=\beta_{2л}=\beta_{л}=\text{const};\quad F_{1m\,ш}=F_{2m\,ш}=F_{m\,ш}=\text{const};$$
$$L_{cp}\sin\beta_{л.cp}=l_{oc};\quad \operatorname{tg}\beta_{л.p}=\frac{S}{\pi D_p}, \tag{7.172}$$

where S - pitch of helical line of screw conveyer. Integrals (7.157) and (7.158) are equal to

$$I_1=\left(\frac{\varrho\pi^2 D_p^2 F_{m\,ш}\, l_{oc}}{3600}\,\frac{n}{G'}-\frac{\pi D_p l_{oc}}{60}\right)\frac{dn}{d\tau}; \tag{7.173}$$

$$I_2=\left(\frac{\pi^2 D_p^2\, l_{oc}}{60S}\,\frac{n}{G'}-\frac{\pi^2 D_p^2\, l_{oc}}{\varrho F_{m\,ш} S^2}\right)\frac{dG'}{d\tau}. \tag{7.174}$$

The theoretical head of constant-pitch screw conveyer at steady and transient conditions [equation (7.155)] has the form

$$H_{т.ш}=A_0 n^2-\frac{B_0}{\varrho}\,nG'+\left(\varrho F_0\,\frac{n}{G'}-F_0\right)\frac{dn}{d\tau}-$$
$$-\left(M_0\,\frac{n}{G'}-N_0\,\frac{1}{\varrho}\right)\frac{dG'}{d\tau}, \tag{7.175}$$

where

$$\left.\begin{aligned}
A_0&=\frac{\pi^2 D_p^2}{3600};\\
B_0&=\frac{\pi^2 D_p^2}{60 F_{m\,ш}};\\
E_0&=\frac{\pi^2 D_p^2 F_{m\,ш}\, l_{oc}}{3600};\\
F_0&=\frac{\pi D_p l_{oc}}{60};\\
M_0&=\frac{\pi^2 D_p^2\, l_{oc}}{60S};\\
N_0&=\frac{\pi^2 D_p^2\, l_{oc}}{F_{m\,ш} S^2}.
\end{aligned}\right\} \tag{7.176}$$

for constant-pitch screw conveyers $i_{\text{ш}} = 5°-8°$;

for variable-pitch screw conveyers $i_{\text{ш}} = 1°-3°$;

for centrifugal pumps $i_{\text{ц}} = 6°-12°$.

The existing methods of calculation of hydraulic losses in pumps give the possibility of obtaining analytical expressions for determination of actual head characteristics under steady state, which provide accuracy acceptable for practical purposes.

It is established that losses on hydraulic friction against walls of the flow passage of a pump are not the determining among other types of losses. The flow in impeller channels of a vane pump is essentially nonuniform, due to which considerable losses of energy take place. Furthermore, as a result of the effect of a finite number of impeller vanes the liquid enters the outlet device after exiting the impeller channels by portions. There occurs exchange by pulses of particles of liquid of different energies, which is accompanied by substantial losses of head. The feed admission especially shows up under conditions of the small pump capacities. Hydraulic losses in the outlet devices of vane pumps are the determining among other types of losses.

7.22. Model of Motion of Liquid in the Elements of the Hydraulic Passage of Screw Forepump

The hydraulic passage of a screw forepump consists of the following basic elements:

- impeller (screw conveyer);

- inlet device;

- outlet device;

- housing of forepump.

The impeller of forepump with z vanes can be made both according to the law of forced circulation

$$\frac{C_U}{r} = \text{const}, \tag{7.179}$$

and according to the law of natural circulation

$$C_U r = \text{const}. \tag{7.180}$$

In axial-flow impellers, made according to law (7.179), the greatest values of peripheral and relative velocities of motion of liquid in impeller channels and past the impeller take place in peripheral layers, where static pressures are maximum along the cross section. Consequently, in axial-flow forepumps, made according to law (7.179), a mechanism is automatically created for cavitation suppression in peripheral layers of liquid (both in channels of axial-flow impeller and at some distance after exiting it). For reliable cavitation suppression in screw conveyers it is recommended to increase the cascade thickness to values τ = 1.8-2.3.

In axial-flow forepumps, made according to law (7.180), the liquid velocity reaches the greatest values on the impeller hub, where static pressures are minimum and the danger of cavitation stall of such impellers is considerable.

Inlet device is made with arbitrary change of the cross-sectional area along the length of the passage.

The motion of liquid in the inlet device can occur both without entry spin of flow, when there are no devices for flow spin (C_{1U} = 0), and with entry spin, when there are special devices, which create spin ($C_{1U} \neq 0$).

Outlet device is made either in the form of a smooth-wall cylindrical tube, in which liquid moves according to the law of natural circulation, or represents a section with variable cross-sectional area and local resistances, intended for decrease of intensity of counterflows. The law of natural circulation in this case is not fulfilled.

The housing of screw forepump is made in accordance with the shape of the generating line of screw conveyer and can be cylindrical, conical, stepped, etc.

At transient operating conditions of the forepump the distribution of pressures and velocity fields in every section of the passage varies with time, which substantially hampers computations. Therefore, the calculation of hydraulic losses in the flow passage of screw forepump in transient conditions is performed in quasi-stationary formulation.

Figure 7.6 shows the diagram of motion of liquid in screw conveyer channels. The exit velocity triangles are combined with inlet velocity triangles.

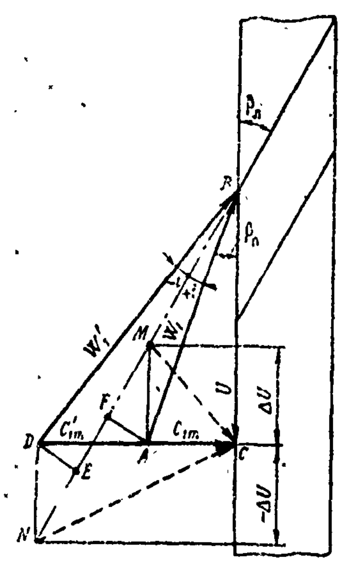

Fig. 7.6. For calculation of screw forepump.

7.23. Calculation of Hydraulic Losses in Screw Forepump

The losses of head in the forepump are made up of losses in the impeller ($\Delta h_{\text{р.к}}$), losses in inlet ($\Delta h_{\text{вх}}$) and outlet ($\Delta h_{\text{вых}}$) devices and in the housing ($\Delta h_{\text{к}}$) of forepump:

$$\sum_i \Delta h_i = \Delta h_{\text{р.к}} + \Delta h_{\text{вх}} + \Delta h_{\text{вых}} + \Delta h_{\text{к}}. \tag{7.181}$$

Hydraulic losses in the impeller

The basic types of hydraulic losses of head in the impeller of forepump are:

a) hydraulic shock losses, proportional to quantity AF:

$$\Delta h_1 = \frac{\xi_{\text{уд}}}{2}\left(U^2 \sin^2\beta_{\text{л}} - 2UC_{1m}\sin\beta_{\text{л}}\cos\beta_{\text{л}} + C_{1m}^2\cos^2\beta_{\text{л}}\right), \tag{7.182}$$

where $\xi_{\text{уд}}$ - coefficient of hydraulic shock losses;

b) losses on secondary longitudinal motion of liquid in screw conveyer channels, proportional to quantity $FM = \Delta U \cos\beta_{\text{л}}$:

$$\Delta h_2 = \frac{\xi_{\text{вт}}}{2}\left(U^2\cos^2\beta_{\text{л}} - 2U\frac{C_{1m}}{\mathrm{tg}\,\beta_{\text{л}}}\cos^2\beta_{\text{л}} + \frac{C_{1m}^2}{\mathrm{tg}^2\beta_{\text{л}}}\cos^2\beta_{\text{л}}\right), \tag{7.183}$$

where $\xi_{\text{вт}}$ - coefficient of losses on secondary flows in screw conveyer channels;

c) hydraulic friction losses:

$$\Delta h_3 = \xi\frac{L_{\text{ср}}}{D_{\text{г.ш}}}\frac{W_{\text{ср}}^2}{2}, \tag{7.184}$$

where ξ - coefficient of hydraulic friction losses; $L_{\text{ср}}$ - length of vane along the average diameter of screw conveyer; $D_{\text{г.ш}}$ - hydraulic diameter of screw conveyer;

d) eddy formation losses:

$$\Delta h_4=\xi_{\text{вихр}}\frac{\pi^2 D_{\text{р}}^2}{7200}\left(1-\frac{G'_l}{G'_{\text{ном}}}\right)^2 n^2, \qquad (7.185)$$

where $\xi_{\text{вихр}}$ - coefficient of eddy formation losses;

e) exit conicity losses of screw conveyer channels: for variable-pitch screw conveyer

$$\Delta h_5=\frac{\xi_{\text{д}}}{2\rho^2}\left(\frac{1}{F_{1m}^2\sin^2\beta_{1\text{р}}}-\frac{1}{F_{2m}^2\sin^2\beta_{2\text{р}}}\right)G'^2, \qquad (7.186)$$

where $\xi_{\text{д}}$ - coefficient of exit conicity losses of screw conveyer channels; for constant-pitch screw conveyer (under the assumption of $F_{1m\,\text{ш}}=F_{2m\,\text{ш}}$)

$$\Delta h_6=0. \qquad (7.187)$$

Total hydraulic losses in the screw forepump impeller are obtained by summation of the separate types of losses:

$$\Delta h_{\text{р.к}}=\sum_{i=1}^{6}\Delta h_i. \qquad (7.188)$$

Hydraulic losses in the inlet device of screw forepump

The main types of losses in the inlet device are:

- hydraulic friction losses;

- losses on overcoming local resistances.

Thus,

$$\Delta h_{\text{вх}}=\xi_{\text{вх}}\frac{l_{\text{вх}}}{D_{\text{г.вх}}}\frac{C_{\text{вх}}^2}{2}+\xi_{\text{м}}\frac{C_{\text{вх}}^2}{2}, \qquad (7.189)$$

where $\xi_{вх}$, $\xi_м$ - coefficients of losses on friction and overcoming local resistances respectively; $D_{г.вх}$ - hydraulic diameter of inlet pipe.

Hydraulic losses in the outlet device of screw forepump

The main types of losses in the outlet device are:

- hydraulic friction losses;

- losses on overcoming local resistances;

- losses on compression of vapor-gas volumes;

- losses on mixing of jets of various energies.

Thus,

$$\Delta h_{вых}=\xi_{вых}\frac{l_{вых}}{D_{г.вых}\cos\alpha_2}\frac{C^2_{вых}}{2g}+\xi_м\frac{C^2_{вых}}{2}+\Delta h_{сж}+\Delta h_{см}. \qquad (7.190)$$

7.24. Actual Head of Screw Forepump for Transient and Steady Conditions

The actual head of vane pump is determined either by multiplication of theoretical head by the hydraulic efficiency of the pump

$$H=H_т\eta_г, \qquad (7.191)$$

or by subtraction of hydraulic losses in the pump from theoretical head

$$H=H_т-\sum_i\Delta h_i. \qquad (7.192)$$

In the first case it is necessary to obtain the calculation relationship for hydraulic efficiency, in the second - for total hydraulic losses in the pump.

We will use relationship (7.192).

The expression for actual head of screw forepump in transient and steady states is obtained by subtraction of total hydraulic losses taking into consideration dynamic phenomena in inlet and outlet devices of forepump from the equation for theoretical head of screw conveyer (7.170) or (7.175):

$$H_{ш}=An^2-BnQ-CQ^2+\left(E\frac{n}{Q'}-F\right)\frac{dn}{d\tau}-\left(M\frac{n}{Q'}-N\right)\frac{dQ'}{d\tau}, \quad (7.193)$$

where

$$\left.\begin{array}{l} A=\xi_1 D_p^2; \quad B=\xi_2\frac{D_p}{F_{mш}\,\mathrm{tg}\,\beta_{л.р}}; \quad C=\xi_3\frac{1}{F^2_{mш}\,\mathrm{tg}^2\,\beta_{л.р}}; \\ E=\frac{\pi^2 D_p^2}{3600}F_{mш}l_{oc}; \quad F=\frac{\pi D_p}{60}l_{oc}; \quad M=\frac{\pi^2 D_p^2 l_{oc}}{60\cdot S}; \\ N=\frac{\pi^2 D_p^2 l_{oc}}{F_{mш}S^2}. \end{array}\right\} \quad (7.194)$$

Coefficients ξ_1, ξ_2, ξ_3 are determined by calculation with insertion of corrections for agreement with experimental data.

For steady state the pressure of screw forepump is determined from equation (7.193) at condition

$$\frac{dn}{d\tau}=\frac{dQ}{d\tau}=0$$

and in coordinates $\frac{H}{n^2}=f\left(\frac{Q}{n}\right)$ is written so:

$$\frac{H_{ш}}{n^2}=\xi_1 D_p^2-\xi_2\frac{D_p}{F_{mш}\,\mathrm{tg}\,\beta_{л.р}}\frac{Q}{n}-\xi_3\frac{1}{F^2_{mш}\,\mathrm{tg}\,\beta_{л.р}}\frac{Q^2}{n^2} \quad (7.195)$$

or

$$\frac{H_{ш}}{n^2}=\xi_1 D_p^2-\xi^*\frac{D_p^2}{F_{mш}S}\frac{Q}{n}-\xi_3^*\frac{D_p^2}{F^2_{mш}S^2}\frac{Q^2}{n^2}, \quad (7.196)$$

and in coordinates $\psi=f_2(q)$ is written so:

$$\psi=\xi_{01}-\xi_{02}q-\xi_{03}q^2, \tag{7.197}$$

where $\psi=\frac{H_{ш}}{U^2}$ - coefficient of forepump head; $q=\frac{Q}{Q_{т0}}$ - flow coefficient; $Q_{т0}=\frac{F_{ш ш}Sn}{60\cdot10^{-3}}$ - capacity at zero theoretical head of screw forepump.

C. GAS TURBINE OF TURBOPUMP UNIT

7.25. Determination of Turbine Power

In liquid-propellant rocket engines single-stage and two-stage gas turbines are applied. In powerful engines, made open configuration, as a rule action two-stage turbines are applied. In low-thrust engines of this configuration single-stage turbines are applied.

In engines with turbogas afterburning single-stage axial-flow reaction and centripetal turbines are used. In certain cases axial-flow action single-stage turbines are applied. Efficiency of such turbines is not high, however, in configurations with gas afterburning this does not lead to lowering of engine thrust.

Many works have been dedicated to the detailed examination of questions of theory of turbines. Within this book we will be limited to examination of the derivation of equations of power and moment, being developed by one stage of an axial-flow turbine.

With derivation of equations of power or moment of the turbine we consider the design features and operating conditions of nozzle rings, rotor wheel vanes, friction between turbogas, the wheel disk and housing walls, discharge conditions of turbogas from the housing, etc.

Initial relationship

The connection between power, developed by the turbine, and moment on the shaft is determined by known equation

$$M=\frac{N}{\omega}. \tag{7.198}$$

Under steady state conditions on the turbine shaft depending on power, transferred by turbogas to the rotor blades of the turbine, it is determined by formula

$$M_{т}=\frac{N_{л}}{\omega}\eta_{л.т}, \tag{7.199}$$

where $\eta_{л.т}$ - efficiency coefficient, considering the losses appearing during transmission of power from blades to the shaft, namely: losses on friction of blades and disk against turbogas, located in the turbine housing; losses caused by parasitic circulation and turbulent motion of turbogas between the disk and housing; mechanical friction in bearings and sealing devices, etc.

At nonsteady state instead of relationship (7.199) we will have

$$M_{т}=\frac{N_{л}}{\omega}-I_{т}\dot{\omega}-m_{г}r_{л}\dot{C}_{г}, \tag{7.200}$$

where $m_{г}$ - mass of turbogas, flowing along the internal cavity of the turbine; $\dot{C}_{г}$ - averaged value of acceleration of motion of turbogas; $r_{л}$ - averaged value of the radius of wheel blades.

The last component in relationship (7.200) can be represented by sum:

$$m_{г}r_{л}\dot{C}_{г}=m_{c}r_{c}\dot{C}_{c}+m_{л}r_{л}\dot{C}_{л}+m_{к}r_{к}\dot{C}_{к}. \tag{7.201}$$

In the right side of equation (7.201) the addend characterizes the effect of inertness of turbogas moving in nozzles, the second component - reflects the inertness of turbogas in the interblade channels and the third component - in the turbine housing.

Calculations show that the third component in equality (7.200) is smaller than the second. Considering the inertness of turbogas flow by coefficient ξ_{Γ}, we obtain

$$M_{\text{т}}=\frac{N_{\text{т}}}{\omega}-\xi_{\text{г}}I_{\text{т}}\dot{\omega}. \tag{7.202}$$

The moment on the turbine shaft can be calculated by formula

$$M_{\text{т}}=P_{U}r_{\text{л}}\eta_{\text{м.т}}, \tag{7.203}$$

where P_U - peripheral force, appearing as a result of interaction between turbogas and rotor wheel blades; $r_{\text{л}}$ - average value of radius of interblade channels.

Let us examine the interblade channel, limited by two blades and walls of the rim and disk. With flow about the profile of blades by turbogas there is revealed an uneven pressure field, acting both on the concave side and the back of blades. The amount of pressure will be changed along the length of the blade (i.e., along the radius) and along the length of the interblade channel.

The moment on blades, calculated for one channel,

$$M_{\text{л1}}=\int_{r_1}^{r_2}\int_{0}^{l} p_{\text{п}}(r,\,l)\cos[\beta_{\text{л}}(r,\,l)]\,r\,dr\,dl-\int_{r_1}^{r_2}\int_{0}^{l} p_{3}(r,\,l)\cos[\beta_{\text{л}}(r,\,l)]\,r\,dr\,dl. \tag{7.204}$$

Here $p_{\text{п}}(r,l)$ - pressure, affecting the concave side of the blade; $\beta_{\text{л}}$ - angle between the tangent to the considered blade and tangent to the circumference of radius r, measured in the plane perpendicular to r; r_1, r_2 - the smallest and largest radii of interblade channel:

$$r_2-r_1=h,$$

where h and l - height and length of interblade channel.

The second term in the right side of equation (7.204) characterizes the magnitude of moment on the blades, caused by the action of turbogas on the blade back.

The amount of moment $M_{\pi 1}$ will depend on the location of the considered interblade channel relative to the turbine nozzles, consequently, $M_{\pi 1}(\varphi)$, where φ - angle of rotation of rotor wheel, which characterizes the place of the location of the considered interblade channel.

Thus, the moment on the shaft

$$M_{\tau}=\frac{\sum M_{\pi 1}(\varphi)}{\eta_{\pi.\tau}}. \qquad (7.205)$$

More precise research of the process of appearance of the moment on the turbine shaft requires the use of equations (7.204) and (7.205). For determination of M_T when performing engineering calculations we use formula (7.203).

Determination of peripheral and radial forces, affecting the blades

In the flow area of a single-stage axial-flow turbine the element of mass dm of turbogas moves with speed C. If the average pressure along the section of flow at the interblade channel inlet is equal to the exit pressure, then

$$dPdt=d(Cdm), \qquad (7.206)$$

where dP - elementary force of action on blades; $dPdt$ - elementary force pulse; Cdm - momentum of elementary mass; $d(Cdm)$ - change of momentum.

Under steady state conditions

$$\frac{dm}{dt}=G=\text{const},$$

consequently,

$$dP=GdC.$$

By integrating, we obtain

$$P=\int GdC.$$

In the direction of motion the blades will be affected by peripheral force

$$P_U=-\int Gd(C\cos\alpha), \qquad (7.207)$$

where α - the angle of direction of motion of turbogas in interblade channels.

The axial force, affecting the blades,

$$P_t=-\int Gd(C\sin\alpha). \qquad (7.208)$$

The right side of (7.207) should be integrated $C_1 \cos \alpha_1$ to $C_2 \cos \alpha_2$ (Fig. 7.7). Having completed integration, we obtain

$$P_U=G(C_1\cos\alpha_1+C_2\cos\alpha_2). \qquad (7.209)$$

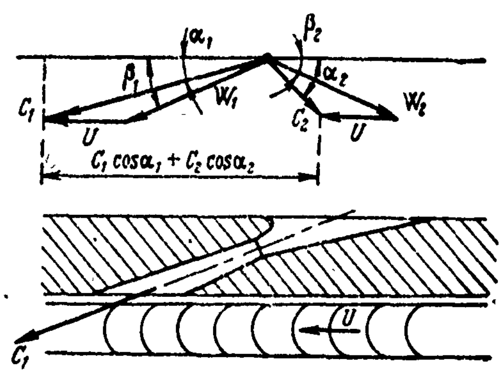

Fig. 7.7. The velocity triangles for the flow area of a turbine.

Derivation of the working formula for determination of turbine power

By using equations (7.199) and (7.203) for determination of power on blades, we obtain

$$N_{\text{л}}=P_U r_{\text{л}} \omega. \qquad (7.210)$$

Whence the power on the shaft

$$N_{\text{т}}=P_U r_{\text{л}} \omega \eta_{\text{в.т}}. \qquad (7.211)$$

By substituting here the value of peripheral force, which affects the rotor wheel blades, from equation (7.209) we find

$$N_{\text{т}}=\omega r_{\text{л}}(C_1\cos\alpha_1+C_2\cos\alpha_2)G\eta_{\text{в.т}}. \qquad (7.212)$$

For further conversions it is necessary to know the motion of turbogas in the flow area of interblade channels. As an example let us examine the results of calculation, presented in the form of velocity triangles in Fig. (7.7), whence it follows that

$$C_2\cos\alpha_2=W_2\cos\beta_2-\omega r_{\text{л}}. \qquad (7.213)$$

The relative exit velocity of working medium from the flow area of interblade channels

$$W_2=\psi\sqrt{2H\rho-W_1^2}, \qquad (7.214)$$

where ψ - coefficient, considering losses in interblade channels; H - heat drop on the turbine; ρ - the degree of reaction of blades; W_1 - relative inlet velocity of working medium into the flow area of the turbine.

From velocity triangles it is evident that

$$W_1=\frac{C_1\cos\alpha_1-\omega r_{\text{л}}}{\cos\beta_1}. \qquad (7.215)$$

The discharge velocity of turbogas from nozzles

$$C_1=\varphi\sqrt{2\Delta i_c+C_0^2}, \qquad (7.216)$$

where ϕ - velocity coefficient, which characterizes losses in nozzles; Δi_c - polytropic change of heat content of turbogas in nozzles.

Now instead of equality (7.212) we will have

$$N_\text{т}=\omega r_\text{д}\left[\varphi\cos\alpha_1\sqrt{2\Delta i_c+C_0^2}+\right.$$
$$\left.+\psi\cos\beta_2\sqrt{2H_Q-\left(\frac{\varphi\cos\alpha_1\sqrt{2\Delta i_c+C_0^2}-\omega r_\text{д}}{\cos\beta_1}\right)^2}-\omega r_\text{д}\right]\eta_{\text{д.т}}G. \qquad (7.217)$$

If for all parameters, which enter (7.217), besides ω and G we take their average values, then the formula of power is reduced to the form

$$N_\text{т}=(A-B\omega)\omega G, \qquad (7.218)$$

where

$$A=r_\text{д}\left[\varphi\cos\alpha_1\sqrt{2\Delta i_c+C_0^2}+\psi\cos\beta_2\times\right.$$
$$\left.\times\sqrt{2H_Q-\left(\frac{\varphi\cos\alpha_1\sqrt{2\Delta i_c+C_0^2}-\omega r_\text{д}}{\cos\beta_1}\right)^2}\right]\eta_{\text{д.т}}. \qquad (7.219)$$

$$B=r_\text{д}^2\eta_{\text{д.т}}. \qquad (7.220)$$

The flow rate of turbogas under steady state conditions

$$G=\frac{a\sum F_{\text{кр.т}}}{\sqrt{RT_\text{т}}}p_\text{т}. \qquad (7.221)$$

Here $\Sigma F_{\text{кр.т}}$ - the total throat area of turbine nozzle rings; R, $T_{\text{т}}$, $p_{\text{т}}$ - parameters of turbogas before the turbine nozzle ring inlet;

$$a=\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\sqrt{2\frac{n}{n+1}}, \tag{7.222}$$

where n - politropic index.

$R, T_T, p_T,$ -

;

(7.222)

SECTION III

POWER PLANTS

CHAPTER VIII

OPERATING CONDITIONS AND CONFIGURATIONS OF POWER PLANTS

8.1. Engine Operating Conditions

The third section is dedicated to examination of some questions, which characterize the interconnection of processes, proceeding in the combustion chamber and in units of the feed system.

It is possible to distinguish three characteristic operation conditions of an engine: starting, operating conditions and shutdown. In fulfilling these conditions both the combustion chamber and units of the feed system take part.

Starting is the complex combination of parallel and series executed operations. These include preparation of tanks and pressurization system, actuation of valves, filling of lines by propellant components, ignition, rise of pressure in the chamber at variable operation conditions of turbopump unit and the character of motion of liquids in flow passages, etc.

Starting is considered according to data of a manufactured or designed engine and with consideration of the features of its operation.

Starting is considered with the aid of a system of ordinary differential equations on analog or digital computers.[1] In a number of organizations engineering methods of calculation have been developed and standard programs compiled for digital computers. The running of a number of programs, terminating in engine starting, is called engine starting.

Under engine operating conditions the engine parameters are changed under the action of commands, proceeding from the control system, under the action of external factors and because of execution of programmed control.

Modern engines are equipped, as a rule, with a tank emptying system (SOB) [СОБ] and an apparent speed regulator (RKS) [РКС].

The SOB can be analog or digital, moreover the most complete fuel consumption will be under the action on the part of the SOB during the entire period of operation of the engine. Under the effect of various actions the component ratio deviates with time from its optimum value, in this case the SOB on the average provides the optimum value of component ratio, i.e.,

$$\frac{1}{t_0}\int_0^{t_0} k_1(t)\,dt \to K_{\text{опт}}.$$

Depending on the accepted program of change of flight velocity of a rocket and the action of disturbing factors of the system the RKS feeds commands, as a result of which the engine thrust, varying with time, brings the apparent velocity of the rocket to its programmed value.

---

[1]Calculation of engine starting with the aid of "Strela" digital computer was performed by the author in 1955; despite the limited quantity of utilized differential equations it was possible then to obtain good agreement of calculation data with results of flight tests [67].

The actual velocity of the rocket

$$\bar{V}=\int_0^{t_k}\dot{\bar{V}}dt;$$

apparent velocity

$$\bar{V}_{\text{к}}=\int_0^{t_k}(\dot{\bar{V}}+\bar{g})\,dt.$$

In some engines cruise condition control is carried out by a system, which includes specialized computers.

Calculation of engine under operating conditions is performed with the aid of algebraic and differential equations, which describe the processes in the combustion chamber and in elements of the feed system.

Depending on the type of equations and the goal of research we use analog or digital computers. The first are applied when processes are described with the aid of ordinary differential and algebraic equations. The solutions are obtained in the form of charts on oscillographs, tables or the course of solution is seen on the screen of an oscilloscope. When using analog computers it is possible to easily change the values of coefficients of equations and look over many variants for a short time.

Digital computers (TsVM) [ЦВМ] are suitable for investigating and both ordinary differential equations and equations in partial derivatives. The accuracy of computation is great, but during prolonged computation the error increases in time so that sometimes even the stability of computation is lost.

The accuracy of calculation does not only depend on the perfection of approximate mathematical methods and computer technology. Sometimes the degree of perfection of the basic mathematical instrument, our inadequate knowledge of the processes themselves, proceeding in the power plant, have decisive value.

The process of engine shutdown, determination of aftereffect pulse and its scattering are examined in a number of works (see, for example, [15], [64]), therefore only the minimum necessary attention is given to this question in the book.

8.2. Configurations of Power Plant

Figure 8.1 depicts the configuration of an engine with tanks, loaded by high pressure. The feed system of such an engine is called pressure, gas pressure or loaded. In the examination of engines, made in the considered configuration, it is established that the relative engine weight (the ratio of the weight of the engine construction to the weight of propellant, which burns up during the time of rocket flight) sharply increased with increase of pressure in the combustion chamber. The relationship of relative weight to pressure in the chamber has asymptotic character: as the pressure in the chamber approaches pressure in the accumulator the relative weight approaches infinity. In connection with the contemporary tendency toward increase of pressure in the chamber the feed system with loaded tanks is not promising from this viewpoint [4], [15].

Calculations show that the relative engine weight is little changed with increase of thrust; with increase of rocket size, mass of useful load and flight distance the weight qualities of the engine are not improved. Consequently, it is most expedient to apply engines equipped with feed systems of the considered type in stand devices, where the relative weight is not the criterion, which characterizes the quality of an engine, and also in small rockets, at relatively low pressure in the combustion chamber.

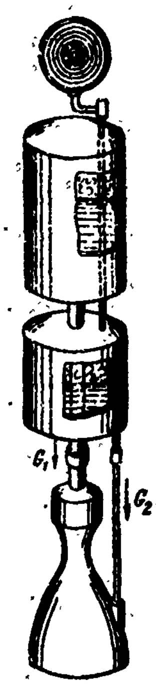

Fig. 8.1. Diagram of liquid-propellant rocket engine with gas pressure feed system.

The specific weight of the engine (ratio of engine weight to thrust) and relative weight are used when determining the qualities and the degree of perfection of the power plant. Bench tests differ from flight tests by the intensity of the action of external mass forces (under bench conditions $\dot{V} = 0$), by the rigidity of connections between the power plant and elements of the stand, by the different operating conditions of SOB and RKS.

When testing an engine under bench conditions several configurations are applied. In one case the tanks, pressurized system and connection of tanks to pumps (or to the combustion chamber) are carried out the same as in a rocket and then the results of bench tests should correspond to the expected results of flight tests to the greatest extent. However, the building of such a stand and its operation are associated with considerable difficulties.

In another case the stand has its own stationary system of tanks, lines, and tank pressurizing system. Here bench tests will differ from flight both at starting and at operating conditions. Starting will not correspond to flying if only because the inertness of the stand hydraulic lines, as a rule, is higher than for the rocket. During starting and under operating conditions in the inlet system of lines of the stand there will appear wave processes, which differ from those observed in the rocket.

For the purpose of approximation of starting of the engine under stand conditions to flight conditions we apply stands with receivers or with continuous flow tanks. However, here the results of bench tests differ from flight. As practice showed, with the presence of receivers it is not possible to obtain agreement of curves $p_{\text{н}}(t)$ in bench and flight tests during the entire starting period, and wave processes on the stand frequently differ from those characteristic for flight conditions.

Figure 8.2 depicts the configuration of a liquid-propellant rocket engine, equipped with pressure feed system, but with liquid-propellant pressure accumulator of liquid-propellant rocket engine. In tanks with basic component reserve G_1 and G_2 there are located small combustion chambers – generators *2*. Fuel is forced into these generators from auxiliary tanks *3* with the aid of compressed gas, located in bottle *1*. The weight characteristic of such an engine is somewhat better than shown in Fig. 8.1, however for heavy rockets, in the chambers of which high pressure prevails, it is inexpedient to apply a pressurizing system with liquid propellant accumulator.

The sequence of operation of the automatic equipment of the main engine lines here in principle does not differ from the case examined above. The automatic equipment of the engine should provide constant (nominal) pressure in tanks after the start of operation of the engine and maintain the required pressure in the combustion chamber.

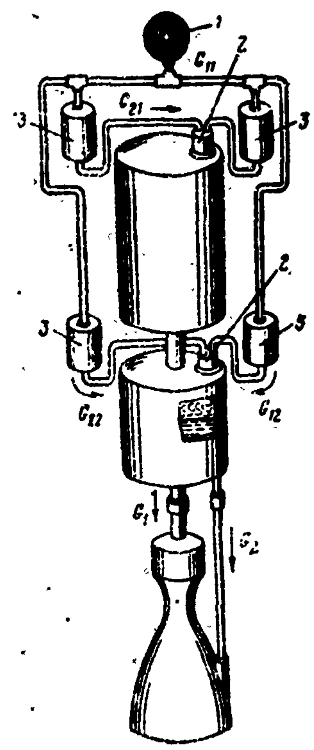

Fig. 8.2. Diagram of liquid-propellant rocket engine with liquid fuel pressure accumulator: *1* — bottle with compressed gas; *2* — liquid fuel accumulators (generators); *3* — fuel tanks of generators.

When designing the construction of hydraulic passages of engines it is also necessary to provide values of hydraulic loss coefficients a_1, a_2 and mass coefficients b_1 and b_2, so that the component ratio of propellant after engine starting would be maintained optimum, providing the greatest reliability and the proper economy.

Figure 8.3 shows the configuration of an engine, equipped with turbopump unit *4* and generator *3*, operating from monopropellant G_4 (hydrogen peroxide) turbogas G_3. To such a configuration was made rocket engine "Fau-2." Research of the considered engine configuration showed that with increase of pressure in the combustion chamber to comparatively high values, the relative engine weight is decreased. Further increase of pressure in the chamber leads to some increase of relative weight. With increase of thrust the relative engine weight is decreased.

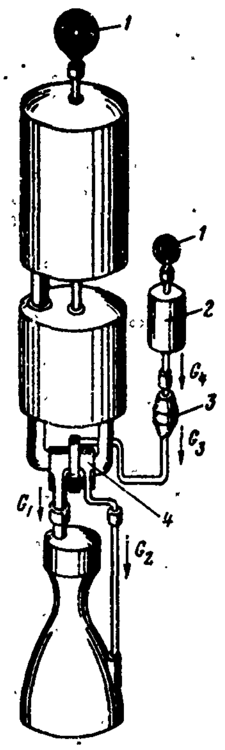

Fig. 8.3. Diagram of liquid bipropellant rocket engine with turbopump unit and single-component gas generator: *1* – bottles with compressed gas; *2* – fuel tank of gas generator; *3* – gas generator; *4* – turbopump unit.

In the process of further perfection of the engine configuration (Fig. 8.4) the pressure feed of propellant (third component) into the gas generator was replaced by pumping. This led to a small improvement of the weight parameters of the engine. The starting time of such engines is somewhat longer than for liquid-propellant rocket engines of earlier described configurations. In the case of preliminary spinup of the turbopump unit shaft the starting depends on the peripheral velocity of the third component pump shaft, being changed directly proportional to the peripheral velocity of rotation of the turbine shaft. For boosting the acceleration of the third component pump between the turbine and pump shafts a special starter can be installed.

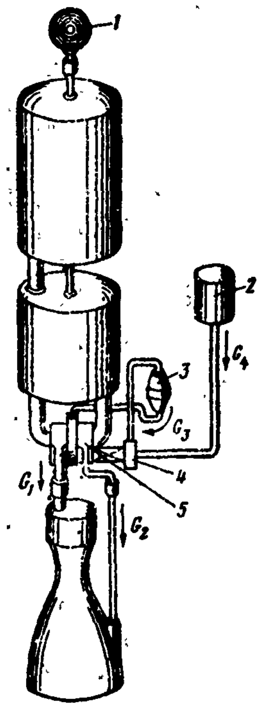

Fig. 8.4. Diagram liquid bipropellant rocket engine with turbopump unit and single-component gas generator: *1* – bottles with compressed gas; *2* – fuel tank of gas generator; *3* – gas generator; *4* – generator pump; *5* – turbopump unit.

From economic and operational considerations we found it more expedient to use the basic propellant components for feeding the gas generator. Figure 8.5 depicts the configuration of an engine, whose turbine working medium generator G_3 operates on basic propellant components G_1 and G_2. If turbine exhaust gas is ejected into the atmosphere or burns up in small auxiliary chambers, then the feed of propellant components into the generator is carried out by the main pumps, moreover the bleeding of propellant is accomplished past their exit throats. If it is proposed to burn up exhaust turbine gas in the main chamber, then between the main pumps and the generator there are installed pumps of the second stage – for additional increase of pressure. More modern configurations of power plant are also possible.

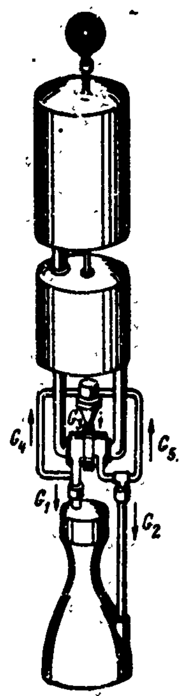

Fig. 8.5. Diagram of engine with turbopump unit and gas generator, operating on the basic propellant.

Figure 1.2 shows the configuration of a power plant, in which one of the components almost completely enters the gas generator of the turbine. There it is gasified and preheated, enters the turbine and from its flow passages – the combustion chamber. The second component is used for cooling the combustion chamber and enters the chamber in liquid state. The power plants, made to such configurations, can operate economically at pressures on the order of 0.1-0.3 MN/m^2.

It is possible to feed both components into the combustion chamber in gaseous form (see Fig. 1.3), moreover each component actuates its own turbine, which revolves one of the pumps. Figure 8.6 shows one of the possible configurations, developed and investigated by the author in 1951.

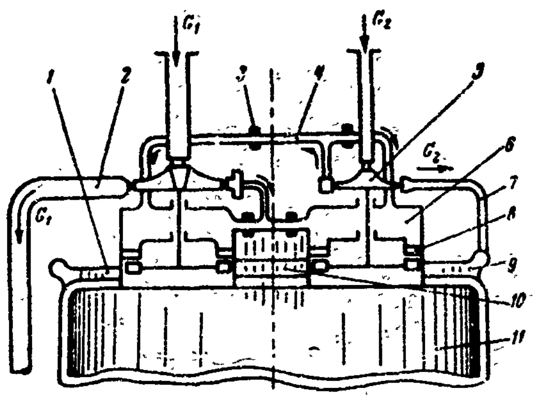

Fig. 8.6. Closed configuration of liquid-propellant rocket engine "gas-gas" with two turbopump units and with afterburning of turbogas in the main combustion chamber: *1*, *9*, *10* – injectors; *2*, *7* – lines, connecting the pumps with the chamber; *3* – throttle; *4* – line feeding component to generators; *5* – pump impeller; *6* – two-component gas generator; *8* – turbine of turbopump unit; *11* – combustion chamber.

8.3. Engine Starting

Here is understood the totality of transient intra-engine processes, which proceed in the period of acceleration to the prescribed amount of thrust, close to stationary.

This idea encompasses the behavior of the whole series of units and their parameters in the starting process, but in the final analysis it is extended to the greatest degree to change of pressure in the combustion chamber and, as a consequence, to change of thrust.

However, the change of thrust, although it is the basic index of the starting process, does not always quite fully characterize it, so in most cases for analysis of the starting indices it is necessary to resort to utilization of intermediate parameters, which directly and indirectly reflect the internal power engineering of the engine. The starting period is characterized by a large variety of processes

and their interaction. The operation of the units can proceed in little studied areas of characteristics, substantially deviating from design and possessing large instability.

It is accepted to unite any deviations of characteristics of structural elements, which lead to corresponding action on the starting characteristics, under the idea of "internal factors." In turn, the idea of "external factors" involves deviations of external medium from standard (or accepted as standard) conditions and deviations of a number of parameters, which are inlet for the engine, from nominal values.

Internal factors should include the temperature of components and construction, inlet starting pressures, operation conditions of elements of automatic equipment, internal mass forces, structural and technological features of the units.

External — include temperature, ambient pressure, external loads and external mass forces. Of course, the temperature of components and the construction also depends on the ambient conditions, therefore the given classification is arbitrary to some extent. Criteria of the quality of occurrence of intra-engine processes in the starting period from an energy point of view are the time of acceleration, i.e., the time of establishment of the prescribed value of thrust or pressure in the combustion chamber, and the provision of optimum value of integral specific thrust during the entire starting period of the engine, i.e., quantity

$$\frac{1}{t_0}\int_0^{t_0} p_{yx}\,dt.$$

The time of acceleration consists of the time of filling of passages with components from starting valves to firing units, during which there is still no acceleration with respect to thrust, and the

time of self-acceleration from the beginning of the processes of ignition and burning to the achievement of the prescribed thrust condition.

The filling time is determined by inlet starting pressures, the geometry of fuel feed lines, adjustment of passages and by the combination of parameters of local hydraulic resistances with characteristics and the operating conditions of starting units and elements of automatic equipment (regulators, starter devices, pumps and others).

The self-acceleration time depends on the internal energy resources of the engine and conditions of their realization, which, in turn, depends on the engine configuration, chemical nature and activity of components.

So, for example, if in an engine with pressure feed system the acceleration time is caused to a considerable extent by the inertness of liquid in fuel feed lines, then with the turbopump fuel feed system this component ceases to play a noticeable role in the shaping of the starting process and the determining parameter becomes power on the turbopump shaft in the acceleration process.

Several programs of engine starting are possible. In certain cases it is expedient to first perform starting of the turbopump unit, and then open the engine starting valves. Two-stage (or multi-stage) acceleration conditions of the turbopump unit are possible. The main fuel valves can be opened approximately simultaneously with the start of acceleration of the turbopump unit. Depending on the operating conditions of automatic equipment and parameters of the feed system starting is rapid or slow. In the first case pressure in the chamber increases to its nominal value in a fraction of a second, in the second – in several seconds.

The pump, located near boundaries of small and large hydraulic resistances, operates in very severe and various conditions and at various modes.

The pumping mode encompasses a rather wide area of characteristics, and in the filling period with the absence of counterpressure on the part of the front of the moving liquid it becomes an area of a very unstable branch of the head characteristic (area of starting conditions). The energy dissipation mode is a continuation of the pumping mode in the filling period at high inlet pressures and low revolutions. This mode is temporarily observed also during filling of a rotating pump with component. The energy dissipation mode is not characteristic for a number of contemporary power plant.

The turbine mode takes place during rotation of pumps by the flow of components without a supply of energy from the turbine which occurs during starting without the use of starter devices. This mode is characteristic for a number of contemporary power plant

After completion of the filling of fuel lines the stage of self-acceleration is started, so the filling period can be considered as the preparatory stage, at which most of the commands are fed to the automatic electrical equipment, by which the creation of the most favorable initial conditions for self-acceleration is attained.

The complex of commands and the priority of their feed are united under the idea of the starting cyclogram.

The search of an acceptable cyclogram is one of the main steps of the firing part of starting.

The stage of self-acceleration and requirements, imposed on it, are determined to a considerable extent by the configuration of the power plant.

In the configuration with pressure feed system the character of starting essentially depends on the kinetics of ignition, the inertial lengths of lines, ambient conditions. "Rigid" startings are accompanied by a "peak" character of change of parameters, which leads to overloads. Measures for the exclusion of effects of the external medium acquire the primary role in the softening of starting (preheating of components, creation of prestarting pressure in the combustion chamber, inhibition of components, etc.).

The startings of engines with turbopump feed system do not exclude the effect of the enumerated factors, however they do not come out already as determining; they become the energy levels of turbine and pumps. In this instance an especially large role is diverted to different types of logical systems (regulators, programmed throttling elements and others), with the aid of which it is possible to actuate the internal power engineering of the engine. With the aid of logical systems the intensification and softening of starting, decrease of overspeeds and overloads of parameters are possible.

In the limit, the development of a complex logical system is possible, which would allow regulating the acceleration process. Basic requirement, which is imposed on logical systems, is the providing of a high degree of reliability and stability of the starting process with the worst combinations of disturbing factors.

Logical systems can be designed on the strength of the principle of the appropriate action on one or several parameters, moreover this action can be either controlling or limiting the growth of the parameter above a certain limit.

The application of logical systems with controlling action is limited to their structural complexity, by difficulties of their development, by the variety of their operation conditions.

Logical systems with limiting action are simpler in structural use and they found wide application.

CHAPTER IX

GRAPHO-ANALYTICAL METHOD OF CALCULATION

9.1. Research of Operating Conditions of an Engine by Grapho-Analytical Method

Graphic or grapho-analytical methods of calculation have been used long and rather widely in engineering practice.

The grapho-analytical method, examined in this paragraph [67], permits constructing graphs, characterizing the connection between engine parameters. In this case several calculation formulas and results from processing experimental data are used. The graphs are constructed with the use of static equations, and therefore they characterize engine operation, as if not having inertial components. As a result of such approach to construction of the solution, the time factor drops from calculation; it is always useful to preface the study of dynamics with examination of equations of statics.

The grapho-analytical method of calculation of an engine is a good supplement to known analytical methods. By providing sufficient accuracy for engineering calculations, with noticeable distortions of results of processing experimental data it permits performing calculation in rather short periods, graphically representing the connection between parameters during engine operation at partial load conditions. The graph-analytical method of calculation gives

the possibility of preliminarily (only from the qualitative side) evaluating the static stability of the engine and separate units, studying the engine capabilities at considerable deviations of parameters from their calculated values, evaluating the effect of peripheral velocity of rotation of the turbopump unit shaft, consumption of generation means and pressurizing in tanks at operating conditions of the engine. With the aid of the grapho-analytical method it is possible to determine the effect of external influences and industrial tolerances on the value of main engine parameters. The graph-analytical method is used for calculation or investigation of a fully determinate engine, for which there should be known the pneudraulic circuit, the sequence of operation and the value of a number of parameters. During discussion of the content of this section we will be guided by the engine configuration, shown on Fig. 8.3.

Hydraulic circuits

Figure 9.1 for an example shows the pressure balance in the main hydraulic circuits of the engine. In the right side of the graph is plotted the relationship of change of engine parameters to the oxidizer consumption G_1, and in the left – to the fuel consumption G_2. There are plotted the relationships of hydraulic losses in lines from the tank to the lower injector edge in the form of curves $\Delta p_1 = f_1(G_1)$ and $\Delta p_2 = f_2(G_2)$. The graphs of the examined functions can be constructed both by calculation and on the basis of processing experimental data – by results of plant hydraulic pressure drop tests or according to data of firing tests. Total hydraulic losses along the circuit

$$\Delta p = \sum_{1}^{n} \Delta p_i, \tag{9.1}$$

where Δp_i – losses of pressure in an element of the circuit.

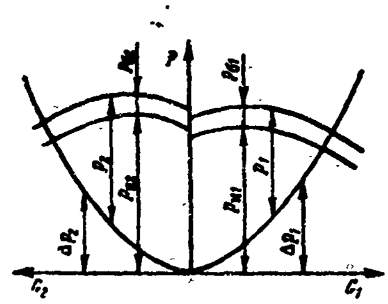

Fig. 9.1. Pressure balance in the main hydraulic circuits of the engine.

The expression for determination of total hydraulic losses depends on the configuration of the hydraulic circuit of the considered engine. For the configuration of oxidizer circuit accepted by us

$$\Delta p_1 = \Delta p_{б-н1} + \Delta p_{н-к1} + \Delta p_{кл1} + \Delta p_{ф1}, \qquad (9.2)$$

where $\Delta p_{б-н1}$ – hydraulic losses on the section from the tank to pump; $\Delta p_{н-к1}$ – the same from the pump to the chamber, but without losses in the valve; $\Delta p_{кл1}$ – the same in the main valve; $\Delta p_{ф1}$ – the same in the head and on injectors. We will consider that for the hydraulic circuit of fuel

$$\Delta p_2 = \Delta p_{б-н2} + \Delta p_{н-к2} + \Delta p_{кл2} + \Delta p_{пр2} + \Delta p_{ф2}, \qquad (9.3)$$

where $\Delta p_{пр2}$ – hydraulic losses in the flow passage of the cooling system.

On the same graph are plotted the head characteristics of the pumps. They represent the relationship of excess pressure, created by the pump, to the consumption of component, moreover each curve $p_{H1} = f_1(G_1)$ or $p_{H2} = f_2(G_2)$ corresponds to a certain value of peripheral velocity of the turbopump unit shaft. On Fig. 9.1 for each hydraulic circuit only one curve is constructed. In order to get the complete idea of engine operation at cruise conditions, calculation should be performed at a number of values of ω. The head characteristics of pumps are constructed by calculation or from results of processing experimental data. Excess pressure, created by the pump, is equal to

$$p_{н} = p_{e} - p_{0}. \qquad (9.4)$$

where p_e – pressure at pump exit; p_0 – pressure at pump inlet.

Figure 9.1 shows curves $(p_{H\,i} + p_{б\,i})$ with respect to G_i. These curves are equidistant with curve $p_{H\,i}\,(G_i)$ when reduced pressure in the tanks does not depend on the consumption. We will consider that

$$p_{б} = p_{б.с} + (p_{j} + p_{g}), \qquad (9.5)$$

where $p_{б.с}$ – pressure in the upper cavity of the tank, created by the pressurizing system; $p_j + p_g$ – pressure of the liquid column, measured from the middle point of the liquid mirror in the tank to the lower injector edge.

Quantity $p_j + p_g$ depends on the height of the column and the density of liquid, the amount of acceleration of flight, acceleration of gravity and the direction of flight. Reduction pressure can vary in the process of flight. Therefore, each calculation graph corresponds to a certain moment of time of rocket flight.

The equations of pressure balance for circuits will be written so:

$$p_1 = p_{н1} + p_{б1} - \Delta p_1; \qquad (9.6)$$
$$p_2 = p_{н2} + p_{б2} - \Delta p_2. \qquad (9.7)$$

Under steady-state conditions p_1 and p_2 represent pressures in the chamber. Inasmuch as both hydraulic circuits feed the same chamber, p_1 should certainly be equal to p_2. This circumstance is used for conducting further calculation. If on Fig. 9.1 we plot p_1 and p_2 equal to the actual pressure in the chamber, then by the graph it will be easily possible to find values G_1 and G_2 corresponding to steady state. When performing engineering calculations for construction of the graph, given in Fig. 9.1, it is recommended to take a sheet not less than 500 mm in size. Scales G_1 and G_2 must be selected so that the length of the segment, which characterizes the oxidizer consumption at design steady state, would be approximately equal to the length of the segment, which corresponds to analogous fuel consumption.

Pressure in the combustion chamber depends on the quantity of propellant, which entered the combustion chamber $G_1 + G_2$, equal under steady state conditions to the consumption of combustion products through the nozzle throat, and on the ratio of specific pressure pulse β to the nozzle throat area $F_{кр}$, moreover

$$\beta = \frac{\sqrt{RT_к}}{a}, \qquad (9.8)$$

where R and $T_к$ – gas constant and temperature of combustion products in the engine chamber; a – function of politropic index;

$$a = \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \left(2\frac{n}{n+1}\right)^{0.5}; \qquad (9.9)$$

here n — the averaged value of politropic index.

The specific pulse depends little on pressure, but is considerably changed with change of component ratio k_1.

The parameters, entering formula (9.8), are determined from results of thermodynamic calculation, and the specific pulse pressure can be determined, furthermore, by results of firing tests. The quantity of propellant, which entered the chamber, depends on the hydraulic circuit parameters and the amount of resistance, shown by the combustion chamber, and the flow rate of gases — on pressure in the chamber $p_{\text{н}}$, specific pulse β and the nozzle throat area $F_{\text{кр}}$.

Combustion chamber

Let us use the graph given in Fig. 9.1 to establish the connection between pressure in the chamber $p_{\text{н}}$ and the inflow of propellant into the chamber $G_1 + G_2$. Let us proceed with construction of the graph, provided in Fig. 9.2. Let us take the right side of the graph from Fig. 9.1 and for a number of randomly selected values of G_1 let us determine the values of p_1.

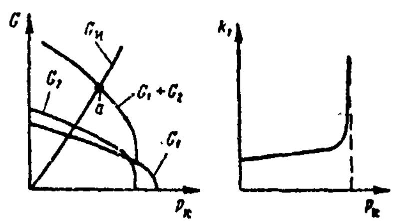

Fig. 9.2. Relationship of flow rates and ratio of components to pressure in the chamber.

Inasmuch as we are considering steady state, we should consider that p_1 is $p_к$. By plotting the values of G_1 and values of $p_к$ corresponding to them on Fig. 9.2, we receive curve G_1 with respect to $p_к$. In perfect analogy to, but being guided by the left side of the graph (Fig. 9.1), let us construct curve G_2 with respect to $p_к$, representing the relationship of inflow of the second component into the chamber to the pressure in it. Now it is possible for any pressure to determine what the inflows of components into the chamber will be.

On Fig. 9.2 let us note the somewhat randomly selected values of $p_к$. For each of them it is possible to determine the sum of $G_1 + G_2$ and by it to construct curve; $G_1 + G_2$ with respect to $p_к$, which will represent the relationship of the inflow of propellant into the chamber to the pressure in it.

The next stage of calculation is reduced to determination of ratio k_1 for the whole range of pressures in the combustion chamber. For any artibrary value of $p_к$ we know G_1 and G_2; this is sufficient for construction of graph k_1 in terms of $p_к$ presented in Fig. 9.2, on which for any value of $p_к$ we have the value of k_1. For construction of the curve of relationship of the flow rate of gases $G_к$, escaping the chamber, to pressure $p_к$ let us use known formula

$$G_к = \frac{F_{кр}}{\beta} p_к. \qquad (9.10)$$

For a number of arbitrary values of $p_к$ by the graph of Fig. 9.2 we find corresponding k_1. Then by results of thermodynamic calculation or processing of experimental data for each obtained value of k_1 we determine the magnitude of specific pressure pulse. It remains to select, assign or determine by measurement the nozzle throat area. Now for selected values of $p_к$ the amount of flow rate of gas, escaping from the chamber, is known.

The results of calculation by formula (9.10) are presented in Fig. 9.2 in the form of curve $G_{к}$. At point *a* of intersection of curves $G_1 + G_2$ and $G_{к}$ the inflow of propellant into the chamber is equal to the flow rate of gases (the combustion products of propellant) from the chamber. Therefore, point *a* corresponds to steady state of operation of the engine at rpm of the turbopump unit shaft selected for calculation. In the conclusion of calculation we find the values of $p_{к}$, G_1, G_2 and k_1. Inasmuch as G_1 and G_2 are known, by the graph of Fig. 9.1 we find the hydraulic losses and excess pressure, created by pumps, corresponding to the selected rpm. Having completed similar calculations for other values of rpm, it is possible to construct a graph of the relationship of found parameters to the rpm of the turbopump unit shaft. In order to determine which revolutions of the shaft are operating, it is necessary to calculate the turbopump device and the generator.

Turbopump unit

It is known that power, required by the pump, is equal to

$$N_{н i}=\frac{p_{н i}G_i}{\varrho_i\eta_{н i}}, \qquad (9.11)$$

where $p_{н i}$ — excess pressure, created by the pump; G_i — mass flow rate per second of component through the pump; ϱ_i — density of component; $\eta_{н i}$ — the pump efficiency.

As a result of calculation of hydraulic circuits for the selected revolutions we determined both excess pressure, created by pumps, and component consumption. If we accept $\eta_{н i}$ = const or if the relationship of $\eta_{н i}$ to pump parameters is known, then for every selected value

of rpm it is possible to calculate power, required by each pump. If there is the possibility, then $N_{H\,i}$ in terms of ω should be obtained experimentally. By summing the values of powers of all the pumps, let us construct the graph of relationship of power, required by these pumps, to the value of the velocity of rotation ω of the turbopump unit shaft (Fig. 9.3).

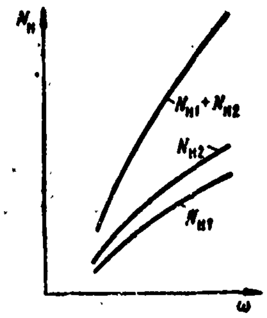

Fig. 9.3. Relationship of the power of pumps to the peripheral velocity of rotation of the turbopump unit shaft.

On the basis of additional calculation or by using results of processing of experimental data, let us construct a family of curves, representing the relationship of turbine power to the velocity ω of rotation of turbopump unit shaft at various consumptions of propellant, entering the gas generator, G_3 (Fig. 9.4).

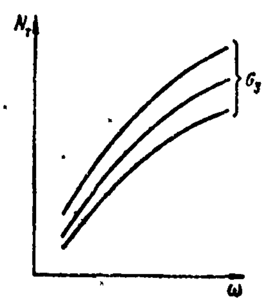

Fig. 9.4. Relationship of turbine power to the peripheral velocity of rotation of the turbopump unit shaft with different consumptions of gas-producing propellant.

It is known that in the first approximation the power, developed by the turbine, is equal to

$$N_T = (A - B\omega)\omega G_3, \tag{9.12}$$

where G_3 - the mass flow rate of the working medium of turbine per second; A, B,- parameters, which depend on the construction and operating conditions of the turbine. They can be calculated, but they can be determined more precisely by results of processing experimental data. One should bear in mind that they depend on G_3 and ω. Each curve, presented on Fig. 9.4, corresponds to a certain composition G_3. While having graphs $N_{\text{н}1}(\omega)$ and $N_{\text{т}}(G_3,\omega)$, let us construct the graph of the power balance of the turbopump unit, for which we combine both graphs (Fig. 9.5). If consumption G_3 is predetermined, then by the appropriate point of intersection of curves $(N_{\text{н}1}+N_{\text{н}2})(\omega)$ and $N_{\text{т}}(G_3,n)$ we find the parameters of steady state. If ω is assigned, then we find the required consumption of working medium of the turbine.

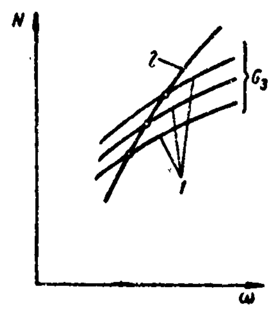

Fig. 9.5. The balance of powers for turbopump unit; *1* - turbine; *2* - pumps.

Generator

In modern engines the most diverse types of generators are applied. If the generator is fed with two propellant components, then its graphic calculation is similar to calculation of the main chamber. The generator, which uses a third component alone (for example, hydrogen peroxide), is calculated somewhat simpler. The character of calculation remains approximately identical regardless of whether hydrogen peroxide (or another monopropellant) will be fed under pressure of compressed gas or by a pump.

Let us examine the calculation of a generator with gas pressure feed of the third component (Fig. 9.6). On a graph let us plot pressure in the generator tank $p_{\text{бз}}$. If from the value of pressure $p_{\text{бз}}$

we subtract the amount of hydraulic losses $\Delta p_з$, then for any assigned flow rate of turbogas it is easy to find pressure in the gas generator $p_{гг}$.

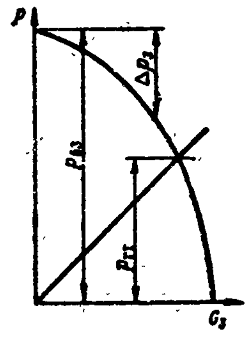

Fig. 9.6. The pressure balance for the generator.

It is possible to solve the inverse problem: by equation

$$p_{гг}=\frac{\sqrt{RT_{гг}}}{a_{гг}\sum_{1}^{n}F_{кр.т}}G_з \qquad (9.13)$$

on the graph of Fig. 9.6 let us draw a straight line, inasmuch as at relatively low temperatures of decomposition of third component

$$\frac{\sqrt{RT_{гг}}}{a_{гг}}\approx\text{const}, \qquad (9.14)$$

where $a_{гг}$ - function of politropic index, determined by formula (9.9). The parameters, entering formula (9.14), are determined by calculation. The sum of $\sum_{1}^{n}F_{кр.т}$ is defined as the sum of nozzle throat areas of the turbine. The point of intersection of lines of Fig. 9.6 will determine the working parameters of the generator.

9.2. Preliminary Evaluation of the Interconnection of Processes in the Power Plant

Let us assume point a (Fig. 9.7) determines the parameters of steady state. Let us examine the vicinity of this point. Let us assume that at some moment of time under the action of some factors some excess quantity of propellant entered the combustion chamber.

As a result of combustion of this portion of propellant the pressure in the chamber is raised and, as a consequence of this - the inflow of propellant to the chamber is decreased. However, with increase of pressure in the chamber the flow rate of gas from the chamber will be greater than calculated. Thus, with increase of pressure in the chamber as a result of combustion of the additional portion of propellant there will simultaneously occur both decrease of inflow of propellant and increase of flow rate of gas. This will contribute to decrease of pressure in the chamber to the calculated value. Thus, point a is stable, since self-regulation occurs in its vicinity. We will arrive at perfectly similar derivations if we trace the process, induced not by increase, but by decrease of inflow of propellant to the chamber.

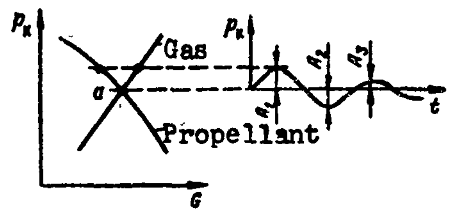

Fig. 9.7. Graphic determination of design conditions of the combustion chamber.

Let us now examine the same process, but with consideration of mass forces and friction forces, affecting the system. Let us assume under the action of a disturbance the pressure in the chamber increased by quantity A_1 (see Fig. 9.7). As a result of such increase, as already mentioned, the inflow of propellant to the chamber is decreased and the flow rate of gas from the nozzle increases, and due to this - pressure in the chamber is reduced. With change of pressures in the chamber both the velocity of propellant, entering the chamber, and the velocity of gas, escaping from the chamber are changed. In view of the inertia of moving liquids and gas the amount of pressure in the chamber at a certain moment of time will be less than nominal by quantity A_2.

If forces of friction are absent in the system, then the amplitudes of pressure fluctuations A_1, A_2, A_3, etc., will be equal to each other, and pressure fluctuations, occurring with constant amplitude will be observed. Thanks to the action of friction forces, which appeared in flows and between liquids and walls of elements of the lines, the magnitude of the amplitude of each subsequent half-wave will be less than the previous; damped oscillations take place in the system. Thus, if the "chamber-hydraulic circuit" system receives disturbance, as a result of which the pressure in the chamber deviates from its nominal value, then in the vicinity of point a damped oscillations will appear. One should bear in mind that such evaluation of stability of the character of motion of the system is tentative, preliminary, since it was accomplished while not allowing for the period of delay and other factors.

If we exclude anomalous cases from examination, then the "chamber-hydraulic circuit" from examination (point a in Fig. 9.7) is always stable. Unstable operation of this system is possible only with very considerable deviations of ratio k_1 from its nominal value.

The situation is somewhat more complex with the turbopump unit, which, depending on the characteristic, can be both stable and unstable, i.e., it can or cannot possess the feature of self-regulation. Let us note, however, that in the overwhelming majority of cases the "turbine-pump" system is stable, and research is reduced to determination of the degree of stability.

Let us examine the balance of powers of the turbopump unit, the turbine of which is fed by a generator with gas pressure feed of third component into it - hydrogen peroxide. At steady state the turbopump unit parameters are determined by point of intersection a (Fig. 9.8). Let us assume that in view of these or other disturbances the peripheral velocity of rotation of the turbopump unit shaft increased from value ω_1 to ω_2 (Fig. 9.8b). In this case the power, required by pumps, will be greater than power developed by the turbine.

Inasmuch as the power, required by pumps, is greater than the power supplied to them, then the peripheral velocity of rotation of the turbopump unit shaft is reduced to nominal value. Therefore, the unit is stable.

Mathematically the conditions of stability of the turbopump unit in the vicinity of point a can be written

$$\left(\frac{\partial \sum N_{\text{н}i}}{\partial \omega}\right)_a > \left(\frac{\partial N_{\text{т}}}{\partial \omega}\right)_a,$$

where $\Sigma N_{\text{н}i}, N_{\text{т}}$ - power of pumps and turbine respectively, calculated by parameters of point a.

If

$$\left(\frac{\partial \sum N_{\text{н}i}}{\partial \omega}\right)_a < \left(\frac{\partial N_{\text{т}}}{\partial \omega}\right)_a,$$

that the system is unstable. In fact, let us influence the system so that calculated velocity ω_1 will grow to ω_2; then power, developed by the turbine, will be greater than power required by pumps (Fig. 9.8a). The presence of excess of power will lead to further increase of velocity of rotation of the turbopump device shaft. If as a result of our influence the velocity will be decreased in comparison with calculated, then as a result of exceeding the power, required by pumps, further decrease of the velocity of rotation of the turbopump unit shaft will occur. Thus, the examined system is unstable.

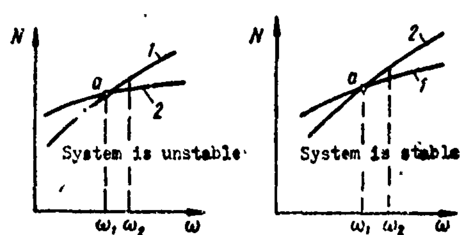

Fig. 9.8. Graphic method of determination of stability of turbopump unit: *1* - turbine; *2* - pump.

Let us compare the stability of two turbopump units. Let us assume the turbine of the first turbopump unit is fed by steam gas, received in the generator with gas pressure feed of hydrogen peroxide, and the turbine of the second turbopump unit - by steam gas, received in the generator with pump feed of hydrogen peroxide. The balance of powers is represented in Fig. 9.9. With gas pressure feed of peroxide line *1-1* is the relationship of power, required by pumps, to the velocity of rotation of the turbine shaft, and lines *3* - characteristics of turbine, which correspond to certain different flow rates of generation means. If the turbine characteristic represented by line 3-IV, then the stability of turbopump unit is determined by angle α.

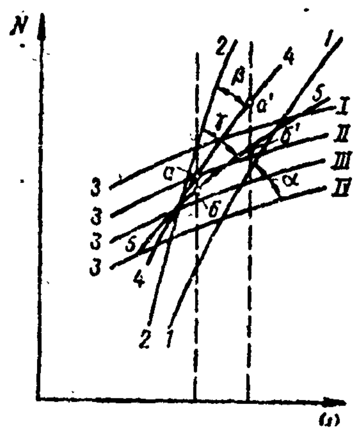

Fig. 9.9. The balance of powers of two turbopump units, fed from different generators.

Let us now examine the pump feed of peroxide. Power, required by pumps, will be greater than in the previous case, due to power, required by the third, additional, pump. Now instead of line *1-1* the relationship of power, required by three pumps, to the angular velocity of rotation of the shaft of the device will be represented by line *2-2*. By special calculation of the generator it is possible to establish the relationship between the velocity of rotation of the pump, feeding peroxide, and the flow rate of peroxide. Let us assume

that in this case the relationship of turbine power to the velocity of rotation of the shaft be represented by line *4-4*, then the stability of turbopump unit will be determined by angle β. The slope of line *4-4* can be changed, if we equip the hydraulic circuit of the generator with a regulator, made, for example, in the form of a throttle valve, actuated by a centrifugal regulator. Line *5-5* is the relationship of N_T to ω with pump feed in the case of action of the regulator.

Let us take point a, which corresponds to operation without a regulator. If we throttle the hydraulic circuit of the generator by turning the throttle to a certain angle, then the flow rate of hydrogen peroxide is decreased; instead of point a the turbine power will be determined by point $б$. At high revolutions without a regulator the turbine power will be determined by point a'; as a result of even more turning of the throttle instead of point a' the turbine power will be determined by point $б'$. Thus, angle β can be increased to value γ, which corresponds to increase the stability of the device.

Let us examine the question about the stability of the turbopump unit with consideration of mass (inertial) forces and friction forces (Fig. 9.10). Under the action of external factors the system leaves the state of equilibrium and the design revolutions are increased by the quantity of initial amplitude A_1. Here $\Sigma N_{Hi}-N_T$ is a positive quantity - the number of revolutions of the turbopump unit shaft is reduced. Inasmuch as the moving wheels of turbine and pumps possess inertia, the revolutions of the shaft are decreased to a quantity, which is less than the calculated values of revolutions.

Under the action of friction forces (friction of liquids and vapors the flow area, friction against the disk in the housing, mechanical friction in seals and others) the amplitude is decreased to quantity A_2. With revolutions less than calculated, as a result of the turbine power exceeding the power of pumps acceleration of the system is observed, which in view of the presence of flywheel masses is terminated at angular velocity of rotation $\omega > \omega_1$. Amplitude A_2 due to the effect of friction forces proves to be less than amplitude A_1.

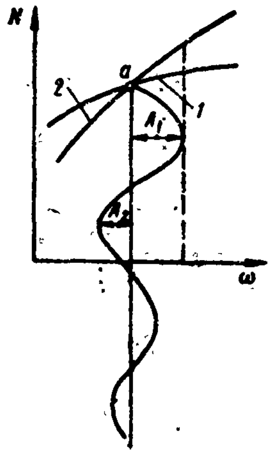

Fig. 9.10. Damping of oscillations of the angular velocity of rotation of turbopump unit shaft. *1* - turbine; *2* - pump.

Another character of motion is possible. Let us increase the intensity of friction forces. Let us assume under the action of external forces the equilibrium is disturbed, and initial amplitude A_1 will be the same as in the preceding case.

If the factors, damping the system, act rather intensively, then instead of the damped oscillations examined above we will observe aperiodic motion. Thus, stable point *a* causes damped oscillations or aperiodic motion if under the action of these or other factors the system left the state of equilibrium. The disturbances of operating condition of the turbopump unit are caused by influences, directed toward the combustion chamber and hydraulic circuits, by the presence of hydraulic shocks, which appear with passage of the centrifugal pump vane past the exit throat of a spiral chamber, periodically acting dynamic loads, which appear in the flow area of the turbine, and also by other factors.

The described grapho-analytical method is one of many, being applied at present in designer offices, in scientific and educational institutions. The methods, worked-out by Berzheron [6], has received wide distribution recently.

Example.

Determine by the grapho-analytical method the averaged values of the main parameters of a stand device after engine starting.

Given.

The stand device of a liquid-propellant rocket engine is equipped with gas pressure feed system. Pressure in the tanks (with consideration of the action of external forces) is kept constant.

With results of hydraulic pressure drop tests the following data have been obtained:

Oxidizer		Fuel	
G_1 kg/s	Δp_1 MN/m^2	G_2 kg/s	Δp_2 MN/m^2
50	1,0	12	1,0
75	2,2	20	3,0
100	4,0	25	5,0
125	6,3	30	7,0
150	9,0	35	9,2
175	12,2	40	11,6
200	16,0	44	14,0

According to results of thermodynamic calculation for the investigated chamber there is obtained

$$\frac{\beta}{F} = 81505\ \frac{\mathrm{MN \cdot s}}{\mathrm{kg \cdot M^2}}.$$

The component ratio

$$k_1 = \frac{G_1}{G_2} = \mathrm{const.}$$

Solution

1. Let us construct the first graph (Fig. 9.11).

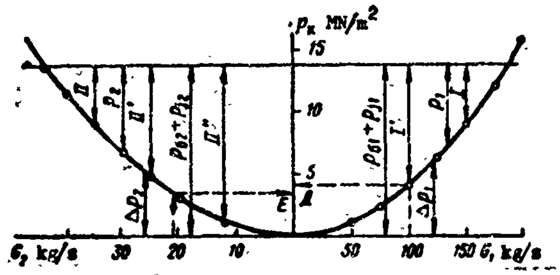

Fig. 9.11. Pressure balance in the main hydraulic circuits of the engine (for the example of calculation).

Let us select a scale for G_1 and G_2 so that segment G_1 = 100 kg/s would be approximately equal to segment G_2 = 22.8 kg/s.

On millimeter graph paper let us plot to scale segments $p_{б1}$ and $p_{б2}$.

Let us draw lines $p_{б1}$ and $p_{б2}$ parallel to the axis of abscissa, inasmuch as pressure in the tanks does not depend on the consumption of propellant components.

By using the results of hydraulic pressure drop tests, let us construct curves Δp_1 with respect to G_1 and Δp_2 with respect to G_2.

Let us construct the second graph (Fig. 9.12). Let us prepare a sheet of millimeter graph paper so that along the axis of ordinates it would be possible to plot values of pressures to 14 MN/m², and along the axis of abscissa - values of the largest possible total consumption of components. With the first graph (see Fig. 9.11) we find: $G_{1\,max} \approx 190$ kg/s and $G_{2\,max} \approx 44$ kg/s. Consequently, $(G_1+G_2)_{max} = G_{max} \approx 234$ kg/s.

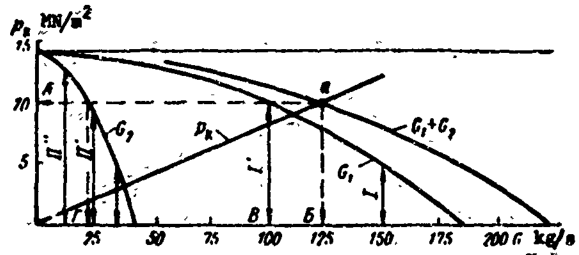

Fig. 9.12. The relationship of flow rates to pressure in the chamber (for the example of calculation).

Let us transfer the values of p_1, corresponding to a number of values of G_1, from the first graph to the second (see Fig. 9.12). Let us connect the obtained points by a smooth curve and we obtain graph G_1 with respect to $p_к$.

For explanation of the method of construction of curve G_1 with respect to $p_к$ on graphs the values of p_1 are marked in the form of segments *I* and *I'*, equal on the first and second graphs respectively.

Similarly there is constructed graph G_2 with respect to $p_к$. For explanation of the method of construction of this graph one should examine respectively segments *II*, *II'* and *II''*, equal to each other, on fields of the first and second graphs.

Having selected on the field of the second graph a number of arbitrary values of p, let us graphically sum up segments G_1 and G_2 and construct graph $(G_1 + G_2)$ with respect to $p_к$.

Let us draw a line, which characterizes the relationship of pressure in the chamber to the flow rate of combustion products through the nozzle throat. Inasmuch as k_1 = const, then $p_к$ with respect to G will be represented in the form of a straight line.

According to the point a of intersection of curve $(G_1 + G_2)$ with respect to $p_к$, which characterizes the inflow of propellant into the chamber, with line $p_к$ with respect to G, which determines the flow rate of gases from the chamber, we find the sought averaged values of parameters:

$p_к = 10$ MN/m^2 (point A); $G = 122.8$ kg/s (point Б); $G_1 = 100$ kg/s (point B); $G_2 = 22.8$ kg/s (point Г).

By Fig. 9.11 for $G_1 = 100$ kg/s we find the amount of hydraulic losses $\Delta p_1 = 4$ MN/m^2 (point D). For $G_2 = 22.8$ kg/s by point E we find hydraulic losses in the fuel circuit.

CHAPTER X

STUDY OF THE CONNECTION OF AVERAGED PARAMETERS

Under operating conditions small deviations of parameters from their calculated (or assigned) values are possible. In certain cases the researcher is interested in the character of change of parameters with time. For the solution of such problems dynamic equations are drawn upon. If it is necessary to determine only finite (new) values, then calculation is performed with the aid of static (algebraic) equations. Inasmuch as we are speaking about the study of an engine under the condition that deviations of parameters are relatively small, then we usually are guided by linear equations, obtained by means of linearization of nonlinear equations, derived and examined in the previous chapters.

10.1. Processes, Proceeding in the Vicinity of Assigned Conditions

The number of dynamic problems includes the study of transient processes, caused by the action of external factors; transient processes, which appear under action of commands, produced with the aid of control systems; emergency transition from one mode to another and shutdown because of the malfunction of some unit of the engine, etc.

The number of static problems includes: determination of results of the action of external factors; determination of deviations of parameters, which appear as a result of errors, permissible during preparation of the engine for operation; the calculation of engine

tuning to the required mode; evaluation of the precision and quality of engine production; selection of the class of precision of manufacture and the cleanness of machining of separate parts, etc.

The initial system of equations for solution of all the enumerated problems is the system of first order linear differential equations of type

$$\dot{x}_k = \sum_1^k a_{ik}\Delta x_k, \tag{10.1}$$

where Δx_k - small deviation of variable parameter x_k; a_{ik} - constant coefficient. The system of equations should be closed, for which it is necessary that the number of equations i be equation to the number of variables k. During solution of static problems

$$\dot{x}_k = 0, \tag{10.2}$$

and the system of equations takes the following form:

$$\sum a_{ik}\Delta x_k = 0. \tag{10.3}$$

Thus, equations of statics are obtained from equations of dynamics. The more comprehensively studied and the more widely used are static equations, the easier it is to operate with equations of dynamics. Therefore, it proves to be expedient to preface the study of the engine by static equations with research of dynamic processes.

Let us examine the equation of the first hydraulic circuit, i.e., oxidizer circuit. By using the pressure balance (6.153) and formulas obtained above, we find

$$\psi_1 = p_{61} + D_1\omega^2 - D_1'\omega G_1 - D_1'' G_1^2 - \frac{\beta}{F_{\text{кр}}}(G_1 + G_2) - a_1 G_1^{n_1} - a_{\kappa 1} G_1^{n_1} - b_1 \dot{G}_1 = 0. \tag{10.4}$$

Let us distinguish three groups of parameters. To the first group let us refer the basic parameters, which determine the operating conditions of the machine: G_1, G_2, G_3, ω, and also - pressure in the chamber p_K and thrust P. Parameters of the second group are those, the values of which change under the action of external factors. They include: a_i, b_i, D_i, β. To the third group belong such parameters, the values of which can be changed at our discretion, for example, pressure in the tank $p_{\sigma 1}$ and the coefficient of hydraulic resistance of the adjustable throttle in equation (10.4).

Basic parameters receive small deviations ΔG_i and $\Delta\omega$ in view of deviation of parameters of the other two groups from nominal. The parameters of the second group can have small deviations, which appear under the action of external factors. Parameters of the third group characterize control or adjustment elements, and their small deviations depend on the degree of influence, being set by the experimenter or by an element of the automatic control system.

Small deviations of basic quantities will depend on the change of other parameters. The external influences, directed to parameters of the second group, appear for reasons of industrial order, operational character and under the action of meteorological factors.

Industrial influences are caused by the presence of tolerances during manufacture and assembly. The deviations of operational character are explained by the influence on structural elements during transport, by inaccuracies, which appear during preparation of the engine for operation, including errors of control measurements and errors, permissible during adjustment of control elements. Influences of meteorological order are caused by changes of ambient temperature and pressure. With change of pressure there are changed the vapor presssure, the boiling point; change of temperature leads to change of heat content and physical properties of components: density, specific heat, viscosity and thermal conductivity. Small deviations of parameters of the third group are usually designated so as to provide nominal conditions, at which small deviations of basic parameters would be equal to zero.

10.2. Derivation of Equation in Small Deviations

All parameters, entering equation (10.4) can receive small deviations. If these deviations are infinitesimal, then the expression of total differential of the considered function will be exactly equal to zero. When writing this expression first let us write down the terms, containing differentials of the basic parameters, then - differentials of parameters of the second group and, finally, the third. We receive

$$d\psi_1=\frac{\partial\psi_1}{\partial G_1}dG_1+\frac{\partial\psi_1}{\partial G_2}dG_2+\frac{\partial\psi_1}{\partial\omega}d\omega+\frac{\partial\psi_1}{\partial a_1}da_1+\frac{\partial\psi_1}{\partial D_1}dD_1+$$
$$+\ldots+\frac{\partial\psi_1}{\partial p_{61}}dp_{61}+\frac{\partial\psi_1}{\partial a_{x1}}da_{x1}=0. \qquad (10.5)$$

Parameter p_{61} refers to parameters of the second group, if deviation Δp_{61} is caused by the action of external factors, and to parameters of the third group, if deviation Δp_{61} is used for control of the operating conditions of the engine. With transition to small finite deviations one should remember that unlike $d\psi_1 = 0$ deviation $\Delta\psi_1$ is nonzero and equal to error c_1, which appears as a result of disregarding nonlinear remainders. When using equations in small finite deviations the quantity c_1 is frequently computed with consideration of errors, which appear when determining the values of constant coefficients, entering equation (10.4), by calculation or experiment. Instead of equation (10.5) now we have

$$\frac{\partial\psi_1}{\partial G_1}\Delta G_1+\frac{\partial\psi_1}{\partial G_2}\Delta G_2+\frac{\partial\psi_1}{\partial\omega}\Delta\omega+\frac{\partial\psi_1}{\partial a_1}\Delta a_1+\frac{\partial\psi_1}{\partial D_1}\Delta D_1+$$
$$+\ldots+\frac{\partial\psi_1}{\partial p_{61}}\Delta p_{61}+\frac{\partial\psi_1}{\partial a_{x1}}\Delta a_{x1}=c_1. \qquad (10.6)$$

In expression (10.6) there are considered small deviations from assigned or steady state. If we designate the current values of parameters G_i and ω, and their assigned or steady values will be G_{i0} and ω_0, then small deviation will be

$$\Delta G_1=G_1-G_{10};\quad \Delta G_2=G_2-G_{20};\quad \Delta\omega=\omega-\omega_0. \qquad (10.7)$$

Inasmuch as expression (10.6) is a neighborhood equation, all partial derivatives should be calculated by the parameters of assigned or steady state, therefore they will enter the equation in the form of numbers.

Let us designate

$$\frac{\partial\psi_1}{\partial G_1}=a_{11};\quad \frac{\partial\psi_1}{\partial G_2}=a_{12}\text{, etc.} \tag{10.8}$$

The first subscript corresponds to the number of the equation, the second - to the number of parameter. Inasmuch as all are assigned and inasmuch as small deviations of parameters of the second group should also be assigned, sum

$$\Delta V_1=a_{15}\Delta p_{б1}+a_{16}\Delta d_1+a_{17}\Delta D_1+a_{18}\Delta D_1' \tag{10.9}$$

represents the number, which characterizes the disturbance of the considered function of equations. In equation (10.9) it is accepted that in the hydraulic circuit we obtain deviations of parameters $p_{б1}$, a_1; D_1, D_1'.

The sum of terms, containing small deviations of parameters of control, is designated ΔP_i. In our function there is considered one deviation, inasmuch as a conditionally accepted engine is tuned with the aid of throttle washers or is controlled by commands, proceeding from liquid reduction gear, therefore

$$\Delta P_i=a_{i11}\Delta a_{\text{к}i}. \tag{10.10}$$

Finally the equation of small finite deviations in general form can be written in the following manner:

$$\sum a_{ik}\Delta x_k=c_i. \tag{10.11}$$

It is read so: the sum of products of partial derivatives of functions according to parameters by small finite deviations of these

parameters is equal to the error, appearing as a result of disregarding nonlinear terms. The numerical values of partial derivatives are calculated from parameters of the considered mode, from which small deviations are read.

In interpreted form equation (10.11) during solution of problems about determination of the action of external factors will be written so:

$$a_{11}\Delta G_1 + a_{12}\Delta G_2 + a_{13}\Delta\omega + \Delta V_1 = c_1; \qquad (10.12)$$

$$\Delta V_1 = \sum a_{1k}\Delta x_k; \qquad (10.13)$$

During solution of the problem about adjustment instead of expression (10.12) we will write

$$a_{11}\Delta G_1 + a_{12}\Delta G_2 + a_{13}\Delta\omega + \Delta V_1 + \Delta P_1 = c_1; \qquad (10.14)$$

$$\Delta P_1 = a_{1\,11}\Delta a_{k1}, \qquad (10.15)$$

moreover the goal of adjustment will be the provision of $\Delta G_1 = 0$ and $\Delta G_2 = 0$. With solution of the problem about the action of external factors we find numerical values of ΔG_1 and ΔG_2, which correspond to the given ΔV_i.

The quantity and type of equations depend, as already indicated, on the engine configuration and the formulated problem. Let us be limited in accordance with accepted scheme of calculation to the examination of four basic parameters: G_1, G_2, G_3, ω. The closed system will include four equations.

In general the basic parameters are all G_i and ω; therefore, in the closed system the number of equations is equal to $i + 1$. As the second equation we take the equation of hydraulic fuel circuit, which is written so:

$$\psi_2 = p_{62} + D_2\omega^2 - D_2'\omega G_2 - D_2''G_2^2 -$$

$$- \frac{3}{F_{\text{кр}}}(G_1 + G_2) - a_2 G_2^{n'} - a_{\text{д}2} G_2^{n'} = 0. \qquad (10.16)$$

Equation (10.16) can be commented in the same way as equation (10.4).

If we consider the external influences and the effect of adjustment, the equation of small deviations will take the following form:

$$a_{21}\Delta G_1 + a_{22}\Delta G_2 + a_{23}\Delta\omega + \Delta V_2 + \Delta P_2 = c_2; \qquad (10.17)$$

$$\Delta V_2 = \sum a_{2k}\Delta x_k; \qquad (10.18)$$

$$\Delta P_2 = a_{\bar{2}11}\Delta a_{\text{к}2}. \qquad (10.19)$$

The third equation is the equation of the turbopump unit (see Chapter VII). Under steady state conditions it will be written in the following manner:

$$M_{\text{т}} = \sum M_{\text{н}i}. \qquad (10.20)$$

Let us take the moment, developed by the turbine,

$$M_{\text{т}} = 2r_{\text{л}}(m_1 C_1 \cos\alpha_1 - m_2\omega)\, G_3 \eta_{\text{т}}, \qquad (10.21)$$

where $2r_{\text{Л}}$ - diameter of the center line of nozzle exit sections; $\eta_{\text{Т}}$ - efficiency of turbine; C_1 - exit velocity from the nozzle; α_1 - nozzle axis angle; ω - angular velocity of rotation of the turbine shaft; G_3 - mass flow rate of turbogas per second. Let us write expression (10.21) so:

$$M_{\text{т}} = (A - B\omega)\, G_3, \qquad (10.22)$$

where the coefficients, which characterize the operating conditions of the turbine, will be:

$$A=2m_1r_д C_1\cos\alpha_1\cdot\eta_т; \qquad (10.23)$$

$$B=2m_2r_д\eta_т. \qquad (10.24)$$

Coefficients m_1 and m_2 are determined by the number of stages and depend both on the construction of the turbine and its operating conditions. According to equation (7.216) the exit velocity from the nozzle

$$C_1=\varphi_1\sqrt{2\Delta i_т+C_0^2}. \qquad (10.25)$$

The torsional moment, required by the pump, will be equal to

$$M_{нi}=\frac{p_{нi}G_i}{\rho_{жi}\omega\eta_{нi}}, \qquad (10.26)$$

where for engineering calculations it is possible to consider

$$p_{нi}=D_i\omega^2-D_i'\omega G_i-D_i''G_i. \qquad (10.27)$$

Equation (10.20) for the turbopump unit, which has n pumps, takes the following form:

$$(A-B\omega)G_3\eta_т=\sum_1^n\frac{p_{нi}G_i}{\rho_{жi}\omega_i\eta_{нi}}. \qquad (10.28)$$

Equation in small deviations will be written so:

$$\Delta M_т=\sum\Delta M_{нi}; \qquad (10.29)$$

$$\Delta M_{нi}=\frac{\partial M_{нi}}{\partial p_{нi}}\Delta p_{нi}+\frac{\partial M_{нi}}{\partial G_i}\Delta G_i-\frac{\partial M_{нi}}{\partial\omega_i}\Delta\omega_i. \qquad (10.30)$$

As the fourth equation let us draw on the equation of the hydraulic circuit of the generator. If the feed of single-component means of generation is performed by a pump, then

$$\psi_4 = p_{64} + D_4\omega^2 - D'_4\omega G_4 - D''_4 G_4^2 - \frac{\beta_{\text{гг}}}{F_{\text{кр.гг}}} G_4 - a_4 G_4^{n_4} - a_{\text{д}4} G_4^{n_4} = 0, \tag{10.31}$$

where $\beta_{\text{гг}}$ - specific pressure pulse for the generator; $a_{\text{д}4}$ - coefficient of hydraulic resistance on the generator line.

If the generator is fed by bipropellant, then for calculation one should draw on a fifth equation, and in expression (10.31) replace the product of $\frac{\beta_{\text{гг}}}{F_{\text{кр.гг}}}G_4$ by term

$$p_{\text{гг}} = \frac{\beta_{\text{гг}}}{F_{\text{кр.гг}}}(G_4 + G_5). \tag{10.32}$$

When using monopropellant the consumption of steam gas

$$G_4 = f(t), \tag{10.33}$$

and in the case of application of bipropellant

$$(G_4 + G_5) = f_1(t). \tag{10.34}$$

If the basic components are used, then in equations (10.4) and (10.16) up to the place of bleed of components into the generator there should be written $(G_1 + G_4)$ and $(G_2 + G_5)$, and after bleed - G_1 and G_2 respectively (see Fig. 8.5).

The equation in small deviations for expression (10.31) will be written so:

$$a_{43}\Delta\omega + a_{44}\Delta G_4 + \Delta V_4 + \Delta P_4 = c_4; \tag{10.35}$$

$$\Delta V_4 = a_{45}\Delta p_{64} + a_{46}\Delta D_4 + a_{47}\Delta D'_4 + a_{48} D''_4; \tag{10.36}$$

$$\Delta P_4 = a_{4\,11}\Delta a_{\text{д}4}. \tag{10.37}$$

10.3. System of Calculation Equations

Let us examine several schemes and write the systems of equations for them in small deviations. During soltuion of problems about the effect of external factors and when determining errors c_i in all equations one should take $P_i = 0$.

Engine with loaded tanks and gas pressure accumulators

The propellant components are forced from the tanks by gas, entering the tanks from accumulators (see Fig. 8.1). The engine is equipped with control elements, installed in hydraulic circuits, therefore

$$\left.\begin{aligned} a_{11}\Delta G_1 + a_{12}\Delta G_2 + \Delta V_1 + \Delta P_1 = c_1; \\ a_{21}\Delta G_1 + a_{22}\Delta G_2 + \Delta V_2 + \Delta P_2 = c_2. \end{aligned}\right\} \quad (10.38)$$

During engine tuning we assume

$$\Delta G_1 = \Delta G_2 = 0. \quad (10.39)$$

whereupon we immediately find ΔP_1 and ΔP_2.

Engine with loaded tanks and liquid propellant pressure accumulators

The propellant components are forced from the tanks by the combustion products of the basic components, which enter the tanks from generators (see Fig. 8.2). Control elements are available in all hydraulic circuits. The system will contain two equations of main hydraulic circuits, two equations of hydraulic circuits of the oxidizer tank generator and two equations of hydraulic circuits of the fuel tank generator:

$$\left.\begin{aligned}
a_{11}\Delta G_1+a_{12}\Delta G_2+a_{13}\Delta G_{11}+a_{14}\Delta G_{21}+\Delta V_1+\Delta P_1&=c_1;\\
a_{21}\Delta G_1+a_{22}\Delta G_2+a_{23}\Delta G_{12}+a_{24}\Delta G_{22}+\Delta V_2+\Delta P_2&=c_2;\\
a_{33}\Delta G_{11}+a_{34}\Delta G_{21}+\Delta V_3+\Delta P_3&=c_3;\\
a_{43}\Delta G_{11}+a_{44}\Delta G_{21}+\Delta V_4+\Delta P_4&=c_4;\\
a_{53}\Delta G_{12}+a_{54}\Delta G_{22}+\Delta V_5+\Delta P_5&=c_5;\\
a_{63}\Delta G_{12}+a_{64}\Delta G_{22}+\Delta V_6+\Delta P_6&=c_6;
\end{aligned}\right\} \tag{10.40}$$

where G_1 - inflow of oxidizer into chamber; G_2 - inflow of fuel into chamber; $G_{1\,1}$ - inflow of oxidizer into oxidizer tank; $G_{2\,1}$ - inflow of fuel into oxidizer tank; $G_{1\,2}$ - inflow of oxidizer into fuel tank; $G_{2\,2}$ - inflow of fuel into fuel tank.

Engine with turbopump unit

Variant No. 1

The basic components are fed to the chamber by pumps. The means of generation is accomplished from the third component in the generator with loaded tank. All three hydraulic circuits are equipped with control elements (see Fig. 8.3):

$$\left.\begin{aligned}
a_{11}\Delta G_1+a_{12}\Delta G_2+a_{13}\Delta\omega\qquad\qquad\quad +\Delta V_1+\Delta P_1&=c_1;\\
a_{21}\Delta G_1+a_{22}\Delta G_2+a_{23}\Delta\omega\qquad\qquad\quad +\Delta V_2+\Delta P_2&=c_2;\\
a_{31}\Delta G_1+a_{32}\Delta G_2+a_{33}\Delta\omega+a_{34}\Delta G_3+\Delta V_3+\Delta P_3&=c_3;\\
a_{44}\Delta G_3+\Delta V_4+\Delta P_4&=c_4.
\end{aligned}\right\} \tag{10.41}$$

During adjustment all $\Delta G_i = 0$; the fourth equation is used for determination of ΔP_4, the third - for determination of $\Delta\omega$ when $\Delta P_3 = 0$, and by the first and second we find ΔP_1 and ΔP_2.

Variant No. 2

Unlike the previous scheme to the hydraulic circuit of the generator is connected a pump ($G_3 = G_4$) (see Fig. 8.4):

$$
\left.
\begin{aligned}
&a_{11}\Delta G_1+a_{12}\Delta G_2+a_{13}\Delta\omega \qquad +\Delta V_1+\Delta P_1=c_1;\\
&a_{21}\Delta G_1+a_{22}\Delta G_2+a_{23}\Delta\omega \qquad +\Delta V_2+\Delta P_2=c_2;\\
&a_{31}\Delta G_1+a_{32}\Delta G_2+a_{33}\Delta\omega+a_{34}\Delta G_3+\Delta V_3 \qquad =c_3;\\
&\qquad\qquad a_{43}\Delta\omega+a_{44}\Delta G_3+\Delta V_4+\Delta P_4=c_4.
\end{aligned}
\right\} \tag{10.42}
$$

Here it is accepted that $\Delta P_3 = 0$.

For realization of adjustment we take

$$\Delta G_1=\Delta G_2=\Delta\omega=0. \tag{10.43}$$

By the third equation we find ΔG_3, by the fourth - the value of ΔP_4, which provides the required value of ΔG_3. Let us determine values of ΔP_1 and ΔP_2 by the first and second equations.

Variant No. 3

The turbine is fed from the generator, operating on basic components (see Fig. 8.5); the system of equations takes the following form:

$$
\left.
\begin{aligned}
&a_{11}\Delta G_1+a_{11}\Delta G_2+a_{13}\Delta\omega+a_{14}\Delta G_4 \qquad +\Delta V_1+\Delta P_1=c_1;\\
&a_{21}\Delta G_1+a_{22}\Delta G_2+a_{23}\Delta\omega \qquad +a_{25}\Delta G_5+\Delta V_2+\Delta P_2=c_2;\\
&a_{31}\Delta G_1+a_{32}\Delta G_2+a_{33}\Delta\omega+a_{34}\Delta G_4+a_{35}\Delta G_5+\Delta V_3 \qquad =c_3;\\
&a_{41}\Delta G_1 \qquad +a_{43}\Delta\omega+a_{44}\Delta G_4+a_{45}\Delta G_5+\Delta V_4+\Delta P_4=c_4;\\
&\qquad a_{52}\Delta G_2+a_{53}\Delta\omega+a_{54}\Delta G_4+a_{55}\Delta G_5+\Delta V_5+\Delta P_5=c_5.
\end{aligned}
\right\} \tag{10.44}
$$

Adjustment is reduced to determination of $\Delta\omega$ by the third equation and P_i - by the remaining equations. Let us note that analogous systems can be easily written for any other schemes.

10.4. Determination of Coefficients of Equations

Let us determine the coefficients by equation (10.4), which we rewrite in the following manner:

$$\psi_1 = p_{б1} + D_1\omega^2 - D'_1\omega G_1 - D''_1 G_1^2 - \frac{\xi}{F_{кр}}(G_1 + G_2) - a_1 G_1^{n_1} - a_{21} G_1^{n_1} = c_1. \qquad (10.45)$$

When performing engineering calculations we take $n_1 = n_2 = 2$. Let us recall that

$$a_{ik} = \frac{\partial \psi_i}{\partial x_k}; \qquad (10.46)$$

for example, coefficients

$$a_{11} = \frac{\partial \psi_1}{\partial G_1}; \quad a_{12} = \frac{\partial \psi_1}{\partial G_2}; \quad a_{13} = \frac{\partial \psi_1}{\partial \omega}. \qquad (10.47)$$

When computing partial derivatives all the parameters, furthermore, with respect to which the derivative is taken, are considered constant. Derivatives according to basic parameters will be:

$$a_{11} = \left(-D'_1\omega - 2D''_1 G_1 - \frac{\xi}{F_{кр}} - n_1 a_1 G_1^{n_1 - 1} - n_1 a_{21} G_1^{n_1 - 1}\right)_*. \qquad (10.48)$$

The asterisk means that the numerical values of quantities, enclosed in brackets, are determined for steady state.

Thus, the asterisk is equivalent to the following:

$$D'_1 = D'_{10}; \quad D''_1 = D''_{10}; \quad \xi = \xi_0, \text{ etc.} \qquad (10.49)$$

Let us calculate coefficients

$$a_{12} = -\left(\frac{\xi}{F_{кр}}\right)_* = -\left(\frac{p_к}{G_1 + G_2}\right)_*; \qquad (10.50)$$

$$a_{13} = (2D_1\omega - D'_1 G_1)_*. \qquad (10.51)$$

In order to determine V_1, it is necessary to use formula (10.13). For the considered case it is written so:

$$\Delta V_1 = a_{1\,5}\Delta p_{61} + a_{1\,6}\Delta D_1 + a_{1\,7}\Delta D_1' + a_{1\,8}\Delta D_1'' + a_{1\,9}\Delta\beta + a_{1\,10}\Delta a_1. \tag{10.52}$$

Small deviations are calculated by the results of tests. For coefficients we find

$$\left.\begin{aligned} & a_{1\,5} = \frac{\partial\psi_1}{\partial p_{61}} = 1; \text{ if } p_{61} = \text{const}; \\ & a_{1\,6} = \frac{\partial\psi_1}{\partial D_1} = (\omega^2)_*; \; a_{1\,7} = -(\omega G_1)_*; \\ & a_{1\,8} = -(G_1^2)_*; \; a_{1\,9} = -\left(\frac{G_1+G_2}{F_{\text{кр}}}\right)_*, \\ & a_{1\,10} = -(G_1^2)_*. \end{aligned}\right\} \tag{10.53}$$

If it is necessary to take into account the deviation of power n_1 from nominal, then to the right side of expression (10.52) one should add $a_{1\,12}\Delta n_1$,

$$a_{1\,12} = \frac{\partial\psi_1}{\partial n_1} = \left[(a_1 + a_{\text{в}1})G_1^{n_1}\ln(G_1)\right]_*. \tag{10.54}$$

Of great interest is the study of the effect of temperature and deviation of sizes of various parts from their nominal values at engine operating conditions.

These problems are solved with the attraction of equations similar to those examined, and with utilization of the relationship of density and viscosity of liquids to temperature.

We will consider that constant coefficients of equations can be determined experimentally. We distinguish individual and average-statistical values of parameters. Individual is a value, obtained by results of treatment of the experiment with one engine or its unit. From the data of tests it is possible to obtain the individual values of the following parameters: a_1, a_2, D_1, D_2, D_1', D_2', etc.

During firing tests we determine the parameters, the values of which were not calculated from results of shop tests, for example, $p_{6\,i}$, β, G_i and ω.

The average values of any parameter, including disturbance ΔV_i, are calculated on the basis of mathematical treatment of statistical data of results of shop, bench or flight tests of many engines and their units. All objects, the quantity of which (and, consequently, the number of tests) n, should refer to one batch, i.e., be identical in construction and test conditions.

When conducting engine tests the value of the parameter, obtained during one test x_i, is a random discrete quantity and to each such quantity corresponds probability $p_i = \frac{m}{n}$, where m - number of obtained identical values of x_i. Always $p_i > 0$, and $\Sigma p_i = 1$. Function $p_i = \phi(x_i)$, called the law of distribution, is represented in a table or in the form of a graph. The treatment of a large number of engine tests shows that the law of distribution is close to standard, which is characterized by equation

$$\varphi(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x_i-\bar{x})^2}{2\sigma^2}\right], \qquad (10.55)$$

where σ - mean square deviation:

$$\sigma=\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}; \qquad (10.56)$$

$$\bar{x}=\frac{\sum x_i}{n}; \qquad (10.57)$$

$$\int_{-\infty}^{+\infty}\varphi(x)\,dx=1. \qquad (10.58)$$

The curve of standard distribution is symmetric relative to ordinate $x = \bar{x}$ and asymptotically approaches the axis of abscissa when $x = \pm\infty$. When $x = \bar{x}$ the curve has a maximum; at point $x = \bar{x} \pm \sigma$ has inflection. As one of the characteristic quantities we take the value of 3σ, at which probability $p < 0.003$. The smaller σ is, the more closely arranged the curve of distribution is near the axis of ordinates, i.e., near axis $\psi(x)$.

When tuning the engine to the required operating conditions it is necessary to determine the mean arithmetic value of disturbance, which we designate $\Delta\bar{V}_S$. Subscript "S" corresponds to the number of equation of any of the systems from (10.38) to (10.44).

Let us examine equation

$$\sum a_{Sk}\Delta x_k + \Delta V_S = 0, \tag{10.59}$$

where in accordance with systems (10.38)-(10.44) subscript "k" - the number of variable parameter. When conducting shop (or firing) tests of n engines we obtain n values of a_{iSk}. The mean arithmetic value of each a_{Sk}:

$$\bar{a}_{Sk} = \frac{1}{n}\sum_i a_{iSk}, \tag{10.60}$$

where i - the serial number of the engine, which underwent shop tests. When conducting firing tests we measure the flow rates G_i and the angular velocity of rotation of the turbopump unit shaft and record their deviations from nominals Δx_k. The mean arithmetic value of each Δx_k:

$$\overline{\Delta x}_k = \frac{1}{m}\sum_l \Delta x_{lk}, \tag{10.61}$$

where m - the number of firing tests; l - serial number of engine, which underwent firing tests. The mean arithmetic value of disturbance

$$\Delta\bar{V}_S = -\sum_k \overline{a_{Sk}\Delta x_k}. \tag{10.62}$$

By using the expression for the mean value of the product of two random quantities, we obtain

$$\Delta\overline{V}_S = -\sum_k (\overline{a}_{Sk}\overline{\Delta x}_k + K); \quad (10.63)$$

$$K = r_{a,\Delta x}\sigma_a\sigma_{\Delta x}. \quad (10.64)$$

Here r - coefficient of correlation; σ - mean square deviation.

10.5. Calculation of Power Plant

Engine adjustment

Now it is possible to write the system of equations for engine adjustment:

$$a_{i1}\Delta G_1 + a_{i2}\Delta G_2 + a_{i3}\Delta\omega + a_{i4}\Delta G_3 + \Delta\overline{V}_i + \Delta P_i = c_i \quad (10.65)$$
$$(i = 1,\ 2,\ 3,\ 4).$$

In equations (10.65) the values of a_{ik}, $\Delta\overline{V}_i$ and $\overline{c}_i$ are calculated from results of processing experimental data. For adjustment of a new engine it is necessary in the beginning to assume

$$\Delta G_i = \Delta\omega = 0, \quad (10.66)$$

inasmuch as providing condition (10.66) is the purpose of adjustment.

Thus, for determination of ΔP_i we have

$$\Delta\overline{V}_i + \Delta P_i = c_i \quad (10.67)$$
$$(i = 1,\ 2,\ 3,\ 4).$$

In order to take into account the specific features of the engine being tuned, it is necessary to calculate $\Delta\overline{V}_S$ by expression (10.62), having accepted a_{ik} from data for the series of engines tested earlier and having substituted values of Δx_k, characteristic for engines being prepared for tests. For other methods of adjustment see [17], [64].

Determination of results of the effect of external factors

The values of small deviations of basic parameters are calculated at $P_i = 0$. Computation of these numbers is called the solution of the problem about the effect of external factors. The calculation system has the following form:

$$\bar{a}_{i1}\Delta G_1+\bar{a}_{i2}\Delta G_2+\bar{a}_{i3}\Delta G_3+\bar{a}_{i4}\Delta\omega=\bar{c}_i-\Delta\bar{V}_i \quad (i=1,\ 2,\ 3,\ 4). \tag{10.68}$$

In many cases it is possible to represent the system by three equations. This can be achieved by solution of the fourth equation relative to ΔG_3 and replacement of ΔG_3 in the third equation by the obtained numerical value. If for example we are guided by equation (10.41), then the system takes the following form:

$$\left.\begin{aligned}\bar{a}_{11}\Delta G_1+\bar{a}_{12}\Delta G_2+\bar{a}_{13}\Delta\omega&=\Delta\bar{v}_1;\\ \bar{a}_{21}\Delta G_1+\bar{a}_{22}\Delta G_2+\bar{a}_{23}\Delta\omega&=\Delta\bar{v}_2;\\ \bar{a}_{31}\Delta G_1+\bar{a}_{32}\Delta G_3+\bar{a}_{33}\Delta\omega&=\Delta\bar{v}_3,\end{aligned}\right\} \tag{10.69}$$

moreover

$$\Delta\bar{v}_1=\bar{c}_1-\Delta\bar{V}_1; \tag{10.70}$$

$$\Delta\bar{v}_2=\bar{c}_2-\Delta\bar{V}_2; \tag{10.71}$$

$$\Delta\bar{v}_3=\bar{c}_3-\Delta\bar{V}_3-\frac{\bar{a}_{34}}{\bar{a}_{44}}\left(\bar{c}_4-\Delta\bar{V}_4\right). \tag{10.72}$$

Solutions, as is known, have the following form:

$$\Delta c_i=\frac{D_i}{D}. \tag{10.73}$$

The principal determinant:

$$D=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}. \tag{10.74}$$

Additional determinants:

$$D_1 = \begin{vmatrix} \Delta v_1 & a_{12} & a_{13} \\ \Delta v_2 & a_{22} & a_{23} \\ \Delta v_3 & a_{32} & a_{33} \end{vmatrix}; \tag{10.75}$$

$$D_2 = \begin{vmatrix} a_{11} & \Delta v_1 & a_{13} \\ a_{21} & \Delta v_2 & a_{23} \\ a_{31} & \Delta v_3 & a_{33} \end{vmatrix}; \tag{10.76}$$

$$D_3 = \begin{vmatrix} a_{11} & a_{12} & \Delta v_1 \\ a_{21} & a_{22} & \Delta v_2 \\ a_{31} & a_{32} & \Delta v_3 \end{vmatrix}. \tag{10.77}$$

Solutions (10.73) are not precise. The coefficients are accepted as constant, although in actuality some of them depend on the sought variables or vary with time. Consequently, in actuality research should be based on nonlinear equations.

With transition to small finite deviations the nonlinear remainders were dropped which led to decrease of the accuracy of solution. In engine equations we encounter components, containing variables to the second or third power or the product of variables. Let us assume there will be assigned function

$$\varphi = ax^2. \tag{10.78}$$

By dropping the nonlinear remainders, we obtain

$$\Delta\varphi = 2ax\Delta x. \tag{10.79}$$

By applying expansion into Taylor series, we find

$$\Delta\varphi = 2ax\Delta x + a(\Delta x)^2, \tag{10.80}$$

where $(\Delta x)^2$ - nonlinear remainder. Expression (10.80) can be written so:

$$\Delta\varphi = \varepsilon 2ax\Delta x. \tag{10.81}$$

By comparing expressions (10.80) and (10.81), we find that the correction factor

$$\varepsilon=1+\frac{1}{2}\frac{\Delta x}{x}. \tag{10.82}$$

The smaller the relative value of small deviation, the less is the error, which appears as a result of disregarding the nonlinear remainders.

In order to exclude errors, one should write the equations of small finite deviations so that components, obtained from functions of type (10.78), would contain factor ε. In the examination of processes in the neighborhood one should be guided by average-statistical value $(\Delta x)^*$ and nominal value x^*. In this case ε will be presented in the form of number ε_*, characteristic for the considered problem:

$$\varepsilon_*=1+\frac{1}{2}\frac{(\Delta x)_*}{x_*}. \tag{10.83}$$

If the function, containing the variable to the third power, is assigned, then for determination of the correction factor we will have

$$\varepsilon_*=1+\frac{(\Delta x)_*}{x_*}+\frac{1}{3}\frac{(\Delta x)_*^2}{x_*^2}. \tag{10.84}$$

Analysis of calculation data shows that consideration of the third component in expression (10.84) rarely leads to noticeable refinement of the results of calculation.

10.6. Dynamic Processes, Proceeding in the Neighborhood of Assigned Conditions

After the engine reaches cruise conditions, as a result of influences on the part of regulation or control systems, and also under the action of external factors small changes in the basic engine parameters can be observed.

The changes of absolute values of parameters are determined by using the equations of statics given in this chapter. For determination of the amount of change of parameters with time there are necessary equations, which characterize the dynamic processes, proceeding in the neighborhood of some assigned condition.

Let us consider for example the equation of the hydraulic circuit of fuel for an engine with gas pressure feed of components in the following form:

$$b\dot{G}=p_6-aG^2-\frac{\beta}{F_{кр}}(1+k_1)G. \tag{10.85}$$

In the neighborhood of static values a_*, G_*, etc., the deviations be equal to

$$\Delta x=G-G_*;\quad \Delta a=a-a_*. \tag{10.86}$$

From expression (10.86) we find that derivative

$$\Delta\dot{x}=\dot{G}. \tag{10.87}$$

Equation (10.85) in this case takes the following form:

$$b\Delta\dot{x}=(p_{6*}+\Delta p_6)-(a_*+\Delta a_*)(G_*+\Delta x)^2-$$
$$-(B_*+\Delta B)(G_*+\Delta x), \tag{10.88}$$

where

$$B_*=\left[\frac{\beta}{F_{кр}}(1+k_1)\right]_*. \tag{10.89}$$

It is obvious that

$$\Delta B=\frac{1+k_1}{F_{кр}}\Delta\beta+\frac{\beta}{F_{кр}}\Delta k_1-\frac{\beta}{F^2_{кр}}(1+k_1)\Delta F_{кр}. \tag{10.90}$$

By subtracting from expression (10.88) the equation of statics

$$p_{6*}-a_*G_*^2-\left[\frac{\xi}{F_{кр}}(1+k_i)\right]_* G_*=0 \qquad (10.91)$$

and disregarding the terms, containing products of small deviations, we obtain

$$b\Delta\dot{x}=-A\Delta x+\Delta\overline{V}_i, \qquad (10.92)$$

where

$$A=2a_*G_*+B_*; \qquad (10.93)$$

$$\Delta\overline{V}_i=\Delta p_6-G_*^2\Delta a-G_*\Delta B. \qquad (10.94)$$

Let us recall that in the accepted meanings disturbance ΔV_i represents small deviation [see equation (10.13)].

Deviation $\Delta\overline{V}_i$, disturbing the assigned condition and leading to the onset of transient process, should be assigned. In a more general case $\Delta\overline{V}_i = f(t)$. If the change of parameters, which enter expression (10.94), proceeds rather rapidly, then the process of onset of disturbance can be considered intermittent.

If $\Delta\overline{V}_i = f(t)$, then expression (10.92) will be written so:

$$\Delta\dot{x}=-\frac{A}{b}\Delta x+\frac{f(t)}{b}. \qquad (10.95)$$

The solution of linear equation (10.95) has the following form:

$$\Delta x=\exp\left(-\frac{A}{b}t\right)\left[C+\frac{1}{b}\int_0^t \Delta f(t)\exp\left(\frac{A}{b}t\right)dt\right]. \qquad (10.96)$$

If we assume $\Delta\overline{V}_i$ = const, then the solution will be written so:

$$\Delta x=\frac{\Delta\overline{V}_i}{A}+C\exp\left(-\frac{A}{b}t\right). \qquad (10.97)$$

For determination of integration constant C let us take initial conditions: when $t = 0$; $\Delta x = 0$. Consequently,

$$C = -\frac{\Delta \bar{V}_l}{A}. \tag{10.98}$$

Finally we receive

$$\Delta x = \frac{\Delta \bar{V}_l}{A}\left[1 - \exp\left(-\frac{A}{b}t\right)\right]. \tag{10.99}$$

Let us examine expression (10.94). Deviation $\Delta p_\text{б}$ can appear as a result of errors in the operation of a reduction gear. On the other hand, the reduction gear could be used for control of the dynamic process. The change of pressure in the upper cavity of the tank requires a certain time, therefore

$$\Delta p_\text{б} = \varphi(t). \tag{10.100}$$

The character of the change of pressure with time can be determined with utilization of the given equations. If we consider that during the period of influence the change of pressure in the tank can be disregarded, then

$$\Delta p_\text{б} = 0. \tag{10.101}$$

Deviation Δa [see equation (10.94)] appears because of the effect of tolerances. Under plant conditions deviation a from nominal is determined by hydraulic pressure drop tests. The coefficient of hydraulic losses is also changed under the action of external factors as a result of change of viscosity $\nu_\text{ж}$. This is observed, for example, with the change in ambient temperature.

The deviation of hydraulic resistance from its nominal value could be used for control of dynamic processes. The movement of working organs of control elements occurs rather rapidly. Therefore, being guided by expression (10.99), we will have

$$\Delta \dot{x}=\frac{G_a^2}{A}\left[1-\exp\left(-\frac{A}{b}t\right)\right]\Delta a. \qquad (10.102)$$

Deviation $\Delta\beta$ [see equation (10.90)] can be caused by change of pressure, ratio and properties of propellant components. Deviation $\Delta F_{\text{кр}}$ is caused by industrial tolerances, and also by the action of power loads and thermal expansion, appearing during the operation of engines.

In expression (10.99) parameter A under a predetermined influence defines both the intensity of damping and the absolute value of deviation.

Decrease of A leads to increase of deviation Δx and acceleration of the dynamic process. For decrease of Δx one should increase A. The mass coefficient b affects only the intensity of damping.

For acceleration of the dynamic process the quantity of coefficient

$$b=\sum_{1}^{n}\int_{0}^{l_i}\frac{dl_i}{F(l)} \qquad (10.103)$$

should be decreased.

According to expression (10.99) the greatest value of deviation

$$\Delta x_{max}=\frac{\Delta\overline{V}_i}{A} \qquad (10.104)$$

and the rate of change of deviation

$$\Delta\dot{x}=\frac{\Delta\overline{V}_i}{b}\exp\left(-\frac{A}{b}t\right). \qquad (10.105)$$

The greatest rate of change corresponds to moment of time $t = 0$; in this case

$$\Delta\dot{x}_{max}=\frac{\Delta\overline{V}_t}{b}. \tag{10.106}$$

The dynamic processes, which proceed in the neighborhood of assigned conditions, for an engine on the whole are characterized by a system of equations, written according to the type of equation (10.92). The quantity and structure of equations will depend on the engine configuration and features of its operating conditions.

Thus, being guided, for example, by system of equations (10.41), for an engine with unloaded tanks we will have

$$\left.\begin{aligned}
\Delta\dot{x}_1&=a'_{11}\Delta x_1+a'_{12}\Delta x_2+a'_{13}\Delta x_3+\Delta\varphi_1(t);\\
\Delta\dot{x}_2&=a'_{21}\Delta x_1+a'_{22}\Delta x_2+a'_{23}\Delta x_3+\Delta\varphi_2(t);\\
\Delta\dot{x}_3&=a'_{31}\Delta x_1+a'_{32}\Delta x_2+a'_{33}\Delta x_3+a'_{34}\Delta x_4+\Delta\varphi_3(t);\\
\Delta\dot{x}_4&=a'_{44}\Delta x_4+\Delta\varphi_4(t).
\end{aligned}\right\} \tag{10.107}$$

For agreement of coefficients of equations (10.107) with coefficients of equations (10.41) it is necessary to use the following formulas:

$$\left.\begin{aligned}
&a'_{1i}=\frac{a_{1i}}{b_1};\quad a'_{2i}=\frac{a_{2i}}{b_2};\\
&a'_{3i}=\frac{a_{3i}}{b_3};\\
&\Delta\varphi_i(t)=\Delta V_i+\Delta P_i-c_i.
\end{aligned}\right\} \tag{10.108}$$

In the examination of many cases, having practical value, it can be considered that the acquisition of a new mode by the generator preceeds dynamic processes, proceeding in the main engine units. System of equations (10.107) is presented in the form of two independent parts. The fourth equation of the system characterizes the change of flow rate of generation means with time, having begun under the effect of pressure $\phi_4(t)$:

$$\Delta\dot{x}_4=a'_{44}\Delta x_4+\varphi_4(t). \tag{10.109}$$

For determination of Δx_4 or Δx_{4max} we use the solutions given in this section.

Now Δx_4 characterizes disturbance; the system of calculation equations will be written so:

$$\left.\begin{aligned}\Delta\dot{x}_1&=a'_{11}\Delta x_1+a'_{12}\Delta x_2+a'_{13}\Delta x_3+\Delta\varphi_1(t);\\ \Delta\dot{x}_2&=a'_{21}\Delta x_1+a'_{22}\Delta x_2+a'_{23}\Delta x_3+\Delta\varphi_2(t);\\ \Delta\dot{x}_3&=a'_{31}\Delta x_1+a'_{32}\Delta x_2+a'_{33}\Delta x_3+\Delta\Phi_3(t),\end{aligned}\right\} \tag{10.110}$$

where

$$\Delta\Phi_3(t)=\Delta\varphi_3(t)-\frac{a'_{34}}{a'_{44}}\Delta\varphi_4(t). \tag{10.111}$$

Equations (10.110) can be used in examination of the following cases.

1. Disturbance of the system is caused only by the change of operating conditions of the generator. In this case

$$\Delta\varphi_1(t)=\Delta\varphi_2(t)=0. \tag{10.112}$$

2. The system is disturbed by the action of external factors. Values of $\phi_1(t)$, $\Delta\phi_2(t)$, $\Delta\Phi_3(t)$ should be assigned.

3. Transient processes are caused both by external factors and by influence on the part of the generator. Here $\Delta\phi_1(t)$ and $\Delta\phi_2(t)$ are assigned, and $\Delta\Phi_3(t)$ is determined from expression (10.111).

4. When $t = 0$ deviations Δx_{i0} are nonzero, but are known according to results of processing experimental data or on the basis of analysis of previous calculations. Values of Δx_{i0} can be used when assigning initial conditions. If during the transient process there is no disturbance, then instead of equations (10.110) we will have a linear homogeneous system:

$$
\left.\begin{aligned}
\Delta\dot{x}_1 &= a'_{11}\Delta x_1 + a'_{12}\Delta x_2 + a'_{13}\Delta x_3;\\
\Delta\dot{x}_2 &= a'_{21}\Delta x_1 + a'_{22}\Delta x_2 + a'_{23}\Delta x_3;\\
\Delta\dot{x}_3 &= a'_{31}\Delta x_1 + a'_{32}\Delta x_2 + a'_{33}\Delta x_3.
\end{aligned}\right\} \qquad (10.113)
$$

Let us examine a case when all a'_{ik} - constant coefficients. System of equations (10.113) has particular solutions of this type:

$$
\Delta x_i = \gamma_i \exp(kt) \quad (i = 1, 2, 3), \qquad (10.114)
$$

where γ_i - pre-exponential factor; k - the root of characteristic equation.

By substituting in system (10.113) the values from solution (10.114), collecting coefficients at γ_i and reducing by $\exp(kt)$, we obtain the following system of algebraic equations:

$$
\left.\begin{aligned}
(a'_{11}+k)\gamma_1 + a'_{12}\gamma_2 + a'_{13}\gamma_3 &= 0;\\
a'_{21}\gamma_1 + (a'_{22}+k)\gamma_2 + a'_{23}\gamma_3 &= 0;\\
a'_{31}\gamma_1 + a'_{32}\gamma_2 + (a'_{33}+k)\gamma_3 &= 0.
\end{aligned}\right\} \qquad (10.115)
$$

If system (10.113) contains n variables, then instead of system (10.115) we will have

$$
\left.\begin{aligned}
(a'_{11}+k)\gamma_1 + a'_{12}\gamma_2 + \ldots \quad &+ a'_{1n}\gamma_n = 0;\\
a'_{21}\gamma_1 + (a'_{22}+k)\gamma_2 + \ldots \quad &+ a'_{2n}\gamma_n = 0;\\
\cdots\cdots\cdots\cdots\cdots\cdots &\cdots\cdots\\
a'_{n1}\gamma_1 + a'_{n2}\gamma_2 + \ldots \qquad &+ (a'_{nn}+k)\gamma_n = 0.
\end{aligned}\right\} \qquad (10.116)
$$

To get the nontrivial solution of (10.114) the determinant of system (10.115) or (10.116) should be equal to zero. The equation obtained in this case is called characteristic. With n variables we will have

$$D=\begin{vmatrix} (a'_{11}+k)\,a'_{12}\ldots & a'_{1n} \\ a'_{21}\,(a'_{22}+k)\ldots & a'_{2n} \\ \ldots\ldots\ldots\ldots & \\ a'_{n1} \quad a'_{n2}\ldots(a'_{nn}+k) & \end{vmatrix} \qquad (10.117)$$

Usually during solution of equations, which describe the dynamic processes of the engine, all roots of characteristic equation prove to be various. In this case to each root k_S corresponds to particular solution:

$$\Delta x_{iS}=\gamma_{iS}\exp(k_S t). \qquad (10.118)$$

The total solution of system (10.113) will be written so:

$$\Delta x_i=\sum_{1}^{S=n} C_S\gamma_{iS}\exp(k_S t), \qquad (10.119)$$

where C_S - constant, determined according to initial conditions.

By analyzing expression (10.119) according to values of C_S, γ_{iS} and k_S, it is sometimes possible to decrease the number of components in solutions.

Contemporary computer methods allow deviating from the use of any kind of approximate solutions and permit investigating equation systems of the type (10.107), written, where possible, more accurately. However, one should not neglect simplifications, which can be accomplished by the results of analysis of computer calculation and experiment, if the accuracy of solutions in this case will remain within the assigned limits.

The utilization of computers allows assigning some values of a'_{ik} in the form of functions of engine parameters, which vary with time.

Example.

It is required to determine the coefficients of the equation of the first hydraulic circuit and to investigate it.

Given.

Values of parameters at calculated conditions:

flow rate of oxidizer G_1 = 100 kg/s;

flow rate of fuel G_2 = 22.8 kg/s;

pressure pulse β = 2600 N·s/kg;

pressure in the chamber $p_{\text{к}}$ = 10 MN/m^2;

pressure losses in hydraulic circuit Δp_1 = 1.3 MN/m^2;

pressure losses on a throttle, installed in hydraulic circuit, $\Delta p_{\text{Д}1}$ = 0.5 MN/m^2;

pressure in the tank $p_{\text{б}1}$ = 0.4 MN/m^2;

pressure, created by external forces $p_1 = 0$;

excess pressure, created by pump $p_{\text{Н}1}$ = 11.4 MN/m^2;

excess pressure, created by pump when $G_1 = 0$, $p_{\text{Н}1\,0}$ = = 12.54 MN/m^2;

the pump characteristic is linear;

rpm of shaft n = 500.

Preparation of material.

Equation of the first hydraulic circuit will be written so:

$$a_{1\,1}\Delta G_1+a_{1\,2}\Delta G_2+a_{1\,3}\Delta n+\Delta V_1+\Delta P_1=0.$$

For determination of the intensity of distrubance we have

$$\Delta V_1=a_{1\,5}\Delta p_{б1}+a_{1\,6}\Delta D_1+a_{1\,7}\Delta D_1'+a_{1\,9}\Delta\beta+a_{1\,10}\Delta a_1-c_1.$$

$a_{1\,8}\Delta D_1''=0$, inasmuch as with linear characteristic $D_1''=0$. Engine adjustment is performed by the throttle, installed in hydraulic circuit. Intensity of adjustment $\Delta P_1=a_{1\,11}\Delta a_{д1}$.

Solution.

Let us calculate the engine coefficients.

The coefficient of hydraulic losses of the first circuit:

$$a_1=\frac{\Delta p_1}{G_1^2}=\frac{1{,}3\cdot 10^6}{10^4}=130\ \ N\cdot s^2/m^2\cdot kg^2.$$

The coefficient of hydraulic losses of adjustment element - throttle

$$a_{д1}=\frac{\Delta p_{д1}}{G_1^2}=\frac{0{,}5\cdot 10^6}{10^4}=50\ \ N\cdot s^2/m^2\cdot kg^2.$$

The nozzle throat area

$$F_{кр}=\frac{\beta}{p_к}(G_1+G_2)=0{,}0319\ \ m^2.$$

The coefficient of pump characteristic. Inasmuch as the characteristic is linear, then

$$D_1=\frac{p_{н\,1\,0}}{n^2}=\frac{12{,}54\cdot 10^6}{500^2}=50{,}16\ \ N\cdot s^2/m^2.$$

Considering the linearity of characteristic, we find

$$D_1' = \frac{p_{\text{н}10} - p_{\text{н}1}}{nG_1} = \frac{12,54 - 11,4}{500 \cdot 100} \cdot 10^6 = 22,8 \quad \text{N} \cdot \text{s}^2/\text{m}^2.$$

Let us compute the coefficients of hydraulic circuit equation:

$$a_{11} = \left[-D_1' n - \frac{\beta}{F_{\text{кр}}} - 2a_1 G_1 - 2a_{\text{з}1} G_1\right]_* = -22,8 \cdot 500 -$$
$$- \frac{2600}{0,0319} - 2 \cdot 130 \cdot 100 - 2 \cdot 50 \cdot 100 = -128\,906 \quad \text{N} \cdot \text{s/m}^2 \cdot \text{kg}.$$

In the considered example the greatest effect on a_{11} has the component, containing pressure pulse;

$$a_{12} = -\left[\frac{\beta}{F_{\text{кр}}}\right]_* = -81\,505 \quad \text{N} \cdot \text{s/m}^2 \cdot \text{kg}.$$

Let us compute $a_{1\,3}$:

$$a_{13} = \left[2D_1 n - D_1' G_1\right]_* = 2 \cdot 50,16 \cdot 500 - 22,8 \cdot 100 = 48\,720 \quad \text{N} \cdot \text{s/m}^2.$$

As one would expect, the numerical value of the first component is greater than the value of the second.

Let us compute the coefficients, entering the expression for determination of the intensity of disturbance.

Taking into account that $\dot{p}_{j1} = 0$, we find

$$a_{15} = 1;$$
$$a_{16} = n_*^2 = 500^2 = 25 \cdot 10^4 \quad \frac{1}{\text{s}^2};$$
$$a_{17} = -[nG_1]_* = -500 \cdot 100 = -5 \cdot 10^4 \quad \text{kg/s}^2;$$
$$a_{19} = -\left[\frac{G_1 + G_2}{F_{\text{кр}}}\right]_* = -\frac{122,8}{0,0319} = -3850 \quad \text{kg/s} \cdot \text{m}^2;$$
$$a_{1\,10} = -G_{1_*}^2 = -10^4 \quad \text{kg}^2/\text{s}^2.$$

Error

$$e_1 = e_1' + e_{1\,\text{оп}},$$

where $c_1^{\prime}$ - error, which appears as a result of not taking into account nonlinear remainders of Taylor series.

For the considered case

$$c_1^{\prime}=D_1(\Delta n)^2-a_1(\Delta G_1)^2-a_{\text{д}1}(\Delta G_1)^2=50{,}16(\Delta n)^2-(180+50)(\Delta G_1)^2.$$

Error $c_{1\text{оп}}$ is determined from results of processing experimental data. During engine designing, if there is still no experimental data, one should assume $c_{1\text{оп}} = 0$.

We calculate

$$\Delta V_1=\Delta p_{\text{б}1}+25\cdot 10^4\Delta D_1-5\cdot 10^4\Delta D_1^{\prime}-3850\Delta\beta-$$
$$-10^4\Delta a_1-50{,}16(\Delta n)^2+230(\Delta G_1)^2.$$

Let us assume that deviations of all parameters are equal to 1% of their calculated values. In this case $p_{\text{б}1} = 0.01\cdot 0.4\cdot 10$ MN/m^2; $\Delta D_1 = 0.01\cdot 50.16$ N$\cdot$s^2/m^2, etc. As a result of calculation of the training engine there is found

$$\Delta V_1=826 \text{ N/m}^2.$$

If deviations of parameters of the second group are equal in percentage, then the greatest disturbance appears with deviation of the coefficient of pump characteristic D_1 from nominal. Somewhat less disturbance appears with deviation β. Deviation $\Delta p_{\text{б}1}$ affects the engine operating conditions less intensively. This is explained by the fact that in engines with turbopump unit pressure $p_{\text{б}i}$ is considerably less than excess pressure $p_{\text{н}i}$. The intensity of the total disturbance (1900 N/m^2) was commensurable with the amount of error (1074 N/m^2). This - a partial result, however it confirms the need for consideration and analysis of nonlinear remainders.

In the considered example there were taken into account the deviations of all parameters, moreover they all were considered positive. If we accept that $\Delta D_1^{\prime} < 0$, $\Delta\beta < 0$ and $\Delta a_1 < 0$, then ΔV

takes the greatest value, equal to 255,826 N/m^2. The error, which appears due to disregarding nonlinear terms, will in this case be approximately 0.5%.

If $\Delta p_{\delta 1} = 0$; $\Delta D_1 = 0$; $\Delta D_1' > 0$; $\Delta\beta > 0$ and $\Delta a_1 > 0$, then $\Delta V_1 = - 129{,}640$ N/m^2.

Thus, in the considered example the intensity of disturbance is changed over wide limits:

$$-129\,640 < \Delta V_1 < 255\,826.$$

For providing the greatest uniformity of engine operation the manufacturing tolerances should be selected so that the value of ΔV_1 would be the smallest. If, for example, on the average for a series deviation ΔD_1 is equal to +0.01, then deviation $\Delta D_1'$ should be equal to (+0.05). In this case

$$\Delta V_1 = 250\,000\Delta D_1 - 50\,000\Delta D_1' = 250\,000 \cdot 0{,}01 - 50\,000 \cdot 0{,}05 = 0.$$

Let us determine the value of $\Delta a_{д1}$, necessary for compensation of the effect of disturbance. If the disturbance is completely compensated, then

$$\Delta P_1 = -\Delta V_1,$$

or

$$\Delta a_{д1} = \frac{\Delta V_1}{a_{1\,11}}.$$

Let us assign conditionally

$$\Delta V_1 = 2360 \text{ N/m}^2.$$

We find

$$\Delta a_{д1} = \frac{2360}{-10^4} = -0{,}236 \text{ N}\cdot\text{s}^2/\text{m}^2\cdot\text{kg}^2.$$

Thus, initial value $a_{д1} = 50\ N{\cdot}s^2/m^2{\cdot}kg^2$ should be reduced by approximately 0.5%. It is easy to see that $\Delta V_1 = 2360\ N/m^2$ corresponds to change of pressure in the tank at $2360\ N/m^2$. One should note that the intensity of disturbance of such order is characteristic for well worn engines.

If the engine is not adjusted, then, by knowing the intensity of disturbance, it is possible to determine the deviations of basic parameters from their calculated values. Let us use equation

$$a_{11}\Delta G_1 + a_{12}\Delta G_2 + a_{13}\Delta n + \Delta V_1 = 0$$

and by substituting the obtained values of the quantity, we find

$$-128\,905\Delta G_1 - 81\,505\Delta G_2 + 48\,720\Delta n + 2360 = 0.$$

The sequence of figures shows that with the presence of disturbances n will receive the greatest change, then G_2, and the least - G_1. If disturbance only affects G_1, then

$$\Delta G_1 = \frac{2360}{128\,905} = 0{,}01831\ \text{kg/s}.$$

With the influence of disturbance only on fuel flow rate G_2 we obtain

$$\Delta G_2 = \frac{2360}{81\,505} = 0{,}02896\ \text{kg/s}.$$

If $\Delta V_1 = 0$, then the calculation equation will be written so:

$$128\,905\Delta G_1 + 81\,505\Delta G_2 = 48\,720\Delta n.$$

Let us assume that the number of revolutions was increased by 1%. If this change affected only the oxidizer flow rate G_1, then we obtain

$$\Delta G_1 = \frac{48\,720}{128\,905}0{,}01\cdot 500 = 1{,}89\ \text{kg/s}.$$

In case of the effect of change of number of revolutions only on the fuel flow rate G_2 we obtain

$$\Delta G_2 = \frac{48\,720}{81\,505} 0{,}01 \cdot 500 = 2{,}99 \text{ kg/s}.$$

Thus, the oxidizer flow rate increased by approximately 1.9%, and the fuel flow rate - by approximately 13%. The consequence of increase of number of revolutions will be not only the increase of flow rate of components, but also the change of component rate k_1.

Before the change of numbers of revolutions

$$k_1 = \frac{100}{22{,}8} = 4{,}386;$$

after increase of the velocity of rotation of turbopump unit shaft by 1%

$$k_1 + \Delta k_1 = \frac{100 + 1{,}89}{22{,}8 + 2{,}99} = 3{,}95.$$

Thus, the value of component ratio was decreased by approximately 10%.

When evaluating the results of the effect of external factors on the basic engine parameters and during the solution of problems on its adjustment, it is necessary to be guided by the complete closed systems of equations, examined in this section. Therefore, the numerical values, obtained in the example, do not characterize the actual operating conditions of the engine.

The described procedure allows taking into account the scattering of parameters, obtained during repeated engine testing. The statistical processing of results of bench tests shows that the parameters examined above have distribution very close to standard. Research, conducted in recent years, confirmed that during the solution of problems about the accuracy of adjustment it is necessary to consider the correlation between engine parameters. However, the

discussion of calculation methods of the engine with consideration of the correlation exceeds the intent of this book.

10.7. One of the Variants of Approximate Calculation of a Power Plant

During research of the starting operation of an engine, equipped with a turbopump unit, and if it is necessary to obtain a rather precise character of change of parameters with time one should use at least four differential equations: two equations of hydraulic circuits, combustion chamber equation and turbopump unit equation. After appropriate conversions it is possible to obtain one nonlinear differential equation of the fourth order. But attempts to find its solution in analytical form did not lead to positive results. Therefore, the considered system of differential equations is usually solved with the aid of analog and digital computers. However, a number of separate problems can be solved analytically with a certain limited degree of accuracy, by applying relationships simplified by appropriate assumptions.

Engine starting operation

During research of the starting operation it is sometimes possible to use equations of the third and even the second order. The order of the differential equation becomes one less, if instead of two equations of hydraulic circuits we use one. Such a method of research is possible in three cases. In the first, when under nonsteady state conditions the component ratio is not changed with time. To the second case corresponds the calculation of an engine with value of k_1 little varying with time, when in the form of an assumption we take k_1 = const. The error appearing in this case should be estimated. To the third case corresponds calculation, being performed with a graph of function $k_1(t)$, assigned on the basis of processing of experimental data. Let us note that with change of component ratio with time for refinement of results one should consider that the specific pressure pulse depends on k_1, i.e., $\beta = f(k_1)$. The effect of pressure on β, as already mentioned, can be disregarded.

Let us examine the equation of hydraulic circuit in the form

$$b_2\dot{G}_2 - D_2\omega^2 + D_2'\omega G_2 - p_{б2} + a_2G_2^2 + p_к = 0 \tag{10.120}$$

and the equation of combustion chamber in the form

$$\varepsilon\dot{p}_к + p_к - \frac{\beta}{F_{кр}}\theta(1+k_1)G_2 = 0. \tag{10.121}$$

Here θ - the relation, characterizing the effect of delaying argument. It is obvious that when $k_1 \neq$ const

$$\theta = \frac{(G_1+G_2)_{-\tau}}{(G_1+G_2)}. \tag{10.122}$$

We solve equation (10.120) relative to $p_к$, then we differentiate it, considering $p_к$, G_2, ω as variables. With the aid of obtained equations we exclude $p_к$ and $\dot{p}_к$ from expression (10.121). We obtain one nonlinear differential equation, containing G_2, ω and their derivatives; by dropping the subscript for simplicity of writing, we obtain

$$\varepsilon b\ddot{G} + 2\varepsilon aG\dot{G} + (\varepsilon D'\omega + b)\dot{G} + aG^2 + \left[\varepsilon D'\dot{\omega} + D'\omega + \frac{\beta}{F_{кр}}\theta(1+k_1)\right]G = 2\varepsilon D\omega\dot{\omega} + D\omega^2 + p_б. \tag{10.123}$$

In order to exclude ω and $\dot{\omega}$ from equation (10.123), it is still necessary to draw upon the differential equation of turbopump unit. As a result of transformations a rather complex third order differential equation will be received. In order not to raise the order of the equation, it is possible to use the graph of function $\omega(t)$, which is constructed by results of processing of experimental data. With sufficient accuracy for engineering calculations for several engines of separate stages we take

$$\omega = \omega_0[1 - \exp(-\alpha t)], \tag{10.124}$$

where α - the coefficient, which characterizes the intensity of change of angular velocity with time.

Consequently,

$$\dot{\omega}=\omega_0\alpha\exp(-\alpha t). \tag{10.125}$$

During processing of experimental data, if it is possible to consider α = const, then for computation of α we use the formula, easily obtained from expression (10.124):

$$\alpha=\frac{1}{t_0}\ln\left(\frac{\omega_0}{\omega_0-\omega_t}\right), \tag{10.126}$$

where ω_0 - the nominal value of angular velocity of rotation of turbopump unit shaft; ω_t - value of ω at some moment of time t_0 of the starting operation of the engine.

Now for calculation of the engine starting operation we will have

$$\begin{gathered}\varepsilon b\ddot{G}+2\varepsilon aG\dot{G}+\{\varepsilon D'\omega_0[1-\exp(-\alpha t)]+b\}\dot{G}+aG^2+\\+\left\{D'\omega_0[1-\exp(-\alpha t)]+\varepsilon D'\omega_0\alpha\exp(-\alpha t)+\frac{\beta}{F_{\text{кр}}}\theta(1+k_1)\right\}G=\\=D\omega_0^2[1-\exp(-\alpha t)]^2+p_6+2\varepsilon D\omega_0^2\alpha[1-\exp(-\alpha t)]\exp(-\alpha t).\end{gathered} \tag{10.127}$$

If the entire period is considered as consisting of several sections, so constant value ω_0 corresponds to each section, then equation (10.127) is simplified and takes the following form:

$$\begin{gathered}\varepsilon b\ddot{G}+2\varepsilon aG\dot{G}+(\varepsilon D'\omega_0+b)\dot{G}+aG^2+\frac{\beta}{F_{\text{кр}}}\theta(1+k_1)G=\\=D\omega_0^2-D'\omega_0G+p_6.\end{gathered} \tag{10.128}$$

For an engine with loaded tanks we will have

$$\varepsilon b\ddot{G}+2\varepsilon aG\dot{G}+b\dot{G}+aG^2+\frac{\beta}{F_{\text{кр}}}\theta(1+k_1)G=p_6. \tag{10.129}$$

If oscillating processes, proceeding in the period of engine cruise

operation, are not considered, then it can be assumed that $\theta = 0$, i.e., $\tau_s = 0$ is accepted.

During solution of equations we consider that when $t = 0$ the following equalities take place: $G = G_0$; $\omega = \omega_0$, $p_к = 0$. The initial value depends on the filling conditions of lines, spinup of turbopump unit shaft and the program of actuation of automatic equipment during starting. The initial value of ω_0 is determined by spinup conditions of the turbopump unit shaft. If with ignition of propellant in the combustion chamber a noticeable increase of pressure occurs, then instead of $p_к = 0$ when determining the initial conditions there should be accepted $p_к = p_{к0}$.

Engine operation at cruise conditions

Inasmuch as after the engine starting operation there can be observed small deviations of parameters from their mean values, then to investigate processes we use equations in small deviations:

$$x = G_2 - G_{20}; \tag{10.130}$$

$$y = \omega - \omega_0, \tag{10.131}$$

where $G_{2\,0}$ and ω_0 - mean (steady) values of flow rate and angular velocity.

By substituting expressions (10.130) and (10.131) into equality (10.123), we obtain the equation in small deviations:

$$\begin{aligned}&\varepsilon b\ddot{x} + 2\varepsilon a(G_{20}+x)\dot{x} + [\varepsilon D'(\omega_0+y)+b]\dot{x} + a(G_{20}+x)^2 + \\ &+\left[\varepsilon D'\dot{y} + D'(\omega_0+y) + \frac{3}{F_{кр}}\theta(1+k_1)\right](G_{20}+x) = \\ &= 2\varepsilon D(\omega_0+y)\dot{y} + D(\omega_0+y)^2 + p_б.\end{aligned} \tag{10.132}$$

Under conditions of statics of deviation $x = y = 0$ and their derivatives $\ddot{x} = \dot{x} = \dot{y} = 0$. The equation of statics takes the following form:

$$aG_{20}^2+\left[D'\omega_0+\frac{\beta}{F_{кр}}\theta(1+k_1)\right]G_{20}=D\omega_0^2+p_б. \qquad (10.133)$$

By subtracting term by term equation (10.133) from equation (10.132) and disregarding term $D'\omega_0 x$, we obtain

$$\begin{aligned}&\varepsilon b\ddot{x}+2\varepsilon a(G_{20}+x)\dot{x}+[\varepsilon D'(\omega_0+y)+b]\dot{x}+ax^2+2axG_{20}+\\&+[\varepsilon D'\dot{y}+D'y](G_{20}+x)+\frac{\beta}{F_{кр}}\theta(1+k_1)x=\\&=2\varepsilon D(\omega_0+y)\dot{y}+2D\omega_0 y+Dy^2.\end{aligned} \qquad (10.134)$$

By dropping the products of small deviations, we find

$$\begin{aligned}&\varepsilon b\ddot{x}+2\varepsilon a(G_{20}+x)\dot{x}+[\varepsilon D'(\omega_0+y)+b]\dot{x}+2aG_{20}x+\\&+\varepsilon D'(G_{20}+x)\dot{y}+D'G_{20}y+\frac{\beta}{F_{кр}}\theta(1+k_1)x=\\&=2\varepsilon D(\omega_0+y)\dot{y}+2D\omega_0 y.\end{aligned} \qquad (10.135)$$

If under cruise conditions ω = const (which usually takes place), then $\dot{y} = y = 0$.

Thus, the calculation equation is written so:

$$\begin{aligned}&\varepsilon b\ddot{x}+2\varepsilon a x\dot{x}+(2\varepsilon aG_{20}+\varepsilon D'\omega_0+b)\dot{x}+\\&+\left[2aG_{20}+\frac{\beta}{F_{кр}}\theta(1+k_1)\right]x=0.\end{aligned} \qquad (10.136)$$

Let us note that when regulating the engine operating conditions with the aid of the working medium generator of a turbine the angular velocity of rotation of the turbine shaft varies with time. However, this change with a well worn engine will occur so slowly that when performing engineering calculations we take the averaged value of ω = const. With the presence of sinusoidal oscillations of flow rate of propellant components

$$x=x_0\sin\omega t; \qquad (10.137)$$

$$\dot{x}=-x_0\omega\cos\omega t; \qquad (10.138)$$

$$x\dot{x} = -\frac{1}{2}x_0^2\omega\sin(2\omega t). \qquad (10.139)$$

When the product of $\varepsilon a x\dot{x}$ can be looked at as the quantity of the second order of smallness, equation (10.136) is simplified and takes the following form:

$$\varepsilon b\ddot{x} + (2\varepsilon a G_{20} + \varepsilon D'\omega_0 + b)\dot{x} + \left[2aG_{20} + \frac{\beta}{F_{кр}}\theta(1+k_1)\right]x = 0. \qquad (10.140)$$

The correctness of assumption that $x\dot{x}$ approaches zero should be checked.

In the case of feeding propellant components in gaseous form into the combustion chamber, it is possible to assume $\tau_{\text{в}} = 0$. In this case $\theta = 1$ and equation (10.136) will be written so:

$$\ddot{x} + \left(2\frac{a}{b}G_{20} + \frac{D'\omega_0}{b} + \frac{1}{\varepsilon}\right)\dot{x} + 2\frac{a}{b}x\dot{x} + \left[2\frac{a}{\varepsilon b}G_{20} + \frac{1}{\varepsilon b}\,\frac{\beta}{F_{кр}}(1+k_1)\right]x = 0. \qquad (10.141)$$

The calculations, performed on an MN-7 nonlinear analog machine, showed that the component, containing $x\dot{x}$, can be disregarded, then expression (10.141) will take the following form:

$$\ddot{x} + \left(2\frac{a}{b}G_{20} + \frac{D'\omega_0}{b} + \frac{1}{\varepsilon}\right)\dot{x} + \left[2\frac{a}{\varepsilon b}G_{20} + \frac{1}{\varepsilon b}\,\frac{\beta}{F_{кр}}(1+k_1)\right]x = 0. \qquad (10.142)$$

Equation (10.136) can be written so:

$$\ddot{x} + Mx\dot{x} + 2h\dot{x} + k^2x = 0, \qquad (10.143)$$

where

$$M = 2\frac{a}{b}; \qquad (10.144)$$

$$2h = \left(2\frac{a}{b}G_{20} + \frac{D'\omega_0}{b} + \frac{1}{\varepsilon}\right); \qquad (10.145)$$

$$k^2=\frac{1}{\varepsilon b}\left[2aG_{20}+\frac{\beta}{F_{\text{кр}}}\theta(1+k_1)\right]. \qquad (10.146)$$

Let us examine the equation, in which there is accepted $x\dot{x} = 0$ and $\tau_g = 0$. With such assumptions we investigate the burning of gaseous propellant, when intensive pressure fluctuations are not observed in the combustion chamber. The obtained equation is as if initial, characterizing the connection of only basic parameters. It is written so:

$$\ddot{x}+2h\dot{x}+k^2x=0. \qquad (10.147)$$

but now

$$k^2=\frac{1}{\varepsilon b}\left[2aG_{20}+\frac{\beta}{F_{\text{кр}}}(1+k_1)\right]. \qquad (10.148)$$

Figure 10.1 shows the effect of the component, containing product $x\dot{x}$, on the character of change of $x(t)$.

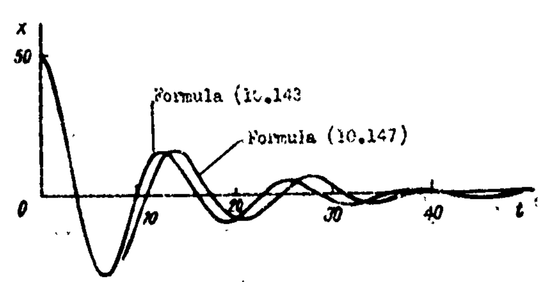

Fig. 10.1. For calculation by formulas (10.143) and (10.147).

Motion will be aperiodic if

$$h^2-k^2>0, \qquad (10.149)$$

or oscillatory if

$$h^2-k^2<0. \qquad (10.150)$$

To the transition from one mode to another corresponds conditio

$$k^2 - k^2 = 0. \qquad (10.151)$$

By using equations (10.145) and (10.148) instead of condition (10.151), we find

$$\left(\frac{a}{b} G_{20}\right)^2 + \frac{1}{4\varepsilon^2} - \frac{3}{F_{\text{кр}}} \frac{1+k_1}{\varepsilon b} - \frac{a}{\varepsilon b} G_{20} = 0. \qquad (10.152)$$

This equation can be solved relative to any other parameter, for example:

$$\beta_{\text{кр}} = \frac{F_{\text{кр}}}{1+k_1} \left[\frac{\varepsilon}{b} \left(\frac{\Delta p}{G_{20}}\right)^2 - \frac{\Delta p}{G_{20}} + \frac{b}{4\varepsilon}\right]; \qquad (10.153)$$

$$p_{\text{к.кр}} = \frac{\varepsilon}{b} \frac{(\Delta p)^2}{G_{20}} - \Delta p + \frac{b}{4\varepsilon} G_{20}, \qquad (10.154)$$

inasmuch as

$$p_{\text{к}} = \frac{3}{F_{\text{кр}}} (1+k_1) G_{20}; \qquad (10.155)$$

$$\Delta p = a G_{20}^2. \qquad (10.156)$$

where Δp - hydraulic losses. With the presence of damped oscillations the solution of (10.147) takes the following form:

$$x = A_0 [\exp(-ht)] \sin(\omega t + \varphi_0), \qquad (10.157)$$

where A_0 - initial amplitude; ϕ_0 - initial phase. Frequency

$$\omega = \sqrt{k^2 - h^2}. \qquad (10.158)$$

If when $t = 0$ we will have $\phi = 0$ and the value of x_0 will be known, initial amplitude

$$A_0 = \frac{\dot{x}_0}{\omega}. \qquad (10.159)$$

Factor $\exp(-ht)$ characterizes the rate of damping. If $h = 0$, then oscillations will be harmonic with period

$$T=\frac{2\pi}{k}. \tag{10.160}$$

If the change of flow rate with time will be aperiodic, then the solution will be written so:

$$x=\frac{\dot{x}_0}{2\sqrt{h^2-k^2}}\left[\exp\left(\sqrt{h^2-k^2}-h\right)t-\exp\left(-\sqrt{h^2-k^2}-h\right)t\right]. \tag{10.161}$$

After obtaining the preliminary approximate solutions, which permit evaluating the basic connections between parameters and the relationships between intensities of various influences, we change to organization of precise calculations, the procedure of accomplishing which is examined in the following chapter.

CHAPTER XI

CALCULATION OF AN ENGINE ON ANALOG AND DIGITAL COMPUTERS

Computer technology is the basis, which provides rapid and high-quality solution of complex systems of equations, which describe the processes in contemporary [ZhRD] (ЖРД) liquid-propellant rocket engines. From the number of computers and methods of research being applied today it is possible to separate three basic groups.

The first group includes computers, intended for algebraic computations (adding machines, tabulators, general-purpose and special digital computers). They are sometimes used during research and calculation of liquid-propellant rocket engines, for example, during solution of algebraic equations.

The second group includes mathematical analog devices or analog computers. With mathematical simulation on the basis of available solutions, describing the investigated processes, electrical analogs of physical processes and phenomena are created. In certain cases electromechanical analogs are created. The widest distribution received analog electronic and electromechanical models. This group includes physical simulation computers, for example hydraulic integrators.

To the third group belong digital computers.

The problems of designing and preparing new articles as a rule are solved by the method of searching for the most acceptable variants from the many possible ones. The final step of the search is the experimental check of a limited number of variants, selected on the basis of analysis of results, obtained on analog and digital computers. Such a method allows finding the optimum solution faster and more accurately and is economically the most justified.

The intensive development of technology is accompanied by complication of the systems being designed, by reduction of the time for development and introduction, by the growth of requirements for quality and economy. In connection with this appeared the need for creation and application of effective methods of optimization of the stages of development, preparation and research of articles, i.e., obtaining the best results with minimum investments of resources and time in the given particular conditions.

Electronic models

The basis of the utilization of electronic models is the circumstance that processes differing in their physical nature can be described by the same (formally identical) differential equations. For example, a second order equation describes the processes, proceeding in the engine, including the combustion chamber, hydraulic circuits and loaded tanks, at condition k_1 = const. Analogous equations describe the processes proceeding in an electrical oscillation circuit. Formally, with respect to writing, these equations do not differ from one another. Perhaps it can be considered that the most widespread in the practice of solution of engineering problems is the electronic type of models, intended for investigation of systems, the processes in which are described by ordinary differential equations. An electronic model can be considered as an integrator for the system of differential equations, since voltages, received at the output of appropriate units, to a certain scale represent the solution of the considered system.

With respect to mathematical principle the electronic models can be models, in which a solution is found by the method of successive integration, when the simulation circuit is constructed on integrating elements, or models, accomplishing solution according to the principle of successive differentiation. The first models have a substantial advantage over the second in reference to decrease of the influence of noises, inasmuch as a circuit with successive integration operates as a smoothing filter. This circumstance is rather important, since the analog devices give less accuracy in comparison with digital computers. If it is required to conduct research with high accuracy, then we are guided by digital computers, but before the beginning of operation on such a computer we look over the character and peculiarities of solution on a display or on the tape of the oscillograph of analog device.

The electronic models, designed to investigate ordinary differential equations, are divided into the three following types.

The first includes general purpose linear models. They are used during research of dynamics of a liquid-propellant rocket engine at operating conditions, when processes are described by linearized differential equations. Some difficulties in the course of research of processes under operating conditions appear when determining the initial conditions, which can be obtained by results of calculation, or with processing of experimental data, or assigned in the form of external disturbances.

The second type includes nonlinear electronic models. They are applied during research of dynamics of an engine at the starting operation. Initial conditions are assigned from results of calculation of processes, preceding the ignition of propellant in the chamber.

The third type - nonlinear models with delay units. They are designed for the study of transient processes in the presence of low-frequency oscillations.

The equations, subject to solution on analog devices, undergo conversions, in the course of which they should always be represented in Cauchy form, i.e., in normal form.

Linear equations are written in the following form:

$$\left.\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots \ \ldots + f_1(t); \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots \ \ldots + f_2(t); \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots \ \ldots + f_n(t), \end{aligned}\right\} \qquad (11.1)$$

where x_i – unknown functions (for example p_H, G_i, ω); $f_i(t)$ – disturbing influence (for example, V_i); a_{ik} – coefficients of the system being integrated.

If the system is nonlinear, then it is written so:

$$\left.\begin{aligned} \dot{x}_1 &= \Psi_1[x_1, x_2, \ldots, f_1(t)]; \\ \dot{x}_2 &= \Psi_2[x_1, x_2, \ldots, f_2(t)]; \\ &\cdots\cdots\cdots\cdots \\ \dot{x}_n &= \Psi_n[x_1, x_2, \ldots, f_n(t)], \end{aligned}\right\} \qquad (11.2)$$

where $\Psi[x_1, x_2, \ldots, f(t)]$ – nonlinear function relationship.

Despite the fact that analog devices do not provide as high accuracy as digital computers, they possess a number of advantages, which include the following: simplility of preparation of the problem, convenience of utilizaiion and the clarity of the obtained solution. During work on an analog device it is possible, while not getting rid of the oscillograph screen, to vary the values of various coefficients and to see now, to what effect the change introduced by the experimenter leads.

Hydraulic integrators

During the solution of some types of equations the utilization of hydraulic integrators proves to be convenient. They are used when a formal analogy (similarity) takes place between the differential equations being investigated and equations, which describe the distribution of heads during laminar flow of liquid in a system with hydraulic resistance and with tanks connected to lines.

Hydraulic integrators received especially wide distribution during the solution of problems of nonstationary thermal conductivity. Let us examine the shown analogy between equations on very simp e examples. The quantity of water Y_1, overflowing for time t from vessel to vessel under laminar conditions through a pipe with predetermined hydraulic resistance and difference of heads, is determined by formula

$$p_1-p_2=aG. \tag{11.3}$$

Consequently,

$$Y_1=\frac{\delta p}{a}G, \tag{11.4}$$

where $\delta p = p_1 - p_2$ — head difference; a — hydraulic resistance, or hydraulic loss coefficient.

The quantity of liquid, accumulated in a vessel with cross-sectional area F as a result of increase of the level by amount $h(t)$, will be

$$Y_2=Fh(t). \tag{11.5}$$

In examining the transient process of heat transfer the quantity of heat Q_1, which flows during time t through the medium with thermal resistance R at temperature difference δT_1, will be

$$Q_1=\frac{\delta T_1}{R}t. \tag{11.6}$$

The total quantity of heat, obtained by the medium, characterized by heat capacity c, as a result of increase of temperature by quantity δT will be equal to

$$Q_3 = c\delta T. \qquad (11.7)$$

Thus, the investigated equations are similar to equations, which describe the motion of liquid in the hydraulic integrator.

11.1. Power Plant with Gas Pressure Feed System of Propellant Components

Let us first examine a rather simple problem. Let us compile a diagram of setting up an analog device and calculate the starting operation of an engine under the condition that the engine at transient conditions provides k_1 = const.

Let us assume that at the beginning of calculation all the hydraulic lines are filled with propellant components, pressure in the tanks is constant and opening of valves (breakthrough of membranes) occurs instantly. Wave processes and the effect of delay on operating conditions are not considered.

Let us consider the solution of the problem on an example.[1]

Example

Given.

$$k_1 = 4{,}386;$$

the coefficient of hydraulic fuel losses $a_2 = 7693\ \mathrm{n \cdot s^2/m^2 \cdot kg^2}$; mass

[1]In the compiling of block diagrams and programming, and also in the solution of given equations there took part colleagues of the laboratory under the direction of G. A. Guberniyev.

coefficient $b_2 = 10^3\ n\cdot s^2/m^2\cdot kg$; pressure in the tank with fuel $p_{б2} = 14\ Mn/m^2$; ratio

$$\frac{\beta}{F_{kp}} = 81{,}505\ Mn\cdot s/kg\cdot m^2;$$

time the gas stays in the chamber $\varepsilon = 0.02$ s; research is performed for a case when $\tau_s = 0$.

Preparation of solution.

The initial system of equations in accordance with the scheme given in Fig. 8.1 has the following form:

$$\left.\begin{aligned} b_2\dot{G}_2 &= p_{б2} - a_2 G_2^2 - p_к; \\ \varepsilon\dot{p}_к &= -p_к + \frac{\beta}{F_{кр}}(1 + k_1)G_2. \end{aligned}\right\} \tag{11.8}$$

For simplicity of writing let us designate $G_2 = x$ and $p_к = y$. The equations will take the following form:

$$\left.\begin{aligned} \dot{x} &= \frac{1}{b_2}(\dot{p}_{б2} - a_2 x^2 - y); \\ \dot{y} &= \frac{1}{\varepsilon}\left[-y + \frac{\beta}{F_{кр}}(1 + k_1)x\right]. \end{aligned}\right\} \tag{11.9}$$

By substituting values of coefficients a_2 and b_2, for physical model we obtain

$$\left.\begin{aligned} \dot{x} &\doteq 14\cdot 10^3 - 7{,}69x^2 - 0{,}001y; \\ \dot{y} &= -50y + 21{,}92\cdot 10^6 x. \end{aligned}\right\} \tag{11.10}$$

Let us study the system with an analog device in order to look over the basic character of the solution.

Let us reduce the system of equations to computer form. When $t = \infty$ we have $x = 22.8$; $y = 10^7$. The expected value of the

characteristic time of starting operation t = 0.01 s. The conversion formulas are written so:

$$x = a_x X;\ y = a_y Y;\ t = a_t T;\ p_6 = a_p P_6. \qquad (11.11)$$

Let us find the value of coefficients. Limiting voltage at the output of all amplifiers of the analog device is equal to 100 V. Therefore,

$$a_x \geqslant \frac{x}{X} > \frac{22{,}8}{100} = 0{,}228.$$

Let us take

$$a_x = 0{,}25.$$

Taking into account that the expected greatest values of y are insufficiently reliable, as applied to the utilized computer, let us take

$$a_y = 5 \cdot 10^{5}.$$

In order to succeed during the action of the analog device in attentively looking over the character of occurrence of the process on the screen of an oscillograph, let us designate T = 5 s. Then

$$a_t = \frac{t}{T} = \frac{0{,}01}{5} = 2 \cdot 10^{-3}. \qquad (11.12)$$

Now, by using conversion formulas, let us write the equations of the physical model so:

$$\left.\begin{aligned} \frac{a_x}{a_t} \dot{X} &= 14 \cdot 10^{-3} - 7{,}69 a_x^2 X^2 - 0{,}001 a_y Y; \\ \frac{a_y}{a_t} Y &= -50 a_y Y + 21{,}92 \cdot 10^6 a_x X. \end{aligned}\right| \qquad (11.13)$$

By substituting values of a_x, a_y, a_t, we find

$$\left.\begin{aligned}\dot{X} &= 14\cdot10^{3}\cdot2\cdot10^{-3}\frac{1}{0,25}-7,69\cdot0,25\cdot2\cdot10^{-3}X^{2}\\ &\quad 0,001\cdot5\cdot10^{5}\cdot2\cdot10^{-3}\frac{1}{0,25},\\ \dot{Y} &= -50\cdot2\cdot10^{-3}Y+21,92\cdot10^{6}\cdot0,25\cdot2\cdot10^{-3}\frac{1}{5}10^{-5}X\end{aligned}\right\}\qquad(11.14)$$

The equations, reduced to computer form, assume the following form:

$$\left.\begin{aligned}\dot{X} &= 28-7,69\cdot10^{-3}X^{2}-8Y;\\ \dot{Y} &= -0,1Y+0,044X.\end{aligned}\right\}\qquad(11.15)$$

Let us use nonlinear model MN-7 to investigate the system of equations.

To the input of the integrator adder let us supply direct-current voltage, equal to 28 V (addend of the right side of the first equation). To variable resistor let us supply a signal $(-X^2)$ and install the resistor so that on the voltmeter of an amplifier we obtain 7.69 with a signal equal to 1 V.

To the second variable resistor let us feed signal $(-Y)$. At the integrator output, with consideration of inversion, we obtain solution $(-X)$ Fig. 11.1.

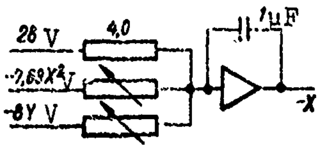

Fig. 11.1. Solution of the first equation (11.15) at analog device MN-7 with consideration of inversion.

Let us direct signal $(-X)$ to a scale converter and with consideration of inversion we obtain solution (+0.1X) Fig. 11.2.

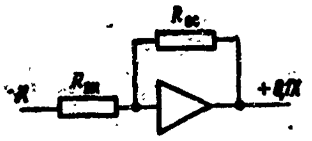

Fig. 11.2. Change of scale and sign of the previous solution in the scale converter.

Inasmuch as $k=\frac{R_{oc}}{R_{BX}}$, to get k = 0.1 let us take R_{oc} = 0.1 MΩ and R_{BX} = 1 MΩ. Signal (+0.1X) is directed to the product unit and to the adder of the second integrator.

Let us examine the diagram of the second integrator (Fig. 11.3). To the adder is fed signal (+0.1X). By using a variable resistor, we obtain signal +0.1X·0.44 = 0.044X, which corresponds to the augend of the right side of the second equation. Let us feed solution (-Y) from the integrator output to the adder-integrator input, to the variable resistor.

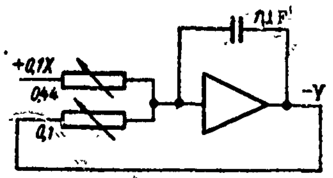

Fig. 11.3. Solution of the second equation (11.15).

Simultaneously with this the solution in the form of (-Y) is fed, as already mentioned, to the input of the first integrator. The final form of the diagram is shown in Fig. 11.4.

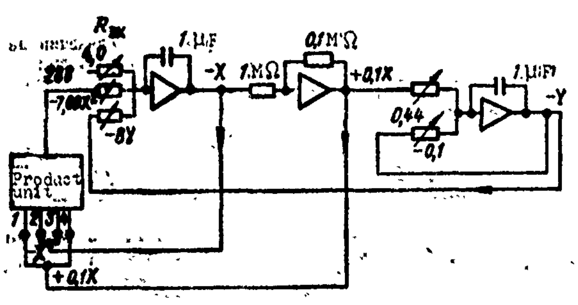

Fig. 11.4. The final form of the block diagram for solution of system (11.15) on MN-7 analog device.

Results of solution are shown in Fig. 11.5. With increase of the initial value of flow rate its "peak" is raised, but insignificantly. The time and character of the starting operation are changed

little. Therefore, it can be considered that the quantity of initial propellant consumption does not have a fundamental effect on the character of change of averaged values of engine parameters with time.

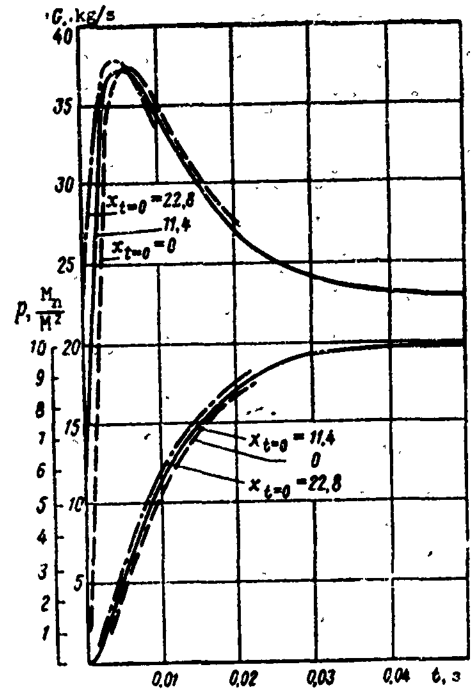

Fig. 11.5. Results of calculation of system of equation (11.15).

A graph of the considered function can be constructed from the results of calculation on a digital computer. However, analysis of obtained results showed that there is no need to repeat the solution for the purpose of obtaining greater accuracy.

If the component ratio under nonstationary conditions varies with time, then it is necessary to use a minimum of three equations: combustion chamber equation and two hydraulic circuit equations.

Let us examine the sequence of compilation of the block diagram of an analog device.

The initial system of equations:

$$\left.\begin{aligned} &\varepsilon\dot{p}_\text{к}+p_\text{к}-\frac{\beta}{F_\text{кр}}(G_1+G_2)=0;\\ &b_1\dot{G}_1=p_{61}-a_1G_1^2-p_\text{к};\\ &b_2\dot{G}_2=p_{62}-a_2G_2^2-p_\text{к}. \end{aligned}\right\} \qquad (11.16)$$

Let us write the equations in a form convenient for simulation:

$$\left.\begin{aligned} &\dot{p}_\text{к}=-\frac{1}{\varepsilon}p_\text{к}+\frac{\beta}{\varepsilon F_\text{кр}}(G_1+G_2);\\ &\dot{G}_1=\frac{1}{b_1}p_{61}-\frac{a_1}{b_1}G_1^2-\frac{1}{b_1}p_\text{к};\\ &\dot{G}_2=\frac{1}{b_2}p_{62}-\frac{a_2}{b_2}G_2^2-\frac{1}{b_2}p_\text{к}. \end{aligned}\right\} \qquad (11.17)$$

We take

$$\left.\begin{aligned} &G_1=a_{G1}U_1;\ G_2=a_{G2}U_2;\\ &p_\text{к}=a_\text{к}U_3;\ p_6=a_6U_4. \end{aligned}\right\} \qquad (11.18)$$

In electrical quantities the equations will be written so:

$$\left.\begin{aligned} &a_\text{к}\dot{U}_3=-\frac{a_3}{\varepsilon}U_3+\frac{\beta}{\varepsilon F_\text{кр}}a_{G_1}U_1+\frac{\beta}{\varepsilon F_\text{кр}}a_{G_2}U_2;\\ &a_{G_1}\dot{U}_1=\frac{a_6}{b_2}U_4-a_1\frac{a_{G_1}^2}{b_1}U_1^2-\frac{a_\text{к}}{b_1}U_3;\\ &a_{G_2}\dot{U}_2=\frac{a_6}{b_2}U_4-a_2\frac{a_{G_2}^2}{b_2}U_2^2-\frac{a_\text{к}}{b_2}U_3. \end{aligned}\right\} \qquad (11.19)$$

Further we obtain

$$\left.\begin{aligned} &\dot{U}_1=\frac{a_6}{a_{G_1}b_1}U_4-a_1\frac{a_{G_1}}{b_1}U_1^2-\frac{a_\text{к}}{a_{G_1}b_1}U_3;\\ &\dot{U}_2=\frac{a_6}{a_{G1}b_2}U_4-a_2\frac{a_{G_1}}{b_2}U_2^2-\frac{a_\text{к}}{a_{G_1}b_2}U_3;\\ &\dot{U}_3=-\frac{1}{\varepsilon}U_3+\frac{\beta a_{G_1}}{\varepsilon F_\text{кр}a_\text{к}}U_1+\frac{\beta a_{G_1}}{\varepsilon F_\text{кр}a_\text{к}}U_2. \end{aligned}\right\} \qquad (11.20)$$

Let us select scales:

$$a_{U_2}=0{,}9;\ a_{U_1}=4;\ a_x=25\cdot10^4.$$

Having substituted the scale factors, we obtain system of equations:

$$\left.\begin{aligned}\dot U_1&=139U_4-0{,}12U_1^2-139U_3;\\ \dot U_2&=137U_4-0{,}122U_2^2-137U_3;\\ \dot U_3&=-50U_3+15{,}2U_1+65U_2.\end{aligned}\right\}\qquad(11.21)$$

Let us select time scale a_t = 0.005; now the system of equations takes the form

$$\left.\begin{aligned}\dot U_1&=0{,}695U_4-0{,}06U_1^2-0{,}695U_3;\\ \dot U_2&=0{,}685U_4-0{,}06U_2^2-0{,}685U_3;\\ \dot U_3&=-0{,}25U_3+0{,}076U_1+0{,}325U_2.\end{aligned}\right\}\qquad(11.22)$$

Let us compile the block diagram of the problem (Fig. 11.6).

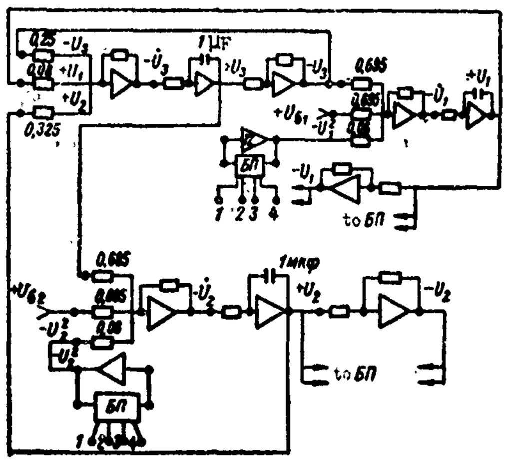

Fig. 11.6. Block diagram of system of equations (11.22).
DESIGNATION: БП = BP = product unit.

11.2. Power Plant with Pressure Chambers

Let us examine the power plant, to the hydraulic lines of which are connected pressure chambers (Fig. 11.7). With the presence of pressure chambers the starting operation is determined basically by parameters of the pressure chamber-combustion chamber line, and operating conditions depend on parameters of the tank-combustion chamber line.

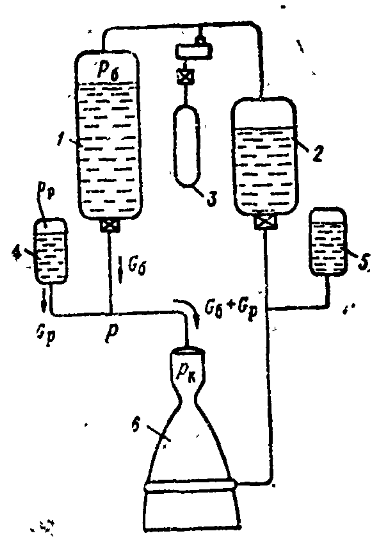

Fig. 11.7. Schematic diagram of liquid-propellant rocket engine with pressure chambers [48], [97], [64]: 1 — oxidizer tank; 2 — fuel tank; 3 — compressed gas bottle; 4, 5 — pressure chambers; 6 — combustion chamber.

Equations of pressure chambers

Pressure chambers can be of two types: closed and flow-through or open (Fig. 11.8).

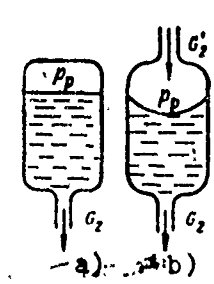

Fig. 11.8. Diagram of pressure chambers: a) closed; b) flow-through

Let us eaxmine a closed pressure chamber.

The equation for determination of volumes has the form

$$V-V_0=\frac{1}{\varrho_{\text{ж}}}\int_0^t G_p^- dt, \tag{11.23}$$

where V_0 – initial free volume of the pressure chamber; G_p – flow rate per second of liquid from the pressure chamber.

By differentiating, we find

$$\dot{V}=\frac{G_p}{\varrho_{\text{ж}}}. \tag{11.24}$$

According to the equation of state

$$V=\frac{Y_0RT}{p}, \tag{11.25}$$

where Y_0 – initial quantity of gas in the pressure chamber.

We will consider that gas constant R = const.

In the isothermal process

$$\dot{V}=-\frac{(Y_0RT_0)\dot{p}}{p^2}. \tag{11.26}$$

Consequently,

$$G_p^-=-\frac{(Y_0RT_0)\,\dot{p}}{p^2}\varrho_{\text{ж}}. \tag{11.27}$$

For adiabatic process

$$T=\frac{T_0}{p_0^m}\,p, \tag{11.28}$$

where

$$m=\frac{k-1}{k}. \tag{11.29}$$

Consequently,

$$G_p^{-}=-(1-m)Y_0R\frac{T_0}{p_0^m}\,\frac{\dot{p}}{p^{2-m}}\,\varrho_\text{ж}. \tag{11.30}$$

For flow-through pressure chamber we find: in isothermal process

$$G_p^{-}-G_p'=-(Y_0RT_0)\varrho_\text{ж}\frac{\dot{p}}{p^2}, \tag{11.31}$$

where G'_p – inflow of liquid into the pressure chamber per second; in adiabatic process

$$G_p^{-}-G_p'=-(1-m)Y_0R\frac{T_0}{p_0^m}\varrho_\text{ж}\frac{\dot{p}}{p^{2-m}}. \tag{11.32}$$

Equations of one hydraulic system of a power plant

To the system let us connect a tank, pressure chamber, combustion chamber and manifolds with elements of hydraulic resistances. With closed pressure chamber we have

$$\left.\begin{aligned}
&p_\text{б}-p=a_1G_\text{б}^2+b_1\dot{G}_\text{б};\\
&p_p-p=a_2G_p^2+b_2\dot{G}_p^{-};\\
&p-p_\text{к}=a_3G_\text{з}^2+b_3\dot{G}_\text{з};\\
&G_p^{-}=-\frac{Y_0RT_0}{p_p^2}\dot{p}_p\varrho_\text{ж};\\
&\ddot{p}_\text{к}+\omega^2p_\text{к}=0;\\
&(G_\text{з}=G_p^{-}+G_\text{б}).
\end{aligned}\right\} \tag{11.33}$$

Here $G_\text{б}$ – flow rate of liquid from the tank.

Preparation of block diagram of an analog device during research of an engine with pressure chambers

Let us examine the compilation of the block diagram of an analog device on an example.

For investigation let us take the following values of parameters:

Designations	a_1	a_2	a_3	b_1	b_2
Quantity	$0{,}827\cdot10^{-4}$	$0{,}413\cdot10^{-5}$	$0{,}413\cdot10^{-4}$	$13{,}2\cdot10^{-4}$	$0{,}713\cdot10^{-4}$

Continuation

Designations	b_3	p_p	p_6	$\varrho_ж$	V_0	$p_к$	T	R
Quantity	$18\cdot10^{-4}$	6	6	0,001	$(17{,}6-3)\cdot10^{6}$	0—6	293° K	29,26

For simplicity of adjustment of the problem on a model let us take $p_н$ = const. Let us reduce equations to a form convenient for simulation:

$$\left.\begin{aligned}
&\dot{G}_6=\frac{1}{b_1}p_6-\frac{1}{b_1}p-\frac{a_1}{b_1}G_6^2;\\
&\dot{G}_p^-=\frac{1}{b_2}p_p^--\frac{1}{b_2}p-\frac{a_2}{b_2}G_p^2;\\
&\dot{p}_p=-\frac{1}{Y_0RT_0}\frac{1}{\varrho_ж}G_p^-p_p^2;\\
&p=p_к+a_3G_з^2+b_3\dot{G}_з;\\
&(G_з=G_p^-+G_6).
\end{aligned}\right\} \tag{11.34}$$

From the equation of state for pressure chamber

$$V=\frac{Y_0RT}{p_p} \tag{11.35}$$

it follows that

$$p_{y0}V'_{y0}=Y_0RT=6\cdot 0{,}1\cdot 17{,}6\cdot 10^6=10{,}6\cdot 10^6.$$

For operation on an electronic analog device it is necessary to change from physical quantities to electrical, i.e., to express everything in voltages.

Let us designate

$$G_6=a_1'U_1;\ G_p^-=a_2'U_2;\ G_3=a_3'U_3;$$
$$p_\beta^-=a_4'U_4;\ p=a_5'U_5;\ p_6=a_6'U_6;\ p_\kappa=a_7'U_7,$$

where a_{1-7}' – scale factors; U_{1-7} – voltages, simulating physical quantities.

Equations in electrical quantities will be written so:

$$\left.\begin{aligned}
&a_1'\dot{U}_1=\frac{a_6'}{b_1}U_6-\frac{a_5'}{b_1}U_5-a_1\frac{(a_1')^2}{b_1}U_1^2;\\
&a_2'\dot{U}_2=\frac{a_4'}{b_2}U_4-\frac{a_5'}{b_2}U_5-a_2\frac{(a_2')^2}{b_2}U_2^2;\\
&a_3'U_3=a_2'U_2+a_1'U_1;\\
&a_4'\dot{U}_4=-\frac{a_2'(a_4')^2}{10{,}6\cdot 10^5\varrho_{\text{ж}}}U_2U_4^2;\\
&a_5'U_5=a_7'U_7+a_3(a_3')^2U_3^2+b_3a_3'\dot{U}_3.
\end{aligned}\right\}\qquad (11.36)$$

For the selection of numerical values of scale factors it is necessary to know the limits of change of physical quantities. A factor is selected so that the voltage, simulating a physical quantity would not exceed 100 V. Usually tentative values of factors are assigned, which after setting up the problem on a model are modified. Let us take

$$a_1'=5;\ a_2'=10;\ a_3'=12;\ a_4'=0{,}12;\ a_5'=0{,}1;$$
$$a_6'=0{,}1;\ a_7'=0{,}1.$$

During computation of coefficients of equations it is necessary to consider that product units of type BP-4, BP-7, utilized in MN-7 device, provide $0.01U^2$ at the output. Therefore, at U_2 the factors must be multiplied by 100. After substitution of numerical values the system of equations takes the form:

$$\left.\begin{aligned}
&\dot{U}_1=15{,}2U_6-15{,}2U_5-31{,}3U_1^2;\\
&\dot{U}_2=168U_4-140U_5-58U_2^2;\\
&U_3=0{,}416U_1+0{,}834U_2;\\
&\dot{U}_4=-11{,}3U_2U_4^2;\\
&U_5=0{,}216\dot{U}_3+5{,}95U_3^2+U_7.
\end{aligned}\right\} \qquad (11.37)$$

By considering the parameters of the device, let us introduce time scale

$$\alpha_t=\frac{t_{маш}}{t_{физ}}.$$

Having accepted

$$\alpha_t=30^{-1},$$

we will have

$$\left.\begin{aligned}
&\alpha_t^{-1}\dot{U}_1=15{,}2U_6-15{,}2U_5-31{,}3U_1^2;\\
&\alpha_t^{-1}\dot{U}_2=168U_4-140U_5-58U_2^2;\\
&U_3=0{,}416U_1+0{,}834U_2;\\
&\dot{U}_4=-11{,}3U_2U_4^2;\\
&U_5=0{,}216\alpha_t\dot{U}_3+5{,}95U_3^2+U_7,
\end{aligned}\right\} \qquad (11.38)$$

where U_i – computer variable, simulating the physical quantity.

Finally after substitution of numerical value of α_t the system will be written so:

$$\dot{U}_1=0{,}506U_6-0{,}506U_5-1{,}025U_1^2; \qquad (11.39\text{-}1)$$

$$\dot{U}_2 = 5{,}6U_4 - 4{,}66U_5 - 1{,}93U_2^2; \quad (11.39\text{-}2)$$

$$U_3 = 0{,}416U_1 + 0{,}834U_2; \quad (11.39\text{-}3)$$

$$\dot{U}_4 = -0{,}377U_2U_4^2; \quad (11.39\text{-}4)$$

$$U_6 = 6{,}48\dot{U}_3 + 5{,}95U_3^2 + U_7. \quad (11.39\text{-}5)$$

The block diagram is constructed according to equations, beginning with the first. However, there is preliminarily explained whether derivatives $\dot{U}_1$ and $\dot{U}_2$ enter other equations. If derivatives are not required, then the quantities, standing in the right side of equation (11.39-1) and (11.39-2), must be fed directly to the input of integration unit. In this case $\dot{U}_1$ and $\dot{U}_2$, simulating derivatives of flow rates $\dot{G}_б$ and $\dot{G}_р$, will be required to get derivatives of total flow rate G_Σ, which enters equation (11.39-3). It is necessary to recall that the quantities, fed to the input of the operating amplifier, change their sign at its output.

Numeration of the inputs and operating amplifiers themselves is performed after the whole block diagram of the system of equations is compiled.

The order of compilation of the block diagram will be the following.

Block diagrams of equations (11.39-1)-(11.39-4), shown respectively in Figs. 11.9-11.12 are constructed in series.

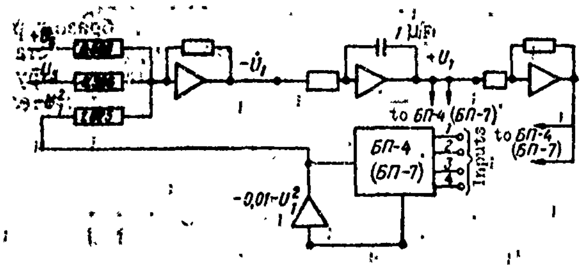

Fig. 11.9. Block diagram of equation (11.39-1).

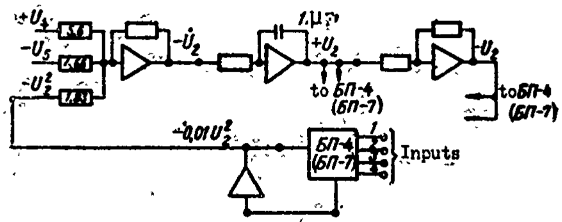

Fig. 11.10. Block diagram of equation (11.39-2).

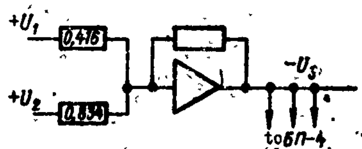

Fig. 11.11. Block diagram of equation (11.39-3).

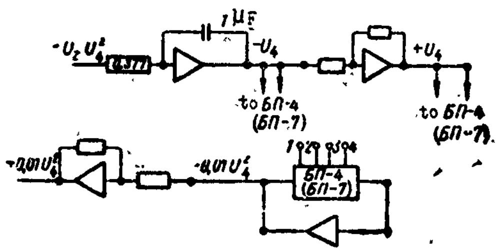

Fig. 11.12. Block diagram of equation (11.39-4).

Inasmuch as the product unit provides only $0.001U\cdot{}^{4}_{2}$, the amount of voltage at the output of unit BP-4 or BP-7 proves to be very small. It is necessary to feed this voltage to the output of one more product unit, in order to obtain $U_2U_4^2$. During the solution large error appears. Therefore, it is expedient to divide coefficient 0.377 into two equal parts.

The system of equations contains 5 nonlinearities. For its realization there is required 4 integrating amplifiers, 5 summing amplifiers and 6 inverting amplifiers, not considering 5 amplifiers, which operate together with product units. The MN-7 analog device contains only 4 nonlinearities and 12 free amplifiers. Thus, to realize prepared block diagram cannot be carried out on an MN-7 analog device. It is necessary to have 2 combined MN-7 analog devices, either EMU-10, or MN-7 and supplemental nonlinear units.

Further there is constructed the block diagram of equation (11.39-5) of Fig. 11.13. In order to realize equation (11.39-5), it is necessary to have U_3^2 and $\dot{U}_3$.

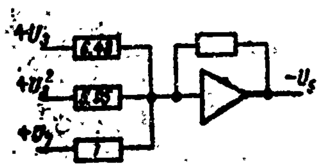

Fig. 11.13. Block diagram of equation (11.39-5).

Quantity U_3 is obtained on a separate adder during supply of $\dot{U}_1$ and $\dot{U}_2$ to the output in the ratio shown in equation (11.39-3) (Fig. 11.14).

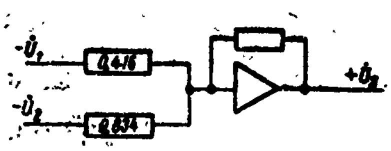

Fig. 11.14. Block diagram for obtaining derivative $\dot{U}$ on a separate adder.

Now we should arrange the block diagrams together on one sheet and connect them. The total block diagram is shown in Fig. 11.15.

Results of some calculations

The operation of a feed system with pressure chambers is described by differential equations, which are distinguished from differential equations characterizing the operation of pressurized feed systems of liquid-propellant rocket engines. Therefore, with the introduction of pressure chambers into the configuration of a stand device the divergence of bench tests with results of tests obtained under flight conditions should be expected earlier. However, as results of bench firing tests show, the introduction of pressure chambers into the configuration of an engine, installed on a rocket, in certain cases can prove to be expedient.

Let us examine the results of computations of some variants of systems with pressure chambers. Figs. 11.16-11.22 show the changes of

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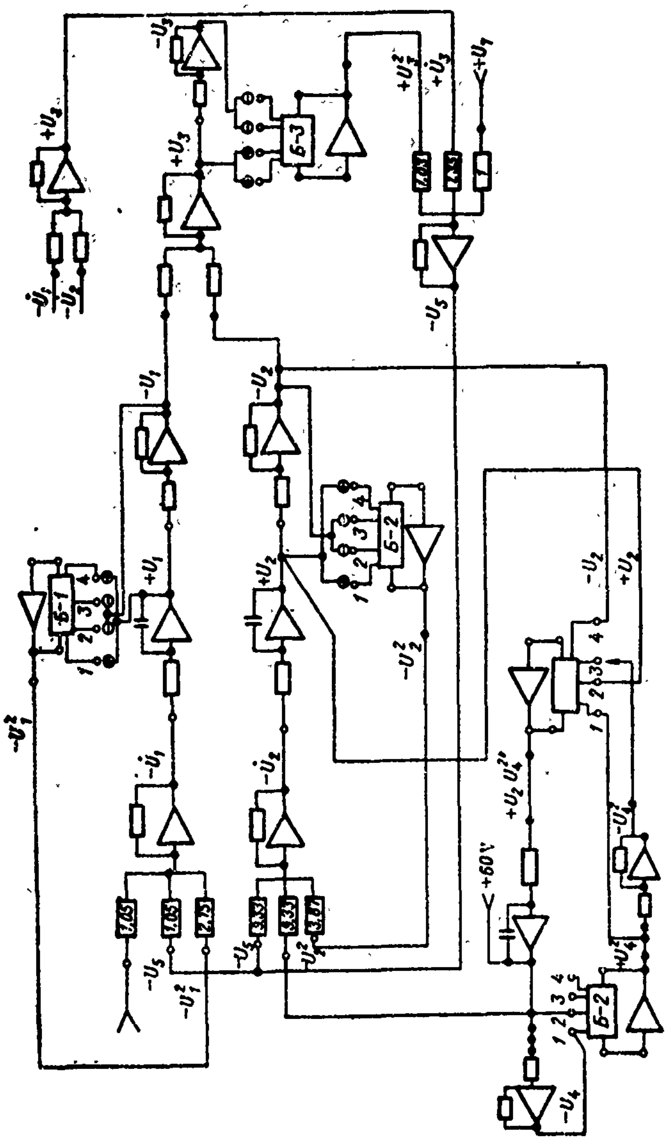

Fig. 11.15. The final form of the block diagram for solution of system of equations (11.39).
DESIGNATION: Б = B = unit.

arbitrary flow rates $G_б$ from the main tank and $G_р$ from the pressure chamber. For partial load operating conditions of the system (Fig. 11.16) the total flow rate of liquid into the tank, in which pressure $p_к$ prevails and which simulates the combustion chamber, is equal to $G_б + G_р$. The flow rate of liquid from the tank is smoothly increased with time and reaches the calculated value, equal to 180 kg/s. Before testing we establish pressure in the pressure chamber, equal to the pressure in the tank. In the process of testing the flow rate from the pressure chamber is at first increased, then drops to zero, which is explained by equalization of pressures in the pressure chamber and at its outlet. The total flow rate of liquid $G_б + G_р$ in the initial starting period is increased so that it exceeds the calculated value, then smoothly approaches nominal. Such a character of change of flow rate with time is convenient when boosting of starting conditions is necessary.

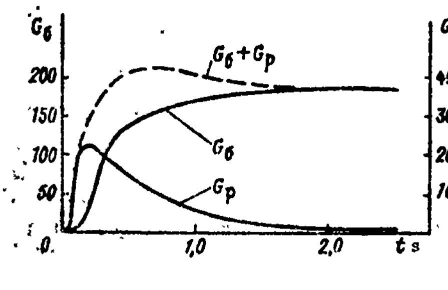

Fig. 11.16. Results of calculation of the starting operation of an engine with pressure chambers at $p_{к \cdot ном}$.

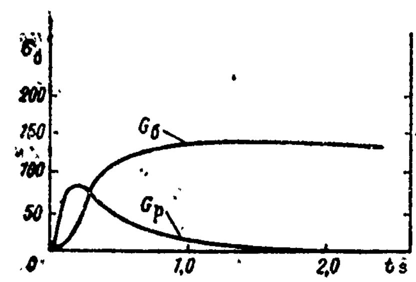

Fig. 11.17. The result of calculation with increased pressure in the combustion chamber $p_к < p_{к \cdot ном}$.

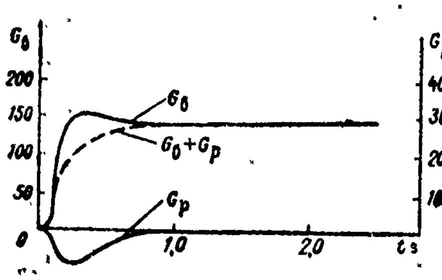

Fig. 11.18. The result of calculation with reduced pressure in the pressure chamber $p_р < p_б$.

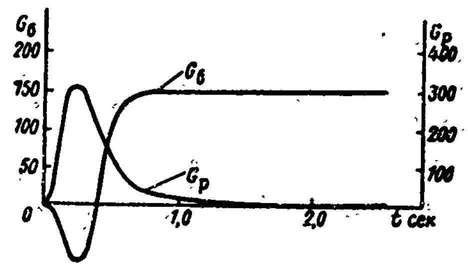

Fig. 11.19. The result of calculation with increased pressure in the pressure chamber $p_p > p_б$.

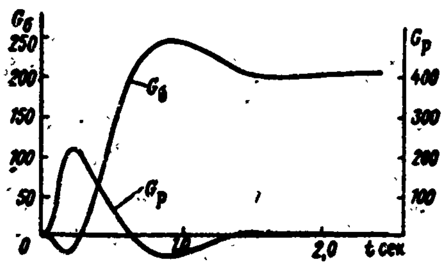

Fig. 11.20. The result of calculation with reduced hydraulic resistance.

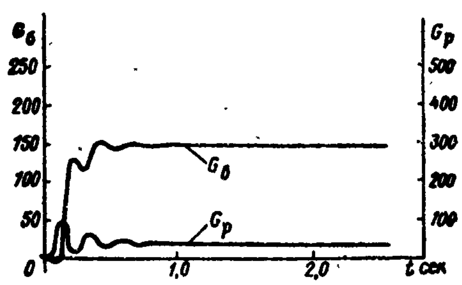

Fig. 11.21. The result of calculation with understated volume of pressure chamber.

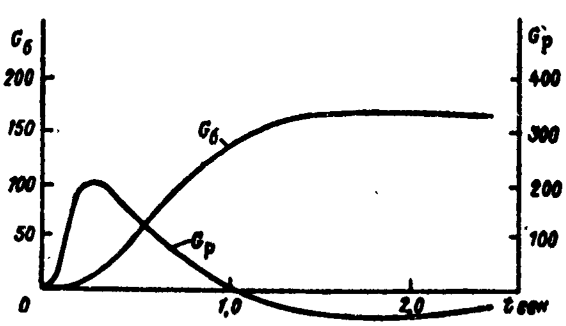

Fig. 11.22. Result of calculation with overstated value of mass coefficient.

Let us assume that pressure in the combustion chamber became higher than nominal, i.e., $p_к > p_{к \cdot ном}$. The result of calculation is shown in Fig. 11.17, which illustrates the decrease of flow rate $G_б$ and G_p with time.

If preliminary pressurization in the pressure chamber is decreased so that $p_{\text{p}} < p_{\text{б}}$, then at operating conditions of the system the liquid from the main line for a certain period of time will make up the pressure chamber (Fig. 11.18). In this case $G_{\text{б}} > 0$, and $G_{\text{p}} < 0$. The above facilitates providing smoother approach of the entire system $G_{\text{б}} + G_{\text{p}}$ to calculated conditions.

If preliminary pressurization in the pressure chamber is increased so that $p_{\text{p}} > p_{\text{б}}$ (Fig. 11.19), then at the initial starting period the feed of the chamber will be carried out only due to the pressure chamber, since $G_{\text{б}} < 0$, moreover a certain time, during which $G_{\text{б}} < 0$, part of the liquid will overflow from the pressure chamber into a tank.

The change of hydraulic resistances in separate components of the system differently influences the flow rates of liquid. With decrease of hydraulic resistances the flow rates grow, and the stability of the system drops. Figure 11.20 shows the result of calcuation of the system for a case when the hydraulic resistance of the line, connecting the tank to the pressure chamber, is decreased 5 times. The considered case is characterized by the beginning of onset of fluctuations of the liquid flow rate.

With decrease of the volume of pressure chamber (Fig. 11.21) the fluctuations of flow rates of liquid are increased.

With increase of inertness of the hydraulic system, achieved due to lengthening of lines or due to decrease of their cross-sectional areas, the time of approach of the system to operating conditions increases (Fig. 11.22), and sometimes fluctuations of liquid flow rate, proceeding from the pressure chamber, are observed.

11.3. Power Plant with Afterburning

Let us examine an engine, in which the gas, consumed in the turbine, is used in the main combustion chamber (Fig. 11.23), and

let us calculate its operation under initial conditions: components filled all the lines, the velocity of their motion is equal to zero, but [TNA] (THA) the turbopump unit shaft still has not started to rotate. Such conditions are far from actual, but they allow writing and investigating the basic equations of a power plant.

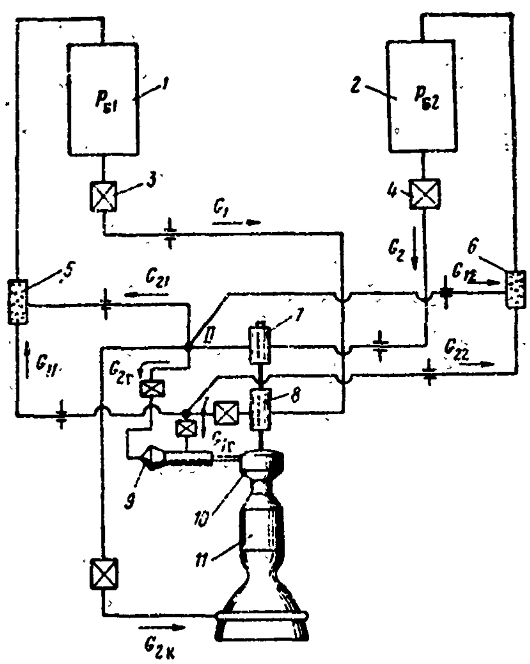

Fig. 11.23. Calculated diagram of liquid-propellant rocket engine with afterburning: *1* – oxidizer tank; *2* – fuel tank; *3, 4* – starting valves; *5* – boost generator (mixer) of oxidizer tank; *6* – boost generator (mixer) of fuel tank; *7* – fuel pump; *8* – oxidizer pump; *9* – gas generator of turbopump unit turbine; *10* – turbine of turbopump unit; *11* – combustion chamber.

The same power plant can be described by different systems of equations, however, in the final analysis the differential equation of high order will always be the same. Therefore, one should select a system of equations, so that it would be convenient for the construction of a block diagram and for solution on an analog device.

Under conditions of the stated problem let us instantly open starting valves 3 and 4, considering the remaining valves already

open. Let us distinguish on the diagram two modal points I and II.

I. The equation of material balance of the oxidizer line:

$$x_1 - x_{1\text{г}} - x_{1\,1} - x_{1\,2} = 0, \tag{11.40}$$

where $x_1 = G_1$ – the total flow rate of oxidizer from tank 1; $x_{1\text{г}} = G_{1\text{г}}$ – the flow rate of oxidizer into generator 9, which feeds turbine 10; $x_{1\,1} = G_{1\,1}$ – the flow rate of oxidizer into pressurization mixer 4 of oxidizer tank; $x_{1\,2} = G_{1\,2}$ – the flow rate of oxidizer into pressurization mixer 5 of fuel tank.

II. The equation of material balance of fuel line:

$$x_2 - x_{2\text{к}} - x_{2\text{г}} - x_{2\,1} - x_{2\,2} = 0; \tag{11.41}$$

where $x_2 = G_2$ – the total flow rate of fuel from tank 2; $x_{2\text{к}} = G_{2\text{к}}$ – the flow rate of fuel into combustion chamber 11 through the cooling passage; $x_{2\text{г}} = G_{2\text{г}}$ – the flow rate of fuel into generator 9, which feeds turbine 10; $x_{2\,1} = G_{2\,1}$ – the flow rate of fuel into pressurization mixer 5 of the oxidizer tank; $x_{2\,2} = G_{2\,2}$ – the flow rate of fuel into pressurization mixer 6 of the fuel tank.

III. The equation of pressure balance from oxidizer tank 1 to point I:

$$p_{\text{б}1} - a_1 x_1^2 + D_1 x_0^2 - D_1' x_0 x_1 - x_{\mathrm{I}} = b_1 \dot{x}_1, \tag{11.42}$$

where $p_{\text{б}1}$ – pressure in tank 1, taken equal to 0.5; a_1 – coefficient of hydraulic losses, taken equal to $4.9 \cdot 10^{-4}$; D_1 – coefficient of pump characteristic 8, taken equal to $8.075 \cdot 10^{-7}$; $x_0 = n$ – number of revolutions of turbopump unit shaft; D_1' – coefficient of pump characteristic 8, taken equal to $4.7\ 10^{-6}$; $x_{\mathrm{I}} = p_{\mathrm{I}}$ – pressure at point I; b_1 – mass coefficient for the considered line.

IV. The equation of pressure balance from point I to turbogas generator 9:

$$x_1 - a_{1\text{г}}x_{1\text{г}}^2 - x_{\text{г}} = b_{1\text{г}}\dot{x}_{1\text{г}}, \tag{11.43}$$

where $a_{1\text{г}}$ – coefficient of hydarulic losses, taken equal to $3.248 \cdot 10^{-3}$; $x_{\text{г}} = p_{\text{г}}$ – pressure in turbogas generator.

V. The equation of pressure balance from generator 9 through turbine 10 to combustion chamber 11:

$$x_{\text{г}} - a_{\text{г.т}}(x_{1\text{г}} + x_{2\text{г}})^2 - x_{\text{т}} - x_{\text{к}} = b_{\text{гг}}(\dot{x}_{1\text{г}} + \dot{x}_{2\text{г}}), \tag{11.44}$$

where $a_{\text{г.т}}$ – coefficient of hydraulic losses form generator 9 to turbine 10, taken equal to $7.11 \cdot 10^{-5}$; $x_{\text{т}} = \Delta p_{\text{т}}$ – pressure drop on turbine 10; $x_{\text{к}}' = p_{\text{к}}$ – pressure in combustion chamber 11.

VI. The equation of pressure balance from point I to pressurization mixer 5 of oxidizer tank:

$$x_1 - a_{11}x_{11}^2 - x_{\text{см}1} = b_{11}\dot{x}_{11}, \tag{11.45}$$

where $x_{\text{см}1} = p_{\text{см}1}$ – pressure in mixer 5; $a_{1\,1} = 156$.

VII. The equation of pressure balance from point I to pressurization mixer 6 of fuel tank:

$$x_1 - a_{12}x_{12}^2 - x_{\text{см}} = b_{12}\dot{x}_{12}, \tag{11.46}$$

In the calculation there is taken $a_{1\,2} = 2500$.

VIII. The equation of pressure balance from mixer 5 to the oxidizer tank:

$$x_{\text{см}1} - a_{\text{см}\,16}(x_{11} + x_{21})^2 - p_{\text{б}1} = b_{\text{см}\,16}(\dot{x}_{11} + \dot{x}_{21}). \tag{11.47}$$

In the calculation there is taken $a_{\text{см }16} = 22.57$.

IX. The equation of pressure balance from mixer 6 to the fuel tank:

$$x_{\text{см}2} - a_{\text{см }26}(x_{12} + x_{22})^2 - p_{62} = b_{\text{см }26}(\dot{x}_{12} + \dot{x}_{22}). \tag{11.48}$$

In the calculation there is taken $a_{\text{см }26} = 51$.

X. The equation of pressure balance from the fuel tank to point II:

$$p_{62} - a_2 x_2^2 + D_2 x_0^2 - D_2' x_0 x_2 - x_{11} = b_2' \dot{x}_2. \tag{11.49}$$

In the calculation there is taken

$$a_2 = 4{,}45 \cdot 10^{-4};\ D_2 = 5{,}639 \cdot 10^{-7};\ D_2' = 6{,}85 \cdot 10^{-6}.$$

XI. The equation of pressure balance from point II to turbogas generator 9:

$$x_{11} - a_{2\text{г}} x_{2\text{г}}^2 - x_{\text{г}} = b_{2\text{г}} \dot{x}_{2\text{г}}. \tag{11.50}$$

In the calculation there is taken $a_{2\text{г}} = 8 \cdot 10^{-2}$.

XII. The equation of pressure balance from point II to mixer 5:

$$x_{11} - a_{21} x_{21}^2 - x_{\text{см}1} = b_{2\text{см}1} \dot{x}_{21}. \tag{11.51}$$

In the calculation there is taken $a_{2\,1} = 25.39$.

XIII. The equation of pressure balance from point II to mixer 6:

$$x_{11} - a_{22} x_{22}^2 - x_{\text{см}2} + b_{2\text{см}2} \dot{x}_{22}. \tag{11.52}$$

In the calculation there is taken $a_{2\,2} = 33.16$.

XIV. The equation of pressure balance from point II to combustion chamber 11:

$$x_{11} - a_{2\text{к}} x_{2\text{к}}^2 - x_\text{к} = b_{2\text{к}} \dot{x}_{2\text{к}}. \tag{11.53}$$

In the calculation for the entire considered line there is taken $a_{2\text{к}} = 2.75 \cdot 10^{-3}$.

XV. The equation of turbopump unit:

$$(A - Bx_0)(x_{1\text{г}} + x_{2\text{г}}) = \frac{30}{\pi} \frac{D_1 x_0^2 - D_1' x_0 x_1}{\varrho_\text{ж} \eta_{\text{н}1} x_0} x_1 + \frac{30}{\pi} \frac{D_2 x_0^2 - D_2' x_0 x_2}{\varrho_\text{ж} \eta_{\text{н}2} x_0} x_2 + \frac{\pi}{30} I \dot{x}_0, \tag{11.54}$$

where A and B are determined from turbine parameters.

The equations of mass for gas capacities without taking into account the delay period have the form

XVI.

$$\frac{\beta_\text{к}}{F_{\text{кр.к}}} (x_{1\text{г}} + x_{2\text{г}} + x_{2\text{к}}) - x_\text{к} = \varepsilon_\text{к} \dot{x}_\text{к}; \tag{11.55}$$

XVII.

$$\mu_\text{г} (x_{1\text{г}} + x_{2\text{г}}) - x_\text{г} = \varepsilon_\text{г} \dot{x}_\text{г}; \tag{11.56}$$

XVIII.

$$\mu_{\text{см}1} (x_{11} + x_{21}) - x_{\text{см}1} = \varepsilon_{\text{см}1} \dot{x}_{\text{см}1}; \tag{11.57}$$

XIX

$$\mu_{\text{см}2} (x_{12} + x_{22}) - x_{\text{см}2} = \varepsilon_{\text{см}2} \dot{x}_{\text{см}2}, \tag{11.58}$$

where μ_i – parameters of generators; ε_i – the time the gases stay in the considered capacity.

The system of equations contains 17 unknowns, namely: flow rates x_1, x_2, $x_{1\text{г}}$, $x_{2\text{г}}$, $x_{2\text{к}}$, $x_{1\,1}$, $x_{1\,2}$, $x_{2\,1}$, $x_{2\,2}$; pressures $x_{\text{к}}$, $x_{\text{г}}$, $x_{\text{см}1}$, $x_{\text{см}2}$, x_{I}, x_{II}, $x_{\text{т}}$ and revolutions x_0.

The first two equations are checked and are not used with computer calculation. Consequently, 17 differential equations remain. The obtained closed system of equations is converted similarly to the above discussed, whereupon the block diagram is constructed.

The examined system of equations describes the process of the engine starting operation only in the period of pressure increase in the combustion chamber. To investigate the entire starting program of the engine it is necessary in stages to introduce into calculation and exclude from calculation the appropriate equations or groups of equations. In this case one should be guided by a starting cyclogram, beginning the calculation with first command, sent to the engine, and finishing it after the starting operation of the engine.

The cyclogram provides for execution of the entire time sequence of transmission of commands in the starting period. Frequently for clarity we construct a network chart, illustrating the formation of parallel following groups of operations and the sequence of their execution inside each group. The description of the method of development of optimum starting cyclogram and the examination of possible groups of equations during calculation of starting exceeds the limits of this monograph. However, the equations and described methods of their solution given in the book are successfully applied during research of the many sides of the problem touched upon.

11.4. Calculation of an Engine on Digital Computers

During the study of modern engines in most cases high accuracy of calculation results is required. Many equations, which describe

the dynamic processes of liquid-propellant rocket engines, for their solution require memory units. Therefore, especially recently, during the study of engine, digital computers, which pertain, as already mentioned, to the mathematical computers of the third group began to be widely applied. They are also convenient in the respect that while rather high-speed, they allow solving with small size of the system, containing tens and hundreds of equations, moreover ordinary differential equations can be nonlinear.[1] In principle on digital computers it is possible to solve equations in partial derivatives, however, it is possible to actually obtain such a solution only when the order of equation is small.

During research of engines on [MDS] (МДС) digital computers it is possible to solve a large group of various questions, as, for example, the following.

Starting operation of the engine – clarification of the character of change of parameters with time, primarily pressure in the chamber and the component ratio, and also derivatives $\dot{G}_i$ and $\dot{k}_i$; determination of the best operating conditions of valves and the selection of their operating sequence; determination of the effect of hydraulic losses on pressure fluctuation in the chamber in the initial period of engine operation; determination of dynamic loads, affecting the construction and research of the combustion process.

Engine operating conditions – research of the character of low-frequency and high-frequency oscillations and the effects of various influences on them; search for means of decrease of the amplitude of oscillations; determination of the effect of external factors and measures for weakening their effect; the establishment of character of change of parameters during the whole operating period of the engine; selection of the best combination of thermodynamic, hydraulic and structural parameters.

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[1]Calculation of an engine at starting operation with utilization of an electronic digital computer was performed by the author for the first time and was finished in 1957.

Transition from one condition to another — research of the change of amplitude and frequency of fluctuations of pressure and flow rates; search for means of weakening the amplitude of the indicated fluctuations with deep engine throttling; optimization of the change of parameters with switching of conditions.

Research of emergency situations, determination of the character of change of parameters of the power plant after the appearance of an emergency, seeking parameters, being changed the most rapidly with the appearance of an emergency, study of engine shutdown conditions after the appearance of emergency with respect to a command from rapidly changing "watch" parameters. As watch parameters in many instances we take time derivatives $\dot{p}$, $\dot{G}$ and $\dot{k}_1$.

Tuning the engine to a prescribed mode — construction of calculation nomograms; evaluation of the precision of tuning; selection of the best method and element of tuning; research on the effect of the quality of production on the precision of engine operation.

The development of the automatic control system — selection of the type of feedback; comparison of various schemes; selection of parameters of the feedback elements; providing the required character of change of engine thrust with time; elimination of pressure fluctuation in the chamber with the aid of a system with feedback.

The shutdown mode — research on the effect of afterburning of propellant in the chamber; selection of the sequence of actuation of elements of the automatic equipment during engine shutdown, etc.

The processing of experimental data — statistical and correlation processing of results of cold and hot bench tests, flying tests; comparison of results of bench tests, conducted at various enterprises

and at a different time; comparison of data of bench tests with results of engine operation in flight.

It is completely natural that besides the enumerated, the solution of also many other problems is possible. Without the wide utilization of analog and digital computers at present the qualitative execution of scientific research works, designing, development of the engine in the designer office and the debugging of engine production at plants are impossible.

Approximate methods of solution are selected depending on the type of differential equations, the features of boundary conditions and requirements with respect to precision and time, being spent on solution of the problem. The Runge-Kutta method received rather wide utilization during solution of ordinary differential equations and the method of finite differences during solution of equations in partial derivatives. The application of other approximate methods of solution is possible [28].

Runge-Kutta Method

The Runge-Kutta method allows calculating with high accuracy and despite the rather high labor input it finds wide utilization with the numerical solution of differential equations with the aid of electronic digital computers. One should note that one of the advantages of this method is also the possibility of performing calculation with variable integration step.

Let us assume there is given differential equation of first order

$$\dot{x}=f(x,t) \tag{11.59}$$

and initial condition

$$x_{(t=0)}=x_0. \tag{11.60}$$

There is selected step h. Let us introduce designations

$$t_i = t_0 + ih; \qquad (11.61)$$

$$x_i = x(t_i) \qquad (11.62)$$
$$(i = 0, 1, 2, \ldots).$$

Into calculation are introduced numbers:

$$k_1^i = hf(x_i, t_i); \qquad (11.63)$$

$$k_2^i = hf\left[\left(x_i + \frac{k_1^i}{2}\right), \quad \left(t_i + \frac{h}{2}\right)\right]; \qquad (11.64)$$

$$k_3^i = hf\left[\left(x_i + \frac{k_2^i}{2}\right), \quad \left(t_i + \frac{h}{2}\right)\right], \qquad (11.65)$$

$$k_4^i = hf\left[(x_i + k_3^i), (t_i + h)\right]. \qquad (11.66)$$

The successive values x_i of sought function x are determined from formula

$$x_{i+1} = x_i + \Delta x_i, \qquad (11.67)$$

moreover

$$\Delta x_i = \frac{1}{6}\left(k_1^i + 2k_2^i + 2k_3^i + k_4^i\right) \qquad (11.68)$$
$$(i = 0, 1, 2, \ldots).$$

It is proved that the error of the method on every step is proportional to h^5. Therefore, with decrease of the step the error decreases very rapidly. For example, during the solution of equations, containing the derivative from the delay period of combustion τ_s, the successful result of computation can be obtained only with a very small step. Effective evaluation of error of the method is difficult,

therefore for checking the correctness of selection of the step from current probable value $x(t_i)$, which should be known, we subtract $x(t_i + 2h)$, having accepted in the beginning step h, and then step $H = 2h$. The divergence of obtained values should not exceed the permissible error.

Let us examine one hydraulic line of an engine with pressure chambers (see Fig. 11.7).

For utilization of the Runge-Kutta method the equations should be solved relative to derivatives. After conversions of equations (11.33), where for convenience of writing there is designated $G_б = G_1$ and $G_р = G_2$, we arrive at the system of equations in the form

$$\dot{G}_1 = \frac{1}{b_1 b_2 + b_2 b_3 + b_3 b_1} \left[(b_2 + b_3) p_б - b_2 p_к - b_3 p_р - \right.$$
$$\left. -(a_1 b_2 + a_3 b_2 + a_1 b_3) G_1^2 - (a_3 b_2 - a_2 b_3) G_2^2 - 2 a_3 b_2 G_1 G_2 \right]; \qquad (11.69)$$

$$\dot{G}_2 = \frac{1}{b_2 + b_3} \left[p_р - (a_2 + a_3) G_2^2 - p_к - a_2 G_1^2 - 2 a_3 G_1 G_2 - b_3 \dot{G}_1 \right]; \qquad (11.70)$$

$$\dot{p}_р = -\frac{1}{\varrho_ж (YRT)_0} G_2 p_р^2 \qquad (11.71)$$

Initial conditions:

$$G_1(0) = 0;\ G_2(0) = 0;\ p_б(0) = 6;\ p_р(0) = 6.$$

By substituting numerical values of coefficients in equations (11.69), (11.70) and (11.71), we obtain

$$\dot{G}_1 = 719 p_б - 27{,}4 p_к - 692 p_р - 0{,}0509 G_1^2 + 0{,}00172 G_2^2 - 0{,}00226 G_1 G_2; \qquad (11.72)$$

$$\dot{G}_2 = 534 p_р - 0{,}0243 G_2^2 - 534 p_к - 0{,}0222 G_1^2 - 0{,}044 G_1 G_2 - 0{,}963 \dot{G}_1; \qquad (11.73)$$

$$\dot{p}_р = 94 \cdot 10^{-5} G_2 p_р^2. \qquad (11.74)$$

Let us select step $h = 0.1$. For each of the equations let us construct a calculation table. First the first column is filled up (see Table 11.1) for $t = 0$. For filling the second columns there are used equations

$$\left.\begin{aligned} G_{11} &= G_{10} + \frac{k_{11}^0}{2}; \\ G_{21} &= G_{20} + \frac{k_{21}^0}{2}; \\ p_{p1} &= p_{p0} + \frac{k_{31}^0}{2}. \end{aligned}\right\} \quad (11.75)$$

For filling the third columns we have

$$\left.\begin{aligned} G_{12} &= G_{10} + \frac{k_{12}^0}{2}; \\ G_{22} &= G_{20} + \frac{k_{22}^0}{2}; \\ p_{p2} &= p_{p0} + \frac{k_{32}^0}{2}. \end{aligned}\right\} \quad (11.76)$$

The quantities of the fourth column are similarly found. After filling Table 11.1 the value of ΔG_1 is calculated.

The values of $\dot{G}_1$, obtained in Table 11.1, are used when filling Table 11.2, which is finished by determination of increment ΔG_2. In Table 11.3 by calculation Δp_p is determined.

With the obtained values in the same sequence we proceed to calcuation of the next step interval to the finite time interval of interest to us. Calculation should be finished when the system approached conditions of the same type, characterized by G_i = const or by oscillations steady in amplitude. With unusual cases the calculation is completed by the obtaining of a characteristic change of parameters, for example, by sharp increase or decrease of pressure in the combustion chamber.

Table 11.1.

No in order	Parameters	Time t $t_0=0$	$t_0+\frac{h}{2}=0{,}05$	$t+\frac{h}{2}=0{,}05$	$t_0+h=0{,}1$
1	p_6	6	6	6	6
2	$719\ p_6$	4310	4310	4310	4310
3	$p_к$	1	1	1	1
4	$27{,}4\ p_к$	27,4	27,4	27,4	27,4
5	p_p	6	6	5,789	5,672
6	$692\ p_p$	4150	4150	4010	3930
7	(2)+(4)+(6)	132,6	132,6	272,6	352,6
8	G_1	0	6,63	7,785	28,58
9	G_1^2	0	44	60,5	820
10	$0{,}0609\ G_1^2$	0	2,68	3,68	49,8
11	G_2	0	126,9	104,3	197,886
12	G_2^2	0	$0{,}161\cdot10^5$	$0{,}109\cdot10^5$	$0{,}392\cdot10^5$
13	$0{,}00172 G_2^2$	0	27,7	18,7	67,5
14	$G_1\cdot G_2$	0	840	814	5660
15	$0{,}00226\ G_1\cdot G_2$	0	1,9	1,84	12,8
16	—(10)+(13)	0	25,02	15,02	17,7
17	(15)—(16)	0	—23,12	—13,18	—4,9
18	$\dot G_1$	132,6	155,72	285,78	357,5
19	$k^0_{11}=h\,\dot G_{1(0)}$	13,26	—	—	—
20	$k^0_{12}=h\,\dot G_{1(0,05)}$	—	15,57	—	—
21	$k^0_{13}=h\,\dot G_{1(0,05)}$	—	—	28,58	—
22	$k^0_{14}=h\,\dot G_{1(0,1)}$	—	—	—	35,75

$$\Delta G_1=\frac{1}{6}(13{,}26+2\cdot15{,}57+2\cdot28{,}58+35{,}75)=22{,}9$$

The described sequence comprises a program for a digital computer. The type of machine should be selected with consideration of the complexity of the problem so that the operation of digital computer in the course of solution would be profitable.

For checking the computer calculation it is expedient to calculate for the first step interval manually.

One of the complex processes, investigated with the aid of an [EVM] (ЭВМ) electronic computer, is, as already mentioned, the starting process. For determination of the character of change of parameters, selection of starting cyclograms, research of the effect of various factors on the engine characteristics there is constructed a mathematical model of the engine starting.

Table 11.2.

No in order	Parameters	Time $t_0=0$	$t_0+\frac{h}{2}=0,05$	$t_0+\frac{h}{2}=0,05$	$t_0+h=0,1$
1	p_κ	1	1	1	1
2	$534\, p_\kappa$	534	534	534	534
3	p_p	6	6	5,789	5,672
4	$534\, p_p$	3200	3200	3090	3030
5	$-(2)+(4)$.	2666	2666	2556	2496
6	G_2	0	126,9	104,3	197,885
7	G_2^2	0	$0,161\cdot10^5$	$0,109\cdot10^5$	$0,392\cdot10^6$
8	0,0243 [illegible]	0	392	265	954
9	G_1	0	6,63	7,785	28,58
10	G_1^2	0	44	60,5	820
11	$0,0222\cdot G_1^2$	0	0,976	1,34	18,2
12	$G_1\cdot G_2$	0	840	814	5660
13	$0,044\ G_1\cdot G_2$	0	37	35,8	219
14	$-(8)-(11)$	0	$-392,976$	$-266,34$	$-972,2$
15	$(14)-(13)$	0	-430	$-302,14$	$-1221,2$
16	$\dot G_1$	132,6	155,72	285,78	357,5
17	$0,963\ \dot G_1$	128	150	275	344
18	$\dot G_2$	25,38	2086	1978,86	930,8
19	$k_{2\,1}^0=h\,\dot G_{2(0)}$	253,8	—	—	—
20	$k_{2\,2}^0=h\,\dot G_{2(0,05)}$	—	208,6	—	—
21	$h_{2\,3}^0=h\,\dot G_{2(0,05)}$	—	—	197,886	—
22	$k_{2\,4}^0=h\,\dot G_{2(0,1)}$	—	—	—	93,08

$$\Delta G_2=\frac{1}{6}(253,8+2\cdot208,6+2\cdot197,886+93,08)=193,5$$

Table 11.3.

No in order	Parameters	Time $t_0=0$	$t_0+\frac{h}{2}=0,05$	$t_0+\frac{h}{2}=0,05$	$t_0+h=0,1$
1	p_p	6	6	5,789	5,672
2	p_p^2	36	36	33,5	32,1
3	G_2	0	126,9	104,3	197,886
4	$G_2 p_p^2$	0	4560	3490	6380
5	$\dot p_p$	0	$-4,23$	$-3,28$	$-5,97$
6	$k_{3\,1}^0=h\dot p_{p(0)}$	0	—	—	—
7	$k_{3\,2}^0=h\dot p_{p(0,05)}$	—	$-0,423$	—	—
8	$k_{3\,3}^0=h\dot p_{p(0,05)}$	—	—	$-0,328$	—
9	$k_{3\,4}^0=h\dot p_{p(0,1)}$	—	—	—	$-0,597$

$$\Delta p_p=\frac{1}{6}[0+2(-0,423)+2(-0,328)+(-0,597)]=-0,349.$$

The system of starting equations besides differential equations can also include algebraic and transcendental equations. Known approximate methods (Runge-Kutta, Adams and others) have been developed for the solution of systems of only differential equations, however it proves to be possible to use this mathematical instrument for the solution of the system of starting equations.

Let us examine the utilization of Runge-Kutta method during calculation of starting.[1] In the process of solution the system is expanded into a series of subsystems, which are solved with the aid of matrix calculation. If the quantity of equations in the subsystem is not more than three, it is expedient in this instance to use the method of substitutions. The quantity of equations in the system being solved is changed, which leads to a change of matrices. In this case it is necessary to see the the components of the right side of the system of equations

$$\dot{\bar{Y}}=f(t,Y), \tag{11.77}$$

where

$$\bar{Y}=\begin{vmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_k \end{vmatrix}$$ is the system of functions f_i,

were arranged in such a way that the rank of the matrix of each subsystem would equal the rank of "expanded" matrix of this subsystem, moreover the rank of the matrix should not be equal to zero.

The system of equations of engine starting can include equations with delayed argument τ (for example, equation of combustion chamber). In this instance it is expedient to construct two parallel programs of integration (one for integration at moment of time t, the other — at moment of time $t - \tau$) or temporarily introduce the appropriate

[1]Developed by V. V. Merslikin cand. tech. sciences.

values of the parameter being integrated into the memory unit of the computer.

Most parameters have initial values, equal to zero. In the process of solution this sometimes leads to division by zero and, consequently, to cut-off of electronic computer. Therefore, it is necessary to assign initial values of these parameters within not more than 1% of their nominal value. The obtained error remains within permissible limits.

The results of solution of the system of equations by the numerical method are noticeably affected by the amount of integration step h. Experience shows that the error of solution and the expenditures of computer time remain acceptable at $h = 10^{-2}$-10^{-3} s. For decrease of expenditures of computer time it is expedient to make the integration step variable, namely: $h = 10^{-2}$ s – with integration of equations of filling of hydraulic lines and $h = 10^{-3}$ s – with connection of equations to the subsystem being solved, which describe the operation of the combustion chamber, gas generator, turbopump unit and regulating devices.

During programming it is expedient to convert the system of equations and its subsystem in such a way that in the right sides of differential equations there would not be variables, being determined from algebraic or transcendental equations. If similar transformation is not possible, then it is expedient to proceed in the following manner.

Let us assume the system of functions $\overline{Y}$ of equations (11.77) contains a system of k algebraic and transcendental equations, where k – whole numbers, moreover

$$1 \leqslant k \leqslant n.$$

If with the combined solution of the system of differential, algegraic and transcendental equations first we solve the differential equations

and after each i-th integration step we determine the values of $\overline{Y}_k^i$, then as a result of delay in the computation of values of $\overline{Y}_k^i$ the error in the process of solution will always be accumulated, which can lead to inadmissible errors even with small integration step or to divergent solutions. For decrease of this error it is necessary to construct the program of solution in such a way that values of $\overline{Y}_k^i$ would be found not after each step of integration of the system of differential equations, but in the process of determination of quantity $\overline{Y}_m^i$, where

$$\dot{\overline{Y}}_m = F(t, \overline{Y}_m) \tag{11.78}$$

is the system of differential equations. This means that quantity $\overline{Y}_k^i$ should be calculated after each value of $\overline{Y}_{m,\,j}^i$ $(j = 1, 2, 3, 4)$ on the i-th integration step:

$$\overline{Y}_{m,1}^i = \overline{Y}_m^i + \frac{1}{6} k_1^i; \tag{11.79}$$

$$\overline{Y}_{m,2}^i = \overline{Y}_{m,1}^i + \frac{1}{3} k_2^i; \tag{11.80}$$

$$\overline{Y}_{m,3}^i = \overline{Y}_{m,2}^i + \frac{1}{3} k_3^i; \tag{11.81}$$

$$\overline{Y}_{m,4}^i = \overline{Y}_{m,3}^i + \frac{1}{6} k_4^i, \tag{11.82}$$

where k_1^i, k_2^i, k_3^i, k_4^i – the system of numbers, determined by formulas (11.63)-(11.66).

The provided method of solution on an electronic computer of a system of differential, algebraic and transcendental equations makes it possible to obtain satisfactory results.

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